

Solutions for class #3 from Yosunism website

Yosunism website: <http://grephysics.yosunism.com>

Problem 9:

Electromagnetism \Rightarrow } Current Directions

The opposite currents cancel each other, and thus the induction (and field) outside is 0.

YOUR NOTES:

Problem 10:

Electromagnetism \Rightarrow } Image Charges

The conductor induces image charges $-q$ and $-2q$ since it is grounded at $x=0$. Since these are (mirror) image charges, each charge induced is the same distance from the conducting plane as its positive component.

The net force on q is just the *magnitude* sum of the positive charge $2q$ and the two induced charges,

$$\sum F = \frac{q^2}{4\pi\epsilon_0} \left\{ \frac{1}{a^2} + \frac{2}{(2a)^2} + \frac{2}{a^2} \right\} = \frac{7q^2}{4\pi\epsilon_0 a^2}$$

Why is it the magnitude sum? Well, recall that $\vec{F} = \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}^2} \hat{r}_{ij}$, where \hat{r}_{ij} is the vector pointing from the charge one is interested in to the field charge, i.e., the other charge. If one has two positive charges, then the force points along that unit vector. However, if one has a minus charge and a positive charge, the force points in the other direction. Thus, all the force quantities are additive, and one might as well just take the magnitude sum.)

YOUR NOTES:

Problem 11:

Electromagnetism \Rightarrow RC Circuit

The energy of a capacitor C with voltage V across it is given by $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$. ($Q = CV$ derives the other variations of the energy.)

From Ohm's Law, one arrives at the relation between charge and time,

$Q/C + \dot{Q}R = 0 \Rightarrow \frac{Q}{RC} = -\frac{dQ}{dt} \Rightarrow -\frac{dQ}{RC} = \frac{dQ}{dt}$. Integrating both sides, one finds that
 $Q(t) = Q_0 e^{-t/(RC)}$.

Plugging this into the energy equation above, one has $U \propto Q(t)^2 \propto e^{-2t/(RC)}$. Twice time required for the energy to dissipate by 2 is thus given by

$1/2 = e^{-t/(RC)} \Rightarrow t_{1/2} = RC \ln(2)$. Divide it by 2 to get choice (E).

YOUR NOTES:

Problem 12:

Electromagnetism \Rightarrow } Potential

A Potential V is related to the electric field E by $\vec{E} = -\nabla V$.

Since the problem supplies the approximation tool that the planes are quite large, one can assume the field is approximately constant. The remaining parameter that can't be thrown out by this approximation is the angle, and thus the only choice that yields $\frac{d}{d\phi} V = \text{constant}$ is choice (B).

YOUR NOTES:

Problem 13:

Electromagnetism \Rightarrow } Maxwell's Equations

Magnetic monopoles remain a likeable (even lovable) theoretical construct because of their ability to perfectly symmetrize Maxwell's equations. Since the curl term has an electric current, the other curl term should have a magnetic current.

($\nabla \cdot \mathbf{B} \neq 0$ is taken to be obvious in presence of magnetic charge.) The answer is thus (D), and the revised equations are,

YOUR NOTES:

Problem 18:

Lab Methods \Rightarrow } Coax Cable

Elimination time. The first-pass question to answer is *why is it important that a coax cable be terminated at an end:*

(A) Perhaps...

(B) Probably not. Terminating the cable at an end would not help heat dissipation and thus should not prevent overheating.

(C) Perhaps...

(D) Probably not, since termination should attenuate the signal rather than to prevent it.

(E) Probably not, since image currents should be canceled by the outer sheath.

Choices (A) and (C) remain. Now, use the second fact supplied by ETS. The cable

should be terminated by its characteristic impedance. Characteristic impedance has to do with resonance. Thus, it should prevent reflection of the signal.

YOUR NOTES:

Problem 32:

Electromagnetism \Rightarrow }Circuits

Power is related to current and resistance by $P = I^2 R$. The resistor that has the most current would be R_1 and R_{eq} (the equivalent resistance of all the resistors except for R_1), since all the other resistors share a current that is split from the main current running from the battery to R_1 . Since $R_{eq} < R_1$, the most power is thus dissipated through R_1 , as in choice (A).

YOUR NOTES:

Problem 33:

Electromagnetism \Rightarrow }Circuits

One can find the voltage across R_4 quite easily. The net resistance of all resistors except R_1 is $R_{eq} = ((1/R_3 + 1/R_4)^{-1} + R_5)^{-1} + 1/R_2 = 25 \Omega$. Kirchhoff's Loop Law then gives $V = I(R_1 + R_{eq}) \Rightarrow I = 3/75 A$.

Now that one knows the current, one trivially finds the voltage across R_2 to be $I R_2 = 1V$. $I'(R_{34} + R_5) = 1$, since the resistors are in parallel.

Since $R_{34} = 1/R_3 + 1/R_4 = 1/60 + 1/30 = 20 \Omega$, the current $I' = 1/(R_{34} + R_5) = 1/50$.

The voltage across either R_3 or R_4 is just $1 - I' R_5 = 1 - 30/50 = 0.4$, as in choice (A).

YOUR NOTES:

Problem 34:

Electromagnetism \Rightarrow } TEM Waves

The full formalism of a conducting cavity can be solved via TEM (transverse electromagnetic) wave guides. However, to solve this problem, one needs only the two boundary conditions from the reflection at a conducting surface, $\Delta E_{||} = 0$ and $\Delta B_{\perp} = 0$.

The electric field parallel to the cavity is the transverse field, and thus one has choice (D), exactly the conditions above.

YOUR NOTES:

Problem 36:

Electromagnetism \Rightarrow }Boundary Conditions

The conductor perfectly reflects the incoming wave, and none is transmitted. The electric field is thus reversed. However, since E and B are perpendicular (related to each other by the Poynting Vector where the direction of propagation is given by the direction of $\vec{E} \times \vec{B}$), the magnitude of B is increased by 2, but its direction stays the same.

Search on the GRE Physics Solutions homepage with keyword *conductors* for more on this.

YOUR NOTES:

Problem 54:

Electromagnetism \Rightarrow }Faraday Law

The induced current would act, according to Lenz Law, to oppose the change. In this case, since the field is decreasing (the wire is being pulled away from the field), the induced current would act to increase the field. On the side closest to the long wire, it would thus point in the same direction as the current from the long-wire. This eliminates all but choices (D) and (E).

Now, since the rectangular loop wire *cannot* induce a force on itself, the force is due to the field from the long wire. To the left of the loop, the long wire has a field pointing into the page, and thus the force there is left-wards. One can check again that choice (E) is right by right-hand-ruling the field on the right side of the loop. Since the field due to the long wire is again into the page, the force here is towards the right (since the current runs down the page on the right side of the loop).

YOUR NOTES:

