Quantum Mechanics – Problem Set # 2

TABLE OF INFORMATION

Rest mass of the electron	$m_e = 9.11 \times 10^{-31}$ kilogram $= 9.11 \times 10^{-28}$ gram
Magnitude of the electron charge	$e = 1.60 \times 10^{-19}$ coulomb $= 4.80 \times 10^{-10}$ statcoulomb (esu)
Avogadro's number	$N_0 = 6.02 \times 10^{23}$ per mole
Universal gas constant	$R = 8.31$ joules/(mole · K)
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ joule/K $= 1.38 \times 10^{-16}$ erg/K
Speed of light	$c = 3.00 \times 10^{8}$ m/s $= 3.00 \times 10^{10}$ cm/s
Planck's constant	$h = 6.63 \times 10^{-34}$ joule · second $= 4.14 \times 10^{-15}$ eV · second
	$\hbar = h/2\pi$
Vacuum permittivity	$\epsilon_0 = 8.85 \times 10^{-12}$ coulomb^2/(newton · meter^2)
Vacuum permeability	$\mu_0 = 4\pi \times 10^{-7}$ weber/(ampere · meter)
Universal gravitational constant	$G = 6.67 \times 10^{-11}$ meter^3/(kilogram · second^2)
Acceleration due to gravity	$g = 9.80$ m/s^2 $= 980$ cm/s^2
1 atmosphere pressure	1 atm $= 1.0 \times 10^{5}$ newton/meter^2 $= 1.0 \times 10^{5}$ pascals (Pa)
1 angstrom	1 Å $= 1 \times 10^{-10}$ meter
	1 weber/m^2 = 1 tesla = 10^4 gauss

Moments of inertia about center of mass

Rod	$\frac{1}{12}M\ell^2$
Disc	$\frac{1}{2}MR^2$
Sphere	$\frac{2}{5}MR^2$

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17. The wave function for a particle constrained to move in one dimension is shown in the graph above ($\Psi = 0$ for $x \leq 0$ and $x \geq 5$). What is the probability that the particle would be found between $x = 2$ and $x = 4$?

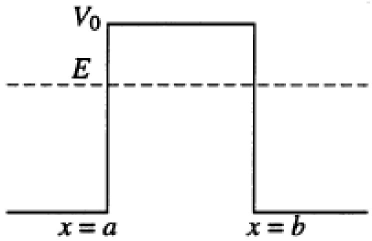
(A) 17/64

(B) 25/64

(C) 5/8

(D) $\sqrt{5/8}$

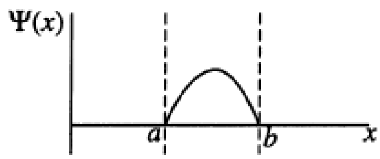
(E) 13/16



18. Consider a potential of the form

$$V(x) = 0,\ \ x \le a$$
$$V(x) = V_0,\ \ a < x < b$$
$$V(x) = 0,\ \ x \ge b$$

as shown in the figure above. Which of the following wave functions is possible for a particle incident from the left with energy $E < V_0$?

(A)


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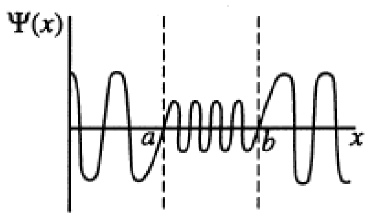

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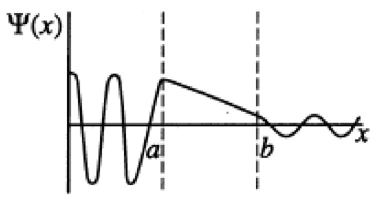
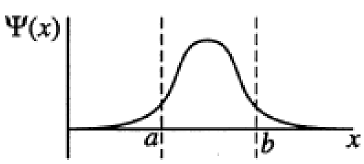
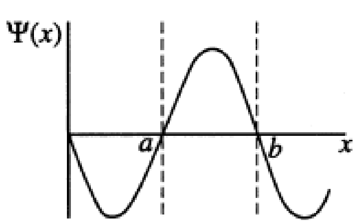


(D)


(E)


33. A diatomic molecule is initially in the state $\Psi(\Theta, \Phi) = (5Y_1^{\,1} + 3Y_5^{\,1} + 2Y_5^{\,-1})/(38)^{1/2}$, where $Y_\ell^{\,m}$ is a spherical harmonic. If measurements are made of the total angular momentum quantum number ℓ and of the azimuthal angular momentum quantum number m, what is the probability of obtaining the result $\ell = 5$?

(A) 36/1444
(B) 9/38
(C) 13/38
(D) $5/(38)^{1/2}$
(E) 34/38

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51. The solution to the Schrödinger equation for a particle bound in a one-dimensional, infinitely deep potential well, indexed by quantum number n, indicates that in the middle of the well the probability density vanishes for

(A) the ground state $(n = 1)$ only
(B) states of even n $(n = 2, 4, \ldots)$
(C) states of odd n $(n = 1, 3, \ldots)$
(D) all states $(n = 1, 2, 3 \ldots)$
(E) all states except the ground state

52. At a given instant of time, a rigid rotator is in the state $\psi(\theta, \phi) = \sqrt{3/4\pi}\ \sin\theta \sin\phi$, where θ is the polar angle relative to the z-axis and ϕ is the azimuthal angle. Measurement will find which of the following possible values of the z-component of the angular momentum, L_z ?

(A) 0
(B) $\hbar/2,\ -\hbar/2$
(C) $\hbar, -\hbar$
(D) $2\hbar,\ -2\hbar$
(E) $\hbar,\ 0,\ -\hbar$

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76. A Gaussian wave packet travels through free space. Which of the following statements about the wave packet are correct for all such wave packets?

I. The average momentum of the wave packet is zero.
II. The width of the wave packet increases with time, as $t \to \infty$.
III. The amplitude of the wave packet remains constant with time.
IV. The narrower the wave packet is in momentum space, the wider it is in coordinate space.

(A) I and III only
(B) II and IV only
(C) I, II, and IV only
(D) II, III, and IV only
(E) I, II, III, and IV

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77. Two ions, 1 and 2, at fixed separation, with spin angular momentum operators $\mathbf{S_1}$ and $\mathbf{S_2}$, have the interaction Hamiltonian $H = -J\,\mathbf{S_1} \cdot \mathbf{S_2}$, where $J > 0$. The values of $\mathbf{S_1}^2$ and $\mathbf{S_2}^2$ are fixed at $S_1(S_1+1)$ and $S_2(S_2+1)$, respectively. Which of the following is the energy of the ground state of the system?

(A) 0

(B) $-JS_1S_2$

(C) $-J[S_1(S_1+1) - S_2(S_2+1)]$

(D) $-(J/2)[(S_1+S_2)(S_1+S_2+1) - S_1(S_1+1) - S_2(S_2+1)]$

(E) $-\dfrac{J}{2}\left[\dfrac{S_1(S_1+1)+S_2(S_2+1)}{(S_1+S_2)(S_1+S_2+1)}\right]$

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97. A particle of mass m has the wave function $\psi(x,\ t) = e^{i\omega t}[\alpha \cos(kx) + \beta \sin(kx)]$, where α and β are complex constants and ω and k are real constants. The probability current density is equal to which of the following? (Note: α^* denotes the complex conjugate of α, and $|\alpha|^2 = \alpha^* \alpha$.)

(A) 0

(B) $\hbar k/m$

(C) $\frac{\hbar k}{2m}(|\alpha|^2 + |\beta|^2)$

(D) $\frac{\hbar k}{m}(|\alpha|^2 - |\beta|^2)$

(E) $\frac{\hbar k}{2mi}(\alpha^* \beta - \beta^* \alpha)$

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98. A particle of mass m is acted on by a harmonic force with potential energy function $V(x) = m\omega^2 x^2/2$ (a one-dimensional simple harmonic oscillator). If there is a wall at $x = 0$ so that $V = \infty$ for $x < 0$, then the energy levels are equal to

(A) $0,\ \hbar\omega,\ 2\hbar\omega,\ \ldots$

(B) $0,\ \frac{\hbar\omega}{2},\ \hbar\omega,\ \ldots$

(C) $\frac{\hbar\omega}{2},\ \frac{3\hbar\omega}{2},\ \frac{5\hbar\omega}{2},\ \ldots$

(D) $\frac{3\hbar\omega}{2},\ \frac{7\hbar\omega}{2},\ \frac{11\hbar\omega}{2},\ \ldots$

(E) $0,\ \frac{3\hbar\omega}{2},\ \frac{5\hbar\omega}{2},\ \ldots$

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99. The electronic energy levels of atoms of a certain gas are given by $E_n = E_1 n^2$, where $n = 1, 2, 3, \ldots$. Assume that transitions are allowed between all levels. If one wanted to construct a laser from this gas by pumping the $n = 1 \rightarrow n = 3$ transition, which energy level or levels would have to be metastable?

(A) $n = 1$ only

(B) $n = 2$ only

(C) $n = 1$ and $n = 3$ only

(D) $n = 1$, $n = 2$, and $n = 3$

(E) None

100. The operator $\hat{a} = \sqrt{\frac{m\omega_0}{2\hbar}}(\hat{x} + i\frac{\hat{p}}{m\omega_0})$, when operating on a harmonic energy eigenstate Ψ_n with energy E_n, produces another energy eigenstate whose energy is $E_n - \hbar\omega_0$. Which of the following is true?

I. $\hat{a}$ commutes with the Hamiltonian.
II. $\hat{a}$ is a Hermitian operator and therefore an observable.
III. The adjoint operator $\hat{a}^\dagger \neq \hat{a}$.

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only