Quantum Mechanics – Problem Set # 1

Rest mass of the electron	$m_e = 9.11 \times 10^{-31}$ kilogram $= 9.11 \times 10^{-28}$ gram
Magnitude of the electron charge	$e = 1.60 \times 10^{-19}$ coulomb $= 4.80 \times 10^{-10}$ statcoulomb (esu)
Avogadro's number	$N_0 = 6.02 \times 10^{23}$ per mole
Universal gas constant	$R = 8.32$ joules/(mole · K)
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ joule/K $= 1.38 \times 10^{-16}$ erg/K
Speed of light	$c = 3.00 \times 10^{8}$ m/s $= 3.00 \times 10^{10}$ cm/s
Planck's constant	$h = 6.63 \times 10^{-34}$ joule · second $= 4.14 \times 10^{-15}$ eV · second
	$\hbar = h/2\pi$
Vacuum permittivity	$\epsilon_0 = 8.85 \times 10^{-12}$ coulomb^2/(newton · meter^2)
Vacuum permeability	$\mu_0 = 4\pi \times 10^{-7}$ weber/(ampere · meter)
Universal gravitational constant	$G = 6.67 \times 10^{-11}$ meter^3/(kilogram · second^2)
Acceleration due to gravity	$g = 9.80$ m/s$^2 = 980$ cm/s^2
1 atmosphere pressure	1 atm $= 1.0 \times 10^{5}$ newton/meter$^2 = 1.0 \times 10^{5}$ pascals (Pa)
1 angstrom	1 Å $= 1 \times 10^{-10}$ meter
	1 weber/m^2 = 1 tesla = 10^4 gauss

1. The wave function of a particle is $e^{i(kx-\omega t)}$, where x is distance, t is time, and k and ω are positive real numbers. The x-component of the momentum of the particle is

(A) 0

(B) $\hbar\omega$

(C) $\hbar k$

(D) $\frac{\hbar\omega}{c}$

(E) $\frac{\hbar k}{\omega}$

27. If a freely moving electron is localized in space to within $\triangle x_0$ of x_0, its wave function can be described by a wave packet $\psi(x, t) = \int_{-\infty}^{\infty} e^{i(kx - \omega t)} f(k)\, dk$, where $f(k)$ is peaked around a central value k_0. Which of the following is most nearly the width of the peak in k?

(A) $\triangle k = \frac{1}{x_0}$

(B) $\triangle k = \frac{1}{\triangle x_0}$

(C) $\triangle k = \frac{\triangle x_0}{x_0^2}$

(D) $\triangle k = \left(\frac{\triangle x_0}{x_0}\right) k_0$

(E) $\triangle k = \sqrt{k_0^2 + \left(\frac{1}{x_0}\right)^2}$

28. A system is known to be in the normalized state described by the wave function

$$\psi(\theta, \varphi) = \frac{1}{\sqrt{30}}(5\, Y_4^3 + Y_6^3 - 2\, Y_6^0),$$

where the $Y_\ell^m(\theta, \varphi)$ are the spherical harmonics. The probability of finding the system in a state with azimuthal orbital quantum number $m = 3$ is

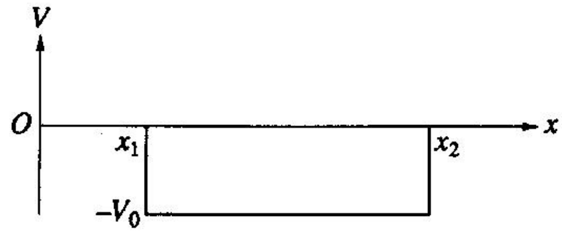
(A) 0

(B) $\frac{1}{15}$

(C) $\frac{1}{6}$

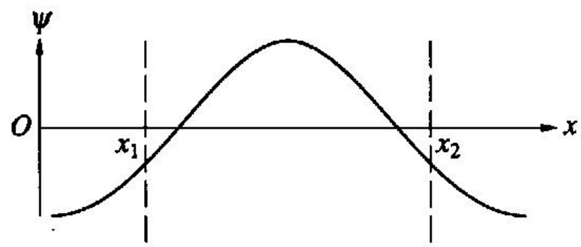
(D) $\frac{1}{3}$

(E) $\frac{13}{15}$



29. An attractive, one-dimensional square well has depth V_0 as shown above. Which of the following best shows a possible wave function for a bound state?

(A)



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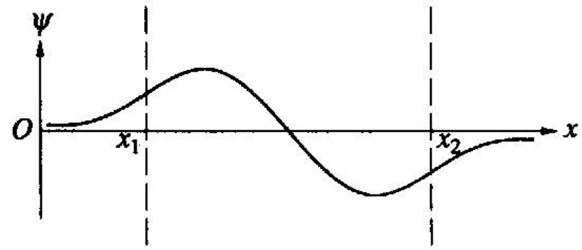

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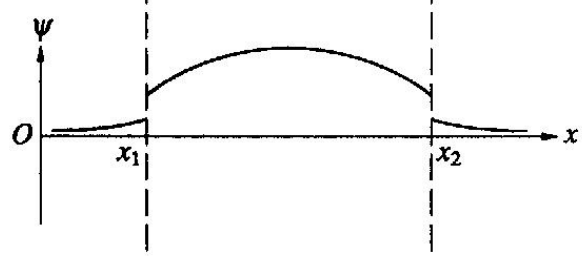

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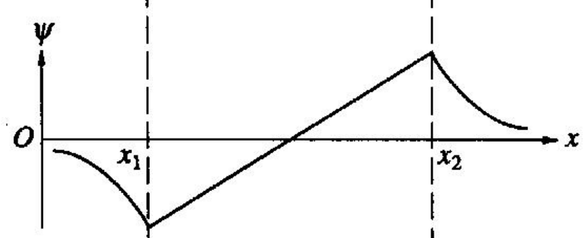
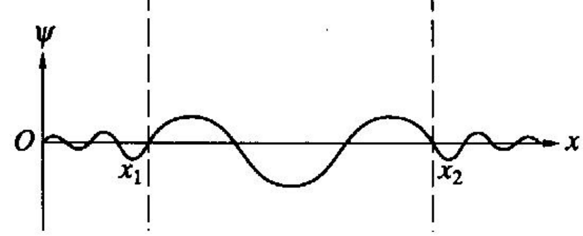


(E)



50. The state of a quantum mechanical system is described by a wave function ψ. Consider two physical observables that have discrete eigenvalues: observable A with eigenvalues $\{\alpha\}$, and observable B with eigenvalues $\{\beta\}$. Under what circumstances can all wave functions be expanded in a set of basis states, each of which is a simultaneous eigenfunction of both A and B?

(A) Only if the values $\{\alpha\}$ and $\{\beta\}$ are nondegenerate
(B) Only if A and B commute
(C) Only if A commutes with the Hamiltonian of the system
(D) Only if B commutes with the Hamiltonian of the system
(E) Under all circumstances

Questions 51-53

A particle of mass m is confined to an infinitely deep square-well potential:

$$V(x) = \infty, x \leq 0, x \geq a$$
$$V(x) = 0, 0 < x < a.$$

The normalized eigenfunctions, labeled by the quantum number n, are $\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$.

51. For any state n, the expectation value of the momentum of the particle is

(A) 0

(B) $\frac{\hbar n\pi}{a}$

(C) $\frac{2\hbar n\pi}{a}$

(D) $\frac{\hbar n\pi}{a}(\cos n\pi - 1)$

(E) $\frac{-i\hbar n\pi}{a}(\cos n\pi - 1)$

52. The eigenfunctions satisfy the condition $\int_0^a \psi_n{}^*(x)\, \psi_\ell(x)\, dx = \delta_{n\ell}$, $\delta_{n\ell} = 1$ if $n = \ell$, otherwise $\delta_{n\ell} = 0$. This is a statement that the eigenfunctions are

(A) solutions to the Schrödinger equation
(B) orthonormal
(C) bounded
(D) linearly dependent
(E) symmetric

53. A measurement of energy E will always satisfy which of the following relationships?

(A) $E \leq \frac{\pi^2\hbar^2}{8ma^2}$

(B) $E \geq \frac{\pi^2\hbar^2}{2ma^2}$

(C) $E = \frac{\pi^2\hbar^2}{8ma^2}$

(D) $E = \frac{n^2\pi^2\hbar^2}{8ma^2}$

(E) $E = \frac{\pi^2\hbar^2}{2ma^2}$

56. If ν is frequency and h is Planck's constant, the ground state energy of a one-dimensional quantum mechanical harmonic oscillator is

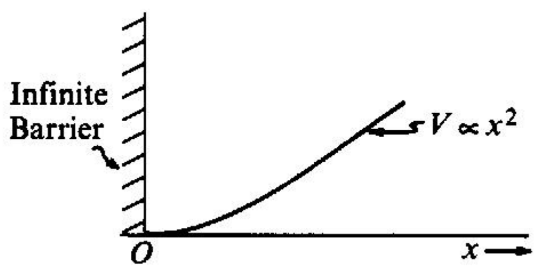
(A) 0

(B) $\frac{1}{3} h\nu$

(C) $\frac{1}{2} h\nu$

(D) $h\nu$

(E) $\frac{3}{2} h\nu$



89. The energy levels for the one-dimensional harmonic oscillator are $h\nu\left(n + \frac{1}{2}\right)$, $n = 0, 1, 2 \ldots$. How will the energy levels for the potential shown in the graph above differ from those for the harmonic oscillator?

(A) The term $\frac{1}{2}$ will be changed to $\frac{3}{2}$.

(B) The energy of each level will be doubled.

(C) The energy of each level will be halved.

(D) Only those for even values of n will be present.

(E) Only those for odd values of n will be present.

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98. The matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

has three eigenvalues λ_i defined by $Av_i = \lambda_i v_i$. Which of the following statements is NOT true?

(A) $\lambda_1 + \lambda_2 + \lambda_3 = 0$

(B) λ_1, λ_2, and λ_3 are all real numbers.

(C) $\lambda_2\lambda_3 = +1$ for some pair of roots.

(D) $\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = 0$

(E) $\lambda_i^3 = +1, i = 1, 2, 3$

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99. In perturbation theory, what is the first order correction to the energy of a hydrogen atom (Bohr radius a_0) in its ground state due to the presence of a static electric field E?

(A) Zero

(B) eEa_0

(C) $3eEa_0$

(D) $\dfrac{8e^2Ea_0^3}{3}$

(E) $\dfrac{8e^2E^2a_0^3}{3}$
