Mechanics – Problem Set # 4

TABLE OF INFORMATION

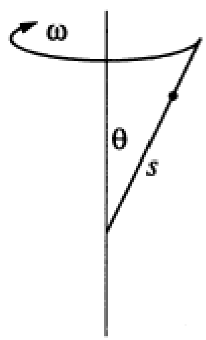
Rest mass of the electron	$m_e = 9.11 \times 10^{-31}$ kilogram $= 9.11 \times 10^{-28}$ gram
Magnitude of the electron charge	$e = 1.60 \times 10^{-19}$ coulomb $= 4.80 \times 10^{-10}$ statcoulomb (esu)
Avogadro's number	$N_0 = 6.02 \times 10^{23}$ per mole
Universal gas constant	$R = 8.31$ joules/(mole $\cdot$ K)
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ joule/K $= 1.38 \times 10^{-16}$ erg/K
Speed of light	$c = 3.00 \times 10^{8}$ m/s $= 3.00 \times 10^{10}$ cm/s
Planck's constant	$h = 6.63 \times 10^{-34}$ joule $\cdot$ second $= 4.14 \times 10^{-15}$ eV $\cdot$ second
	$\hbar = h/2\pi$
Vacuum permittivity	$\epsilon_0 = 8.85 \times 10^{-12}$ coulomb^2/(newton $\cdot$ meter^2)
Vacuum permeability	$\mu_0 = 4\pi \times 10^{-7}$ weber/(ampere $\cdot$ meter)
Universal gravitational constant	$G = 6.67 \times 10^{-11}$ meter^3/(kilogram $\cdot$ second^2)
Acceleration due to gravity	$g = 9.80$ m/s^2 $= 980$ cm/s^2
1 atmosphere pressure	1 atm $= 1.0 \times 10^5$ newton/meter^2 $= 1.0 \times 10^5$ pascals (Pa)
1 angstrom	1 Å $= 1 \times 10^{-10}$ meter
	1 weber/m^2 $= 1$ tesla $= 10^4$ gauss

Moments of inertia about center of mass

Rod	$\frac{1}{12}M\ell^2$
Disc	$\frac{1}{2}MR^2$
Sphere	$\frac{2}{5}MR^2$

66. When it is about the same distance from the Sun as is Jupiter, a spacecraft on a mission to the outer planets has a speed that is 1.5 times the speed of Jupiter in its orbit. Which of the following describes the orbit of the spacecraft about the Sun?

(A) Spiral
(B) Circle
(C) Ellipse
(D) Parabola
(E) Hyperbola



68. A bead is constrained to slide on a frictionless rod that is fixed at an angle θ with a vertical axis and is rotating with angular frequency ω about the axis, as shown above. Taking the distance s along the rod as the variable, the Lagrangian for the bead is equal to

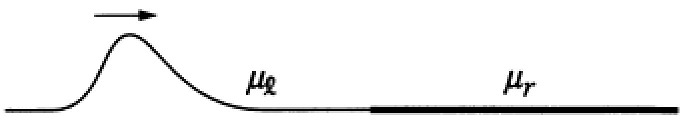
(A) $\frac{1}{2}m\dot{s}^2 - mgs\cos\theta$

(B) $\frac{1}{2}m\dot{s}^2 + \frac{1}{2}m(\omega s)^2 - mgs$

(C) $\frac{1}{2}m\dot{s}^2 + \frac{1}{2}m(\omega s\cos\theta)^2 + mgs\cos\theta$

(D) $\frac{1}{2}m(\dot{s}\sin\theta)^2 - mgs\cos\theta$

(E) $\frac{1}{2}m\dot{s}^2 + \frac{1}{2}m(\omega s\sin\theta)^2 - mgs\cos\theta$



80. A string consists of two parts attached at $x = 0$. The right part of the string ($x > 0$) has mass μ_r per unit length and the left part of the string ($x < 0$) has mass μ_ℓ per unit length. The string tension is T. If a wave of unit amplitude travels along the left part of the string, as shown in the figure above, what is the amplitude of the wave that is transmitted to the right part of the string?

(A) 1

(B) $\dfrac{2}{1 + \sqrt{\mu_\ell/\mu_r}}$

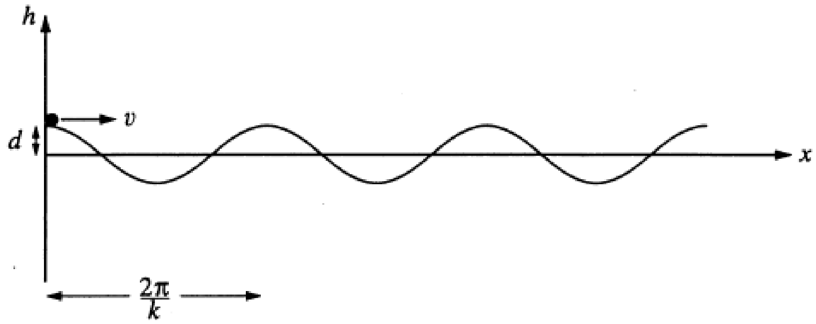
(C) $\dfrac{2\sqrt{\mu_\ell/\mu_r}}{1 + \sqrt{\mu_\ell/\mu_r}}$

(D) $\dfrac{\sqrt{\mu_\ell/\mu_r} - 1}{\sqrt{\mu_\ell/\mu_r} + 1}$

(E) 0

81. A piano tuner who wishes to tune the note D_2 corresponding to a frequency of 73.416 hertz has tuned A_4 to a frequency of 440.000 hertz. Which harmonic of D_2 (counting the fundamental as the first harmonic) will give the lowest number of beats per second, and approximately how many beats will this be when the two notes are tuned properly?

	Harmonic	Number of Beats
(A)	6	5
(B)	6	0.5
(C)	5	0.1
(D)	3	0.372
(E)	2	4.5



83. Consider a particle moving without friction on a rippled surface, as shown above. Gravity acts down in the negative h direction. The elevation $h(x)$ of the surface is given by $h(x) = d\ \cos(kx)$. If the particle starts at $x = 0$ with a speed v in the x direction, for what values of v will the particle stay on the surface at all times?

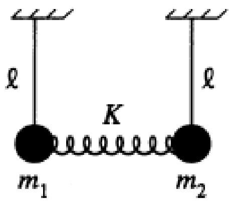
(A) $v \leq \sqrt{gd}$

(B) $v \leq \sqrt{\frac{g}{k}}$

(C) $v \leq \sqrt{gkd^2}$

(D) $v \leq \sqrt{\frac{g}{k^2 d}}$

(E) $v > 0$



84. Two pendulums are attached to a massless spring, as shown above. The arms of the pendulums are of identical lengths ℓ, but the pendulum balls have unequal masses m_1 and m_2. The initial distance between the masses is the equilibrium length of the spring, which has spring constant K. What is the highest normal mode frequency of this system?

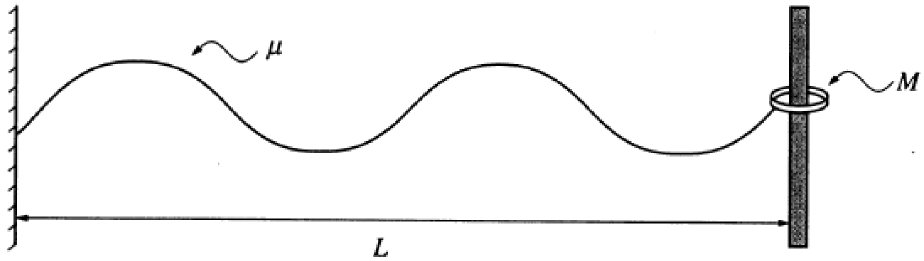
(A) $\sqrt{g/\ell}$

(B) $\sqrt{\dfrac{K}{m_1 + m_2}}$

(C) $\sqrt{\dfrac{K}{m_1} + \dfrac{K}{m_2}}$

(D) $\sqrt{\dfrac{g}{\ell} + \dfrac{K}{m_1} + \dfrac{K}{m_2}}$

(E) $\sqrt{\dfrac{2g}{\ell} + \dfrac{K}{m_1 + m_2}}$



85. Small-amplitude standing waves of wavelength λ occur on a string with tension T, mass per unit length μ, and length L. One end of the string is fixed and the other end is attached to a ring of mass M that slides on a frictionless rod, as shown in the figure above. When gravity is neglected, which of the following conditions correctly determines the wavelength? (You might want to consider the limiting cases $M \to 0$ and $M \to \infty$.)

(A) $\mu / M = \frac{2\pi}{\lambda} \cot \frac{2\pi L}{\lambda}$

(B) $\mu / M = \frac{2\pi}{\lambda} \tan \frac{2\pi L}{\lambda}$

(C) $\mu / M = \frac{2\pi}{\lambda} \sin \frac{2\pi L}{\lambda}$

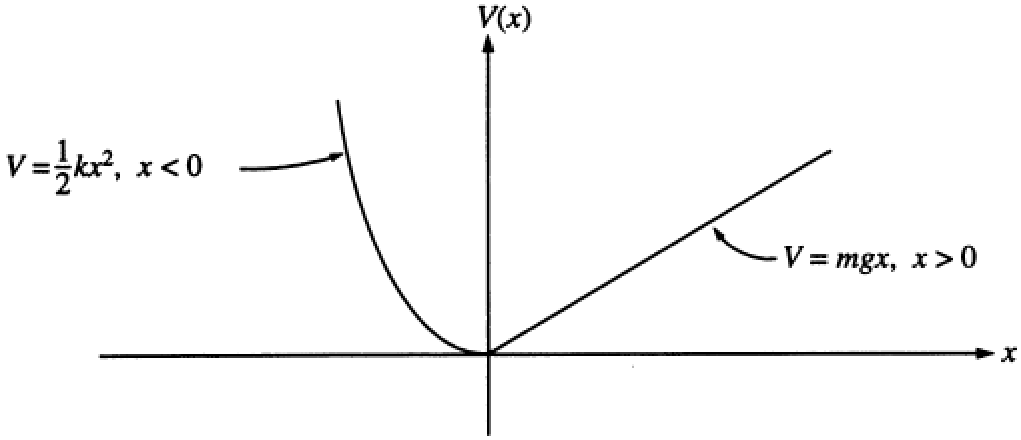
(D) $\lambda = 2L/n, \; n = 1, 2, 3, \ldots$

(E) $\lambda = 2L/(n + \frac{1}{2}), \; n = 1, 2, 3, \ldots$

---

92. A particle of mass m moves in a one-dimensional potential $V(x) = -ax^2 + bx^4$, where a and b are positive constants. The angular frequency of small oscillations about the minima of the potential is equal to

(A) $\pi(a/2b)^{1/2}$
(B) $\pi(a/m)^{1/2}$
(C) $(a/mb)^{1/2}$
(D) $2(a/m)^{1/2}$
(E) $(a/2m)^{1/2}$



93. A particle of mass m moves in the potential shown above. The period of the motion when the particle has energy E is

(A) $\sqrt{k/m}$

(B) $2\pi\sqrt{m/k}$

(C) $2\sqrt{2E/mg^2}$

(D) $\pi\sqrt{m/k} + 2\sqrt{2E/mg^2}$

(E) $2\pi\sqrt{m/k} + 4\sqrt{2E/mg^2}$