

Mechanics - Problem Set #2

Rest mass of the electron	$m_e = 9.11 \times 10^{-31}$ kilogram = 9.11×10^{-28} gram
Magnitude of the electron charge	$e = 1.60 \times 10^{-19}$ coulomb = 4.80×10^{-10} statcoulomb (esu)
Avogadro's number	$N_0 = 6.02 \times 10^{23}$ per mole
Universal gas constant	$R = 8.32$ joules/(mole · K)
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ joule/K = 1.38×10^{-16} erg/K
Speed of light	$c = 3.00 \times 10^8$ m/s = 3.00×10^{10} cm/s
Planck's constant	$h = 6.63 \times 10^{-34}$ joule · second = 4.14×10^{-15} eV · second $\hbar = h/2\pi$
Vacuum permittivity	$\epsilon_0 = 8.85 \times 10^{-12}$ coulomb ² /(newton · meter ²)
Vacuum permeability	$\mu_0 = 4\pi \times 10^{-7}$ weber/(ampere · meter)
Universal gravitational constant	$G = 6.67 \times 10^{-11}$ meter ³ /(kilogram · second ²)
Acceleration due to gravity	$g = 9.80$ m/s ² = 980 cm/s ²
1 atmosphere pressure	1 atm = 1.0×10^5 newton/meter ² = 1.0×10^5 pascals (Pa)
1 angstrom	1 Å = 1×10^{-10} meter
	1 weber/m ² = 1 tesla = 10^4 gauss

44. A particle of mass m on the Earth's surface is confined to move on the parabolic curve $y = ax^2$, where y is up. Which of the following is a Lagrangian for the particle?

(A) $L = \frac{1}{2}m\dot{y}^2\left(1 + \frac{1}{4ay}\right) - mgy$

(B) $L = \frac{1}{2}m\dot{y}^2\left(1 - \frac{1}{4ay}\right) - mgy$

(C) $L = \frac{1}{2}m\dot{x}^2\left(1 + \frac{1}{4ax}\right) - mgx$

(D) $L = \frac{1}{2}m\dot{x}^2(1 + 4a^2x^2) + mgx$

(E) $L = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 + mgy$

45. A ball is dropped from a height h . As it bounces off the floor, its speed is 80 percent of what it was just before it hit the floor. The ball will then rise to a height of most nearly

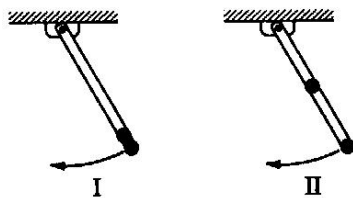
(A) $0.94 h$

(B) $0.80 h$

(C) $0.75 h$

(D) $0.64 h$

(E) $0.50 h$



61. A long, straight, and massless rod pivots about one end in a vertical plane. In configuration I, shown above, two small identical masses are attached to the free end; in configuration II, one mass is moved to the center of the rod. What is the ratio of the frequency of small oscillations of configuration II to that of configuration I?

- (A) $(6/5)^2$
 (B) $(3/2)^2$
 (C) $6/5$
 (D) $3/2$
 (E) $5/3$
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66. A thin uniform steel chain is 10 meters long with a mass density of 2 kilograms per meter. One end of the chain is attached to a horizontal axle having a radius that is small compared to the length of the chain. If the chain initially hangs vertically, the work required to slowly wind it up on to the axle is closest to

- (A) 100 J
 (B) 200 J
 (C) 1,000 J
 (D) 2,000 J
 (E) 10,000 J
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74. Two circular hoops, X and Y , are hanging on nails in a wall. The mass of X is four times that of Y , and the diameter of X is also four times that of Y . If the period of small oscillations of X is T , the period of small oscillations of Y is

- (A) T (B) $T/2$ (C) $T/4$
 (D) $T/8$ (E) $T/16$

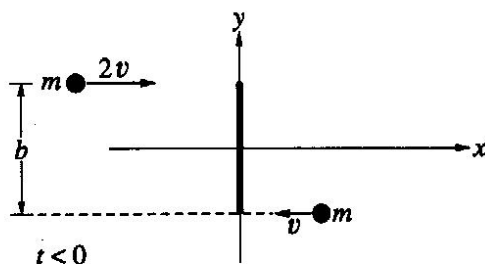


Figure I

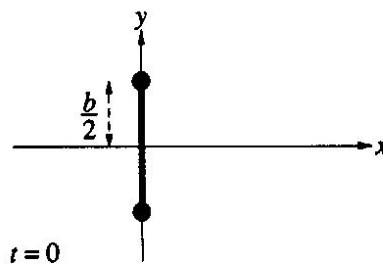
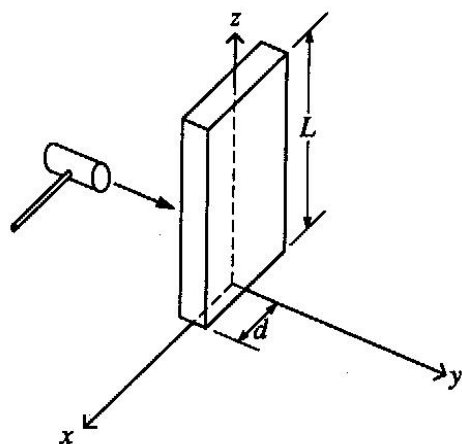


Figure II

78. One ice skater of mass m moves with speed $2v$ to the right, while another of the same mass m moves with speed v toward the left, as shown in Figure I. Their paths are separated by a distance b . At $t = 0$, when they are both at $x = 0$, they grasp a pole of length b and negligible mass. For $t > 0$, consider the system as a rigid body of two masses m separated by distance b , as shown in Figure II. Which of the following is the correct formula for the motion after $t = 0$ of the skater initially at $y = b/2$?

- (A) $x = 2vt, y = b/2$
 (B) $x = vt + 0.5b \sin(3vt/b), y = 0.5b \cos(3vt/b)$
 (C) $x = 0.5vt + 0.5b \sin(3vt/b), y = 0.5b \cos(3vt/b)$
 (D) $x = vt + 0.5b \sin(6vt/b), y = 0.5b \cos(6vt/b)$
 (E) $x = 0.5vt + 0.5b \sin(6vt/b), y = 0.5b \cos(6vt/b)$

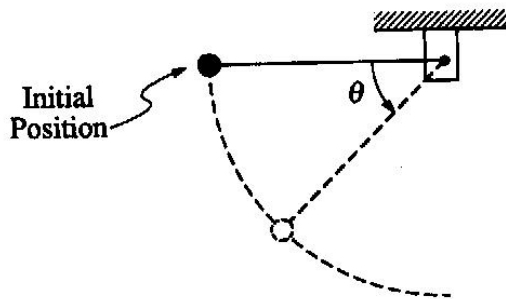


82. A thin plate of mass M , length L , and width $2d$ is mounted vertically on a frictionless axle along the z -axis, as shown above. Initially the object is at rest. It is then tapped with a hammer to provide a torque τ , which produces an angular impulse \mathbf{H} about the z -axis of magnitude $H = \int \tau dt$. What is the angular speed ω of the plate about the z -axis after the tap?

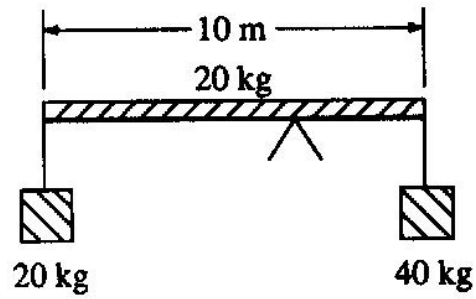
- (A) $\frac{H}{2Md^2}$
 (B) $\frac{H}{Md^2}$
 (C) $\frac{2H}{Md^2}$
 (D) $\frac{3H}{Md^2}$
 (E) $\frac{4H}{Md^2}$

87. A particle of mass M moves in a circular orbit of radius r around a fixed point under the influence of an attractive force $F = \frac{K}{r^3}$, where K is a constant. If the potential energy of the particle is zero at an infinite distance from the force center, the total energy of the particle in the circular orbit is

- (A) $-\frac{K}{r^2}$
- (B) $-\frac{K}{2r^2}$
- (C) 0
- (D) $\frac{K}{2r^2}$
- (E) $\frac{K}{r^2}$



93. The figure above shows a small mass connected to a string, which is attached to a vertical post. If the mass is released when the string is horizontal as shown, the magnitude of the total acceleration of the mass as a function of the angle θ is
- (A) $g \sin \theta$
 - (B) $2g \cos \theta$
 - (C) $2g \sin \theta$
 - (D) $g\sqrt{3 \cos^2 \theta + 1}$
 - (E) $g\sqrt{3 \sin^2 \theta + 1}$



100. A uniform rod of length 10 meters and mass 20 kilograms is balanced on a fulcrum with a 40-kilogram mass on one end of the rod and a 20-kilogram mass on the other end, as shown above. How far is the fulcrum located from the center of the rod?
- (A) 0 m
(B) 1 m
(C) 1.25 m
(D) 1.5 m
(E) 2 m