MECHANICS – Problem Set #1

Questions 4-5

The magnitude of the Earth's gravitational force on a point mass is $F(r)$, where r is the distance from the Earth's center to the point mass. Assume the Earth is a homogeneous sphere of radius R.

4. What is $\frac{F(R)}{F(2R)}$?

(A) 32
(B) 8
(C) 4
(D) 2
(E) 1

5. Suppose there is a very small shaft in the Earth such that the point mass can be placed at a radius of $R/2$. What is $\frac{F(R)}{F\left(\frac{R}{2}\right)}$?

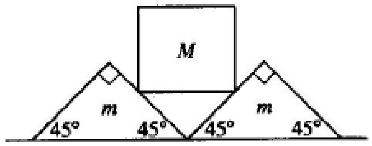
(A) 8

(B) 4

(C) 2

(D) $\frac{1}{2}$

(E) $\frac{1}{4}$



6. Two wedges, each of mass m, are placed next to each other on a flat floor. A cube of mass M is balanced on the wedges as shown above. Assume no friction between the cube and the wedges, but a coefficient of static friction $\mu < 1$ between the wedges and the floor. What is the largest M that can be balanced as shown without motion of the wedges?

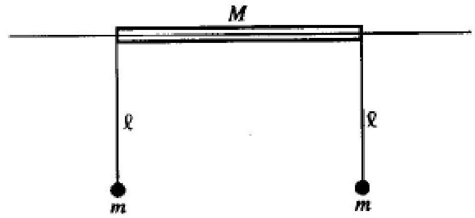
(A) $\frac{m}{\sqrt{2}}$

(B) $\frac{\mu m}{\sqrt{2}}$

(C) $\frac{\mu m}{1 - \mu}$

(D) $\frac{2\mu m}{1 - \mu}$

(E) All M will balance.



7. A cylindrical tube of mass M can slide on a horizontal wire. Two identical pendulums, each of mass m and length ℓ, hang from the ends of the tube, as shown above. For small oscillations of the pendulums in the plane of the paper, the eigenfrequencies of the normal modes of oscillation of this system

are 0, $\sqrt{\frac{g(M + 2m)}{\ell M}}$, and

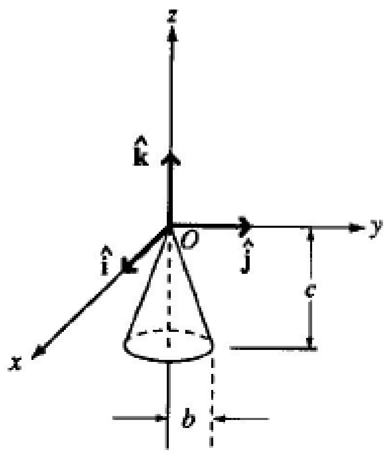
(A) $\sqrt{\frac{g}{\ell}}$

(B) $\sqrt{\frac{g}{\ell}\frac{M + m}{M}}$

(C) $\sqrt{\frac{g}{\ell}\frac{m}{M}}$

(D) $\sqrt{\frac{g}{\ell}\frac{m}{M + m}}$

(E) $\sqrt{\frac{g}{\ell}\frac{m}{M + 2m}}$



8. A solid cone hangs from a frictionless pivot at origin O, as shown above. If $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ a unit vectors, and a, b, and c are positive co which of the following forces $\mathbf{F}$ applied to the the cone at a point P results in a torque τ on cone with a negative component τ_z?

(A) $\mathbf{F} = a\hat{\mathbf{k}}$, P is $(0, b, -c)$

(B) $\mathbf{F} = -a\hat{\mathbf{k}}$, P is $(0, -b, -c)$

(C) $\mathbf{F} = a\hat{\mathbf{j}}$, P is $(-b, 0, -c)$

(D) $\mathbf{F} = a\hat{\mathbf{j}}$, P is $(b, 0, -c)$

(E) $\mathbf{F} = -a\hat{\mathbf{k}}$, P is $(-b, 0, -c)$

19. Which of the following is most nearly the mass of the Earth? (The radius of the Earth is about 6.4×10^6 meters.)

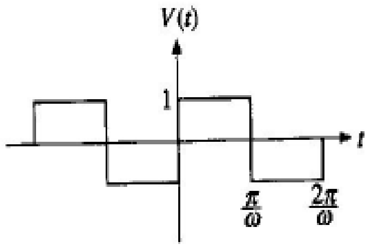
(A) 6×10^{24} kg

(B) 6×10^{27} kg

(C) 6×10^{30} kg

(D) 6×10^{33} kg

(E) 6×10^{36} kg



39. If n is an integer ranging from 1 to infinity, ω is an angular frequency, and t is time, then the Fourier series for a square wave, as shown above, is given by which of the following?

(A) $V(t) = \frac{4}{\pi}\sum_{1}^{\infty}\frac{1}{n}\sin(n\omega t)$

(B) $V(t) = \frac{4}{\pi}\sum_{0}^{\infty}\frac{1}{(2n+1)}\sin((2n+1)\omega t)$

(C) $V(t) = \frac{4}{\pi}\sum_{1}^{\infty}\frac{1}{n}\cos(n\omega t)$

(D) $V(t) = \frac{4}{\pi}\sum_{0}^{\infty}\frac{1}{(2n+1)}\cos((2n+1)\omega t)$

(E) $V(t) = -\frac{4}{\pi} + \frac{4}{\pi}\sum_{1}^{\infty}\frac{1}{n^2}\cos(n\omega t)$

40. A rigid cylinder rolls at constant speed without slipping on top of a horizontal plane surface. The acceleration of a point on the circumference of the cylinder at the moment when the point touches the plane is

(A) directed forward
(B) directed backward
(C) directed up
(D) directed down
(E) zero

Questions 41-42

A cylinder with moment of inertia 4 kg · m^2 about a fixed axis initially rotates at 80 radians per second about this axis. A constant torque is applied to slow it down to 40 radians per second.

41. The kinetic energy lost by the cylinder is

(A) 80 J
(B) 800 J
(C) 4000 J
(D) 9600 J
(E) 19,200 J

42. If the cylinder takes 10 seconds to reach 40 radians per second, the magnitude of the applied torque is

(A) 80 N · m
(B) 40 N · m
(C) 32 N · m
(D) 16 N · m
(E) 8 N · m

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43. If $\frac{\partial L}{\partial q_n} = 0$, where L is the Lagrangian for a conservative system without constraints and q_n is a generalized coordinate, then the generalized momentum p_n is

(A) an ignorable coordinate

(B) constant

(C) undefined

(D) equal to $\frac{d}{dt}\left(\frac{\partial L}{\partial q_n}\right)$

(E) equal to the Hamiltonian for the system