Chapter 10: Nuclear Reactions

Goals of Period 10

     Section 10.1: To describe the half-life of nuclei and radio-carbon dating
     Section 10.2: To further explore the relationship between mass and energy
     Section 10.3: To calculate the binding energy of atomic nuclei

In Period 8 we discovered that unstable isotopes tend to decay into more stable nuclei. In Period 9 we found that unstable isotopes can decay by emitting alpha, beta, or gamma radiation, or by nuclear fission. Next we discuss the rate at which an isotope decays – the half life of the isotope. We then examine the effect on the mass and the energy of a nucleus when ionizing radiation is emitted.

10.1 Rates of Radioactive Decay

Half-Life

The decay of unstable, radioactive nuclei is a random process, and it is impossible to predict when a given nucleus will decay. Nuclei do not all decay at once, nor do those that have lived a long time show signs of aging like people do. The best one can do is to make the statistical statement that half the nuclei in a large sample of an unstable isotope will decay in a time characteristic of that isotope, called the **half-life** of the isotope and denoted by $T_{1/2}$. A similar statistical statement would be that in a large sample of coin tosses, half would come up heads. However, this does not mean that you get 5 heads in every 10 tosses nor does it tell you which of the coins will come up heads and which will come up tails.

In Period 1 we discussed exponential growth and decay. In exponential growth, a quantity doubles its size in a characteristic time called the doubling time. The equation describing exponential growth is

$$N = B \times 2^{t}$$

(Equation 10.1)

where

     N = the amount of the quantity at a given time
     t = the number of time periods elapsed
     B = the initial amount of the quantity

In exponential decay, a quantity decreases by one half in a characteristic time, called the half life. A radioactive isotope is described by exponential decay, in which (statistically) the number of nuclei of the isotope is cut in half in a characteristic time, $T_{1/2}$. The equation for exponential decay is

$$N = B \times 2^{-t} = B \times \frac{1}{2^{t}}$$

(Equation 10.2)

Table 10.1 uses this relationship to compare the number of half lives elapsed to the portion of a sample remaining.

Table 10.1 Half Life

Number of Half Lives	Fraction of Original		Number of Half Lives	Fraction of Original
0	$1 = \dfrac{1}{2^0}$		6	$\dfrac{1}{64} = \dfrac{1}{2^6}$
1	$\dfrac{1}{2} = \dfrac{1}{2^1}$		7	$\dfrac{1}{128} = \dfrac{1}{2^7}$
2	$\dfrac{1}{4} = \dfrac{1}{2^2}$		8	$\dfrac{1}{256} = \dfrac{1}{2^8}$
3	$\dfrac{1}{8} = \dfrac{1}{2^3}$		9	$\dfrac{1}{512} = \dfrac{1}{2^9}$
4	$\dfrac{1}{16} = \dfrac{1}{2^4}$		10	$\dfrac{1}{1024} = \dfrac{1}{2^{10}}$
5	$\dfrac{1}{32} = \dfrac{1}{2^5}$			

(Example 10.2)

A sample of radioactive material has a half life of 15 minutes. If there is 5.0 grams of the material at the beginning of an experiment, how much will be left after 1 hour has passed?

After one hour, four 15-minute half lives has passed. Using Equation 10.2, we find

$$\boldsymbol{N} = \boldsymbol{B} \; x \; \frac{1}{2^t} \;\; = 5.0 \text{ grams } \frac{1}{2^4} \;\; = \;\; 5.0 \text{ g } \frac{1}{16} \;\; = \;\; 0.31 \text{ g}$$

An example of exponential decay in atomic physics, rather than nuclear, is the emission of visible photons (energy a few eV) by glow-in-the-dark materials. When the material is exposed to a bright light, electrons in the atoms are raised to excited states. These atomic excited states also decay exponentially, so that the glow gets weaker and weaker.

In class we will illustrate the exponential decay of radioactive materials using a non-radioactive example, a capacitor. A capacitor stores separated positive and negative charge. When the separated charge is allowed to recombine, the capacitor discharges exponentially. Thus, the discharge of a capacitor is another example of exponential decay. The rate of discharge of a capacitor decreases over time as shown in Figure 10.1. In class, we will use the discharge of a capacitor to model the exponential decay of radioactive isotopes.

Figure 10.1 Exponential Decay of a Capacitor
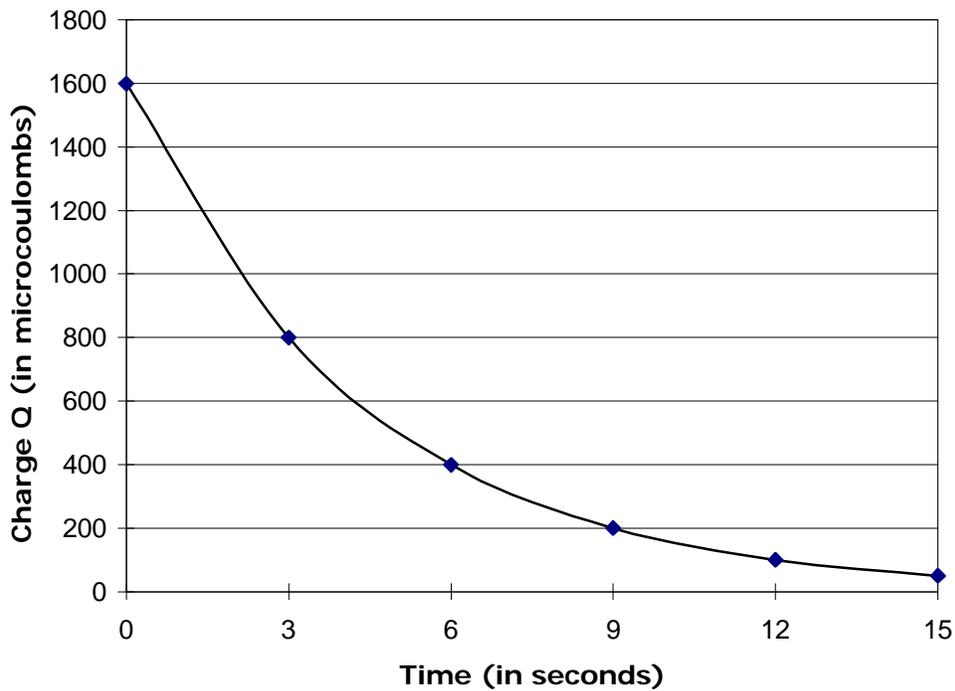
Radioactive Dating

Naturally-occurring radioactive isotopes can be used to determine the age of objects that have incorporated these isotopes. One of the most important isotopes used for such radioactive dating is carbon-14 ($^{14}_{6}C$), because carbon is an important component of living plants and animals. Unstable isotopes such as carbon-14 are continuously produced by nuclear interactions of cosmic rays and are ingested by living organisms such as trees. When the organism dies, no more carbon-14 is ingested. The amount of the radioactive isotope is then reduced by a factor of ½ for every half-life $T_{1/2}$. that has passed since the organism died.

Suppose your sample is a piece of charcoal from an ancient campfire. If you know the carbon-14 counting rate to expect from a fresh sample and what the counting rate is now, you can calculate how many half-lives have elapsed. If the half life of the isotope is known, the elapsed time since the radioisotope ceased to be replenished can be found. For carbon-14, each half-life is known to be 5,568 years.

(Example 10.2)

A sample of charcoal from an archeological site contains one-sixteenth of the carbon-14 expected in a fresh sample. Find the age of the charcoal sample.

Four half-lives must pass for the sample to decay to one-sixteenth of the initial amount.

$$\text{4 half lives} \times \frac{5,568 \text{ years}}{\text{half life}} = 22,272 \text{ years}$$

10.2 Mass as a Form of Energy, $E = Mc^2$

We have learned that energy comes in many forms, but that in any process energy is neither created nor destroyed. In Chapter 8 we saw that in everyday situations, including atomic processes such as chemical reactions, the amount of mass converted to other forms is so tiny as to be unobservable. However, in the nuclear domain, changes in mass are observable. For example, in the alpha decay

$$^{232}_{90}\text{Th} \rightarrow \,^{228}_{88}\text{Ra} + \,^{4}_{2}\text{He}$$

the masses of the decay products Ra-228 and He-4 do not quite add up to the mass of the parent Th-232 nucleus. About 0.4% of a proton mass disappears in the decay. However, when one calculates the energy equivalent of this lost mass by using the equation $E=Mc^2$, this lost mass is found to be exactly equal to the kinetic energy of the decay products.

Exothermic Nuclear Reactions: Fission and Fusion

The nuclear reactors now in widespread use for generation of electric power are **fission** reactors, in which very heavy isotopes release energy by splitting into smaller pieces. Still in the research stage are **fusion** reactors, in which light isotopes release energy by combining (or fusing) into heavier ones. Nuclear fusion is the energy source that fuels stars, including our sun.

How can you extract energy both from fusing nuclei together and also from breaking up nuclei? This is not a great puzzle – the same thing happens with atoms and molecules in chemical reactions. Energy can be released when atoms or molecules combine to form a molecule that is more tightly bound, such as when hydrogen and oxygen combine to form water. But energy can also be released when a large, loosely bound molecule breaks up: an example is the breakup of the explosive nitroglycerine. The key is the relative binding energy of the reactants and the products. When nitroglycerine breaks up, the products (N_2O, O_2, CO_2, $H_2 O$) are more tightly bound than the unstable nitroglycerine molecule. The sum of the binding energies of the products is greater than the binding energy of nitroglycerine, and energy is released in the breakup. By contrast, the binding energy of water is greater than the sum of the binding energies of H_2 and O_2. Therefore energy is released when H_2 and O_2 combine. This occurs in a fuel cell, for example, or when hydrogen burns.

In both of these chemical examples, it is necessary to supply (invest) some *activation energy* to make the exothermic reactions happen. Nitroglycerine molecules must be heated or subjected to shock or collision in order to break the weak chemical bonds so that the atoms can rearrange themselves. Similarly, energy must be supplied (by heating, for example) to break up H_2 and O_2 molecules into the H and O atoms, which recombine to form water. Once the exothermic chemical reactions get started, the activation energy is supplied by the released energy. For example, the heat from combining H and O atoms can break up more H_2 and O_2. The energetic breakup products of one nitroglycerine molecule can bombard others, causing a chain reaction chemical explosion.

10.3 Nuclear Binding Energy

A graph of binding energy per nucleon vs. mass number A is useful for understanding which nuclear isotopes are most tightly bound. On this graph (Figure 10.2), each point represents a stable or nearly stable isotope, with the lightest nuclei on the left, and the heaviest nuclei on the right. A tightly bound nucleus is one in which each nucleon in the nucleus has a large binding energy, regardless of how many nucleons there are. The reason for plotting binding energy per nucleon, rather than just binding energy, is to compare light and heavy nuclei on an equal basis.

Figure 10.2 Nuclear Binding Energy
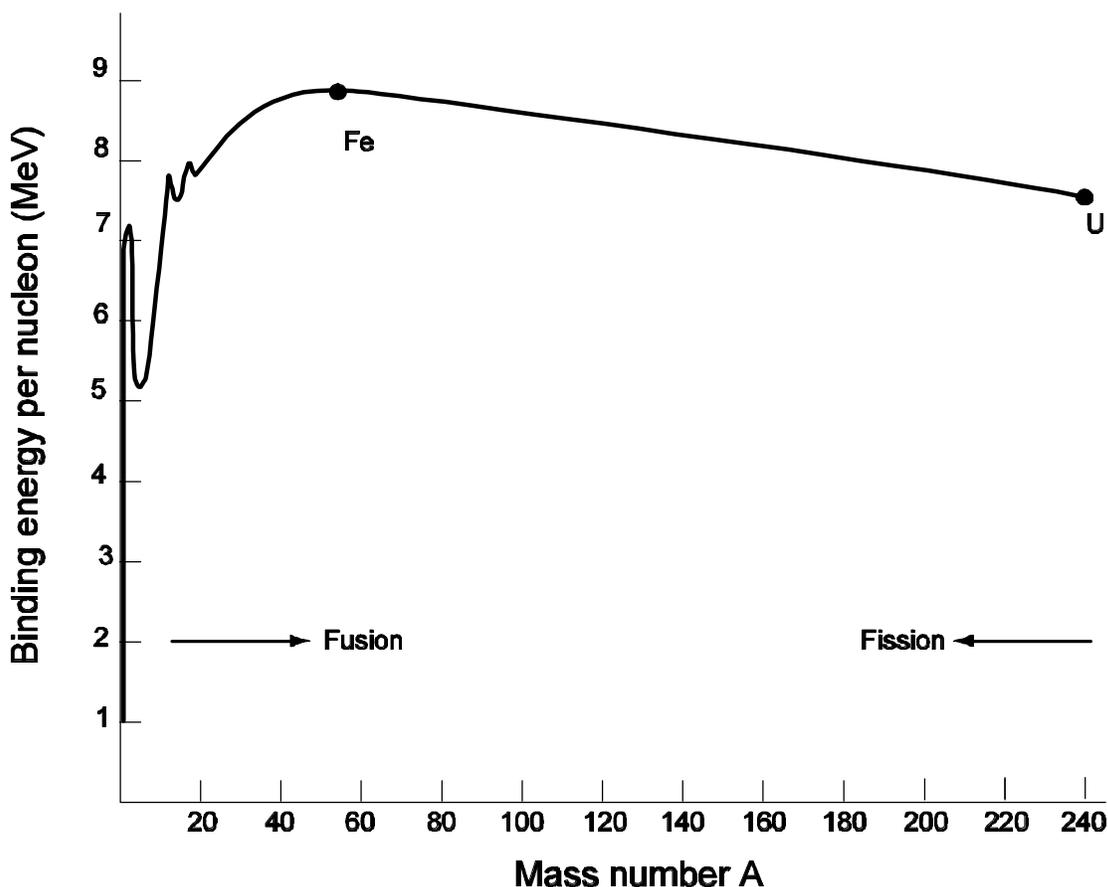
In the graph of Figure 10.2, deuterium ($^2_1$H, A=2, the leftmost point), has a binding energy of about 1 MeV per nucleon; it is quite weakly bound. By comparison, helium ($^4_2$He A=4) is quite tightly bound, with a binding energy per nucleon of about 7 MeV. This means that if one were able to force two $^2_1$H nuclei together to make one nucleus of $^4_2$He, about (7 minus 1) MeV of energy per nucleon would be released. Since there are 4 nucleons in $^4_2$He, the total energy released would be 4 times (7-1) = 24 MeV for each helium nucleus formed. This is an example of nuclear fusion.

On the other end of the graph, uranium-235 ($^{235}_{92}$U, A=235) has a binding energy of about 7.5 MeV per nucleon, while isotopes with an A about half as large have binding energies of about 8.5 MeV per nucleon. Thus, if one were able to induce uranium to split into two pieces, there would be an energy release of about (8.5 – 7.5) MeV = 1 MeV per nucleon. Since there are 235 nucleons in U-235, about 235 MeV is released for each U-235 nucleus split. This is an example of nuclear fission.

In Figure 10.2, the curve of binding energy per nucleon vs. atomic mass (A) for stable and nearly stable isotopes shows strongest binding for medium-sized nuclei such as iron-56 ($^{56}_{26}$Fe), while both very light and very heavy nuclei are less tightly bound. Why is this true? Recall that in a very large nucleus every proton exerts a repulsive electrical force on every other proton. However, in a large nucleus every nucleon does not attract every other nucleon, because some of them are too far away for a nuclear force to operate between them. This is because the range of the attractive nuclear force is so short; this force is large only over distances comparable to the size of a small nucleus. Thus in a large nucleus, the nucleons on one side of the nucleus are too far away to be attracted by those on the opposite side. The result is weaker binding for very large nuclei. This effect is seen on the graph as the gradual decrease of binding energy per nucleon above A=60. The rapid increase in binding energy per nucleon between A=2 and A=40 can also be understood. These nuclei are small enough so that most of the nucleons feel the attractive nuclear force from all the others, so that adding more nucleons tends to make tighter binding.

It is not difficult to calculate the binding energy of a nucleus if its mass is known accurately. Einstein's equation, $E=Mc^2$, says that mass is actually a concentrated form of energy. The difference in energy between an assembled nucleus and its unassembled parts (protons and neutrons) shows up as a difference in mass. The formation of a bound nucleus from protons and neutrons is exothermic. When a bound nucleus is assembled from protons and neutrons, energy is released (as radiant energy, for example) and is lost from the nucleus. Thus the nucleus has less mass than the unassembled protons and neutrons. This difference in mass, when multiplied by c^2, is the binding energy of the nucleus. This relationship is expressed by equation 10.3.

(Equation 10.3)

Binding energy = [(mass of unbound protons + neutrons) − (mass of nucleus)] c^2

or

Binding energy = [Z M$_p$ + (A − Z) M$_n$ − M$_{nuc}$] c²

where

M_p = mass of a free proton = 1.6726×10^{-27} kg

M_n = mass of a free neutron = 1.6749×10^{-27} kg

M_{nuc} = mass of the assembled nucleus in kg

Z = number of protons in the nucleus

A-Z = number of neutrons in the nucleus

c^2 = (speed of light)^2 = $(3 \times 10^8 \text{ m/s})^2$ = 9×10^{16} joule/kg

(Equation 10.4)

$$\text{Binding energy per nucleon} = \frac{\text{binding energy}}{A}$$

where

A = number of nucleons in the nucleus

It may also be useful to convert energy units between MeV and joules:

$$1 \text{ Mev} = 10^6 \text{ eV} = 1.6 \times 10^{-13} \text{ joules}$$

or

$$1 \text{ joule} = 6.25 \times 10^{12} \text{ MeV}$$

(Example 10.3)

The mass of a $^{235}_{92}$U nucleus is 390.206×10^{-27} kg. Calculate the binding energy of an unarium-235 nucleus in joules.

First, find the total mass of the free protons and neutrons that make up the nucleus. The $^{235}_{92}$U nucleus has 92 protons and (235 − 92) 143 neutrons. The mass of one proton = 1.6726×10^{-27} kg and the mass of one neutron = 1.6749×10^{-27} kg.

mass of 92 free protons = $92(1.6726 \times 10^{-27}$ kg) = 153.8792×10^{-27} kg

mass of 143 free neutrons = $143(1.6749 \times 10^{-27}$ kg) = 239.5107×10^{-27} kg

Add the masses of the free protons and neutrons to find the total mass of the unbound nucleons.

$$153.8792 \times 10^{-27} \text{ kg}$$
$$+ \quad \underline{239.5107 \times 10^{-27} \text{ kg}}$$
$$393.3899 \times 10^{-27} \text{ kg}$$

However, when these nucleons combine into a U-235 nucleus, the mass of the nucleus is only 390.206×10^{-27} kg. Subtract the mass of the nucleus from the mass of the unbound nucleons to find the difference in mass.

$$393.3899 \times 10^{-27} \text{ kg}$$
$$- \underline{390.206 \times 10^{-27} \text{ kg}}$$
$$3.1839 \times 10^{-27} \text{ kg}$$

This difference in mass is the amount of mass that has been converted into energy. Use $E = Mc^2$ to find the amount of energy given off when the neutrons and protons combine to form the U-235 nucleus. This is the binding energy of the nucleus.

Binding energy $=$ (mass difference) c^2 $=$ $(3.1839 \times 10^{-27}$ kg $) \times (3 \times 10^8 \text{m/s})^2$

$= (3.1839 \times 10^{-27}$ kg $) \times (9 \times 10^{16} \text{ m}^2/\text{s}^2 = 2.8655 \times 10^{-10}$ J

(Example 10.4)

Convert the binding energy of a $^{235}_{92}$U nucleus into units of megaelectron volts.

Use ratios and the equality: 1 joule $= 6.25 \times 10^{12}$ MeV.

2.8655×10^{-10} joules $\times$ $\dfrac{6.25 \times 10^{12} \text{ MeV}}{1 \text{ joule}}$ $= 1{,}791$ MeV

(Example 10.5)

Find the binding energy per nucleon in MeV of a $^{235}_{92}$U nucleus.

The nucleus has 235 nucleons, so the binding energy per nucleon is

$\dfrac{1{,}791 \text{ MeV}}{235 \text{ nucleons}}$ $= 7.62$ MeV/nucleon

This result agrees with the binding energy per nucleon from the binding energy graph in Figure 10.2.

Concept Check 10.2

A $^{238}_{92}$U nucleus has a mass of 395.2138×10^{-27} kg. Find the binding energy of this nucleus in joules and megaelectron volts following the steps below.

a) Find the mass of all of the protons and all of the neutrons in this nucleus. Add them to find the total mass of the unbound protons and neutrons.

b) Find the difference in mass between the protons and neutrons that make up the nucleus and the given mass of the nucleus, 395.2138×10^{-27} kg.

c) Use $E = Mc^2$ to find the binding energy of this nucleus in joules.

d) Convert this binding energy into units of megaelectron volts.

e) What is the binding energy *per nucleon*?

Another way of using Einstein's equation is to say that the total amount of energy stored as mass in a U-235 nucleus is Mc^2. Only a small fraction of this energy, about 0.1%, is released when the nucleus fissions. The appeal of nuclear power arises from the fact that typical binding energies in nuclei are about a million times greater than those between atoms in molecules. We can therefore extract about a million times more energy per pound of fuel from nuclear processes than from chemical ones.

Period 10 Summary

10.1 Unstable nuclei decay by giving off alpha particles (helium nuclei), beta particles (electrons or antielectrons) or gamma particles (photons of high energy electromagnetic radiation).

The half-life of an unstable element is the time it takes on average for one half of the nuclei in the sample to decay.

Unstable nuclei decay exponentially: $\boldsymbol{N} = \boldsymbol{B} \times 2^{-t}$

Radio-carbon dating uses the half life of naturally-occurring radioactive isotopes to determine the age of objects that have incorporated these isotopes.

Carbon-14 ($^{14}_{6}C$), with a half-life of 5,568 years is used because carbon is an important component of living plants and animals. When the organism dies, no more carbon-14 is ingested. The amount of the radioactive isotope is then reduced by a factor of ½ for every half-life that has passed since the organism died.

10.2 Energy can be released when large loosely bound nuclei break up. In fission reactions, very heavy isotopes release energy by splitting into smaller pieces. Nuclear reactors use fission reactions.

Energy can be released when nuclei fuse to form a nucleus that is more tightly bound. In fusion reactions, light isotopes release energy by combining (or fusing) into heavier ones. Nuclear fusion is the energy source that fuels stars.

10.3 Nuclear binding energy: the energy required to hold nucleons together into atoms.

The most stable nuclei have the greatest binding energy per nucleon. Iron (Fe) has high binding energy, while Uranium (U) has less binding energy.

If an atom splits into two new atoms (a fission reaction), the amount of energy released by the reaction per nucleon is the difference in binding energy between the original atom and the new atoms.

The energy released when nuclei fuse or fission is calculated from $E = Mc^2$

Binding energy = [(mass of unbound protons + neutrons) – (mass of nucleus)] c^2

Period 10 Exercises

E.1 In 1990, a radioactive sample was observed to decay at a rate of 800 counts per minute. In 2002, that same sample was observed to decay with a rate of 100 counts per minute. What is the half-life of the material in the sample?

    a) 2 years
    b) 4 years
    c) 6 years
    d) 8 years
    e) 12 years

E.2 A student measured the radioactive particles given off by a Barium-137 source. In the experiment, the count rate fell from 280 counts per minute to 35 counts per minute in some time interval t. The half-life of Ba-137 is about 2.6 minutes. How long was the time interval t?

    a) 2.6 minutes
    b) 5.2 minutes
    c) 7.8 minutes
    d) 10.4 minutes
    e) You would need to know the mass of the Ba-137 to find the answer.

E.3 In chemical reactions, the amount of mass that might be converted into energy is very small. In nuclear reactions this is not the case. This is because the binding energy in chemical bonds

    a) is much larger than the binding energy in nuclear bonds.
    b) is almost the same as the binding energy in nuclear bonds, except for light nuclei that fuse.
    c) is much smaller than the binding energy in nuclear bonds.
    d) is about the same as in nuclear bonds, but many more chemical reactions take place.
    e) depends on the catalyst present, while nuclear bonds are the same for all nuclei.

E.4 A deuterium nucleus consists of one proton and one neutron. The mass of a deuterium nucleus $= 3.3435 \times 10^{-27}$ kg. What is the binding energy of deuterium in joules? The mass of a proton $= 1.6726 \times 10^{-27}$ kg and the mass of a neutron $= 1.6749 \times 10^{-27}$ kg.

    a) 3.3×10^{-27} J
    b) 3.01×10^{-10} J
    c) 3.6×10^{-10} J
    d) 6.02×10^{-10} J
    e) 2.75×10^{-8} J

E.5   When uranium-235 fissions, about 0.1 % of the uranium mass is converted to
       kinetic energy of the decay products. How many joules of energy are released
       from the fissioning of 1.0 kg of uranium-235 in a nuclear power plant?

       a) 4.5×10^5 J
       b) 4.5×10^{13} J
       c) 9×10^{13} J
       d) 9×10^{16} J
       e) 6.67×10^{18} J

E.6   How many kilowatt-hours of energy are produced by the fission reaction in
       exercise E.5?

       a) 2.5×10^2 kWh
       b) 6.67×10^5 kWh
       c) 2.5×10^7 kWh
       d) 3.2×10^{20} kWh
       e) 3.2×10^{23} kWh

Period 10 Review Questions

R.1   Carbon-14 dating cannot be used for objects older than about 70,000 years.
       Why should this be true? (Hint: the half-life of C-14 is 5568 years.)

R.2   The half life of Ba-137 is about 2.6 minutes. If you took a count rate from the
       Ba-137 30 minutes (about 12 half lives) after it was extracted, could you
       estimate this elapsed time well from such counting rate data? Why or why not?

R.3   In class you used capacitor decay to model radioactive decay and graphed the
       count rate (voltage) per unit time. What was the shape of your graph? Some
       graphs leveled off at a count rate greater than zero. Why was this the case?

R.4   The amount of matter converted into energy in a chemical reaction is much
       smaller than the matter converted into energy in a nuclear reaction. Why is this?

R.5   You know the mass of a nucleus and the number of protons and neutrons that
       make up the nucleus. How would you find the binding energy of the nucleus?