

## Activity 9 Solutions: Mass and Energy

### 9.1 Einstein's Equation Applied to Physical and Chemical Changes

Your instructor will discuss Einstein's equation,  $E = Mc^2$ , which is probably the most important equation of the 20<sup>th</sup> century.

#### 1) Mass and Energy

What is the relationship between mass and energy?

**Mass (or matter) can be converted into energy, and energy can be converted into mass as illustrated by Einstein's equation  $E = Mc^2$ . The constant  $c$  is the speed of light,  $3 \times 10^8$  meters/second. The mass of an object can be increased by giving the object energy, and the mass of an object can be decreased by taking energy away from that object.**

#### 2) Einstein's Equation applied to physical changes: stored potential energy

Should adding energy to a spring by winding it change the mass of the spring?

- What is the mass of the free play radio in kilograms? **\_\_about 1.70 kg\_\_**
- Your instructor will turn the crank to wind the radio's spring. Does the mass of the radio appear to change after the spring is wound? **No**
- How much force is needed to turn the radio's handle at a constant speed? **\_13 N\_**
- Find the distance in **meters** that the end of the handle travels in when it is turned one revolution by measuring the radius of the extended crank handle.

**The handle radius = 0.1 meter. The distance the end of the handle travels is the circumference of the circle =  $2\pi (0.1 \text{ m}) = 0.63 \text{ meters}$**

- Calculate the amount of work done to turn the handle one revolution.

$$\mathbf{W = F D = 13 \text{ N} \times 0.63 \text{ m} = 8.17 \text{ J}}$$

- The work done to turn the radio handle equals the energy added to the radio's spring. Use your result from part f) and Einstein's equation to find the additional mass of the radio as a result of winding its spring.

**use  $M = E/c^2$  where  $E$  is the work you found in part e)**

$$\mathbf{M = 8.17 \text{ J} / (9 \times 10^{16} \text{ m}^2/\text{s}^2) = 9.1 \times 10^{-17} \text{ kg}}$$

- How would the following physical changes affect the energy and mass of a substance?

	<b>Energy Change</b>	<b>Mass Change</b>
1) A book is raised to a shelf.	<b>gravitational energy added</b>	<b>increase</b>
2) A spring is stretched.	<b>strain energy added</b>	<b>increase</b>
3) A car accelerates	<b>kinetic energy added</b>	<b>increase</b>
4) Liquid nitrogen boils.	<b>thermal energy added</b>	<b>increase</b>
5) Water is frozen into ice.	<b>thermal energy removed</b>	<b>decrease</b>

- Group Discussion Question: Would these changes in mass register on a scale? Why or why not?

### 3) Einstein's Equation applied to chemical changes

- a) What is the mass of a light stick in kilograms? **\_approx 0.22 kg\_**
- b) Break the light stick's vial. What type of energy is given off? **\_visible light\_** Do you detect any infrared radiation? **\_No\_**
- c) Examine a night light. What type of energy is given off? **\_visible light\_** Do you detect any infrared radiation? **\_No\_**
- d) Since the light stick and the night light give off similar radiation, we compare the brightness of the light stick to the brightness of two night lights. If the light stick's brightness is 10 times greater than the two night lights, what is the wattage of the light stick? (Hint: the wattage of the night lights is given on their back cover.) **\_1.0 watts\_**
- e) If the light stick radiates energy at this same rate for one hour, how many joules of energy have been emitted? (Hint: one watt = one joule/second)

$$1.0 \text{ J/s} \times 3,600 \text{ s} = 3,600 \text{ J}$$

- f) Calculate the mass loss due to this emitted energy.

$$M = E/c^2 = (3.6 \times 10^3 \text{ J}) / (3 \times 10^8 \text{ m/s})^2 = 4.0 \times 10^{-14} \text{ kg}$$

- g) What is the percent change in the mass of the light stick after it has been radiating for one hour?

$$\frac{\text{mass loss}}{\text{mass of light stick}} = \frac{\text{mass from part f)}}{0.22 \text{ kg}} = \frac{4.0 \times 10^{-14} \text{ kg}}{2.2 \times 10^{-2} \text{ kg}}$$

$$= 1.8 \times 10^{-12} = 1.8 \times 10^{-10} \%$$

- h) Measure the mass of the radiating light stick. \_\_\_\_\_ Is there a noticeable change in mass? **\_no\_** Note: we will measure the mass of the light stick again at the end of this period after it has radiated for a longer time.

### 4) The limits of mass measurement

Watch the video clip of the mass of a pencil dot.

- a) What was the mass of the paper before the "i" was dotted? **\_97.239 mg\_**
- b) What was the mass of the added dot? **\_0.162 mg or 1.62 x 10<sup>-7</sup> kg\_**
- c) What was the percent change in the mass of the paper after the dot was added?

$$\frac{0.162 \text{ mg}}{97.239 \text{ mg}} = 0.00167 = 0.167 \%$$

- d) What was the change in mass in units of energy?

$$E = M c^2 = (1.62 \times 10^{-7} \text{ kg}) \times (3 \times 10^8 \text{ m/s})^2 = 1.46 \times 10^{10} \text{ J}$$

- e) Group Discussion Question: If the scale used in the video could measure one part in one million, would such a scale be helpful in measuring any change in mass of your light stick? Why or why not?

## 9.2 Nuclear Processes

### 5) Atoms and isotopes

- a) What determines which element an atom is?

**The number of protons in the atom's nucleus determines the identity of the atom.**

- b) What is an isotope?

**Isotopes are species of atoms of a given chemical element that have different numbers of neutrons (but the same number of protons). For instance, the most common isotope of carbon, carbon-12, has 6 protons and 6 neutrons, while a rarer isotope, carbon-14, has 6 protons and 8 neutrons.**

- c) How many protons are contained in one atom of Cobalt-60 ( ${}^{60}_{27}\text{Co}$ )?

**One atom has 27 protons. The lower left number indicates the number of protons.**

- d) How many neutrons are contained in one atoms of  ${}^{60}_{27}\text{Co}$ ?

**One atom has  $60 - 27 = 33$  neutrons. The upper left number indicates the total number of nucleons.**

### 6) Nuclear Processes

- a) How can an atom of one element change into an atom of a different element?

**Protons can be added to or removed from atomic nuclei. As we will see in Period 10, a neutron in the nucleus can be changed into a proton, or a proton can be changed into a neutron. Changing the number of protons within a nucleus changes the charge on the nucleus of the atom and it becomes an atom of a different element.**

- b) Is this process a physical, chemical, or nuclear change?

**Changing the number of protons is a nuclear change. Changes in the atomic nucleus involve nuclear reactions.**

- c) What is the difference between a physical change, a chemical reaction, and a nuclear reaction?

**In a physical change, molecules may change phase or rearrange, but no molecules are created or destroyed. In a chemical reaction molecules may be created or destroyed, but atoms are not created or destroyed. In a nuclear process, atoms may be created or destroyed, but nucleons (protons or neutrons) are not created or destroyed.**

### 9.3 Einstein's Equation Applied to Nuclear Processes

From Einstein's equation,  $E = Mc^2$ , we know that changing the energy of a substance changes its mass. We have found this mass change too small to measure for physical changes and chemical reactions. Next, we consider a situation where mass changes can be measured – changes to an atomic nucleus.

#### 7) Energy released when nuclei form

The mass of a  ${}^{238}_{92}\text{U}$  nucleus is  $395.2138 \times 10^{-27}$  kg. Follow the steps below to find the energy that binds together the nucleons of  ${}^{238}_{92}\text{U}$  into a nucleus.

- a) How many protons does the  ${}^{238}_{92}\text{U}$  nucleus have? **\_92 protons\_**  
Find the mass of these protons. (The mass of one proton =  $1.6726 \times 10^{-27}$  kg)  
**mass of 92 protons =  $92(1.6726 \times 10^{-27} \text{ kg}) = 153.8792 \times 10^{-27} \text{ kg}$**
- b) How many neutrons does the  ${}^{238}_{92}\text{U}$  nucleus have? **\_146 neutrons\_**  
Find the mass of these neutrons. (The mass of one neutron =  $1.6749 \times 10^{-27}$  kg.)  
**mass of 146 neutrons =  $146(1.6749 \times 10^{-27} \text{ kg}) = 244.5354 \times 10^{-27} \text{ kg}$**
- c) Find the total mass of these nucleons.

$$\begin{array}{r} 153.8792 \times 10^{-27} \text{ kg} \\ + 244.5354 \times 10^{-27} \text{ kg} \\ \hline 398.4146 \times 10^{-27} \text{ kg} \end{array}$$

- d) What is the difference in mass between the total mass you found in part c) and the mass of the U-238 nucleus, which is  $395.2138 \times 10^{-27}$  kg?

$$\begin{array}{r} 398.4146 \times 10^{-27} \text{ kg} \\ - 395.2138 \times 10^{-27} \text{ kg} \\ \hline 3.2008 \times 10^{-27} \text{ kg} \end{array}$$

- e) Calculate the energy in joules that was given off when the U-238 nucleus was formed.

$$\begin{aligned} \text{Energy} &= (\text{mass difference}) c^2 = (3.2008 \times 10^{-27} \text{ kg}) \times (3 \times 10^8 \text{ m/s})^2 \\ &= (3.2008 \times 10^{-27} \text{ kg}) \times (9 \times 10^{16} \text{ m}^2/\text{s}^2) = 2.8807 \times 10^{-10} \text{ J} \end{aligned}$$

### 9.4 The Binding Energy of Nuclei

#### 8) Binding energy

The energy given off when the uranium nucleus was formed is called the binding energy of the  ${}^{238}_{92}\text{U}$  nucleus.

- a) Find the binding energy *per nucleon* of a  ${}^{238}_{92}\text{U}$  nucleus.

$$\frac{2.8807 \times 10^{-10} \text{ J}}{238 \text{ nucleons}} = \frac{0.0121 \times 10^{-10} \text{ J}}{\text{nucleon}} = \frac{1.21 \times 10^{-12} \text{ J}}{\text{nucleon}}$$

- b) Convert the binding per nucleon from joules into units of megaelectron volts (MeV). (1 joule =  $6.25 \times 10^{12}$  MeV)

$$1.21 \times 10^{-12} \text{ joules} \times \frac{6.25 \times 10^{12} \text{ MeV}}{1 \text{ joule}} = 7.57 \text{ MeV}$$

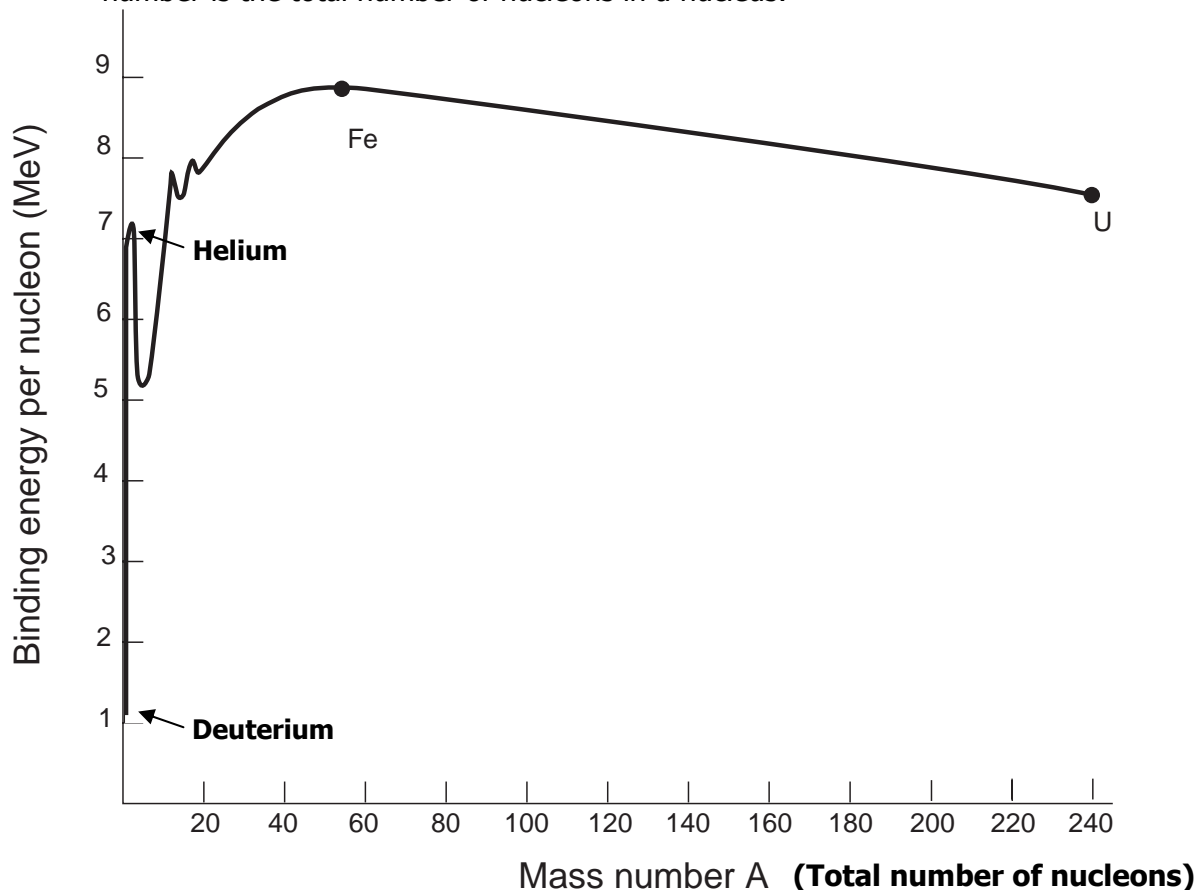
- c) When two protons come together, one proton can change into a neutron. The proton and neutron can then bind together to form one deuterium nucleus. For each deuterium nucleus formed,  $3.52 \times 10^{-13}$  joules of energy are given off. What is this energy called?

**the binding energy of the deuterium nucleus**

- d) What is the process of nucleons binding called? **\_fusion\_**
- e) Calculate the number of electron volts (eV) in  $3.52 \times 10^{-13}$  joules. (Hint:  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ ). **\_2.2 x 10<sup>6</sup> eV\_**
- f) How many megaelectron volts (MeV) is this? **\_2.2 MeV\_**
- g) How many MeV is this per nucleon? **\_1.1 MeV\_**

### 9) Graph of the binding energy of nuclei

The figure below graphs the binding energy per nucleon of the most stable nucleus of a given mass number (that is, the isotope of each element with the largest binding energy) versus the mass number of that nucleus. The mass number is the total number of nucleons in a nucleus.



- a) The leftmost point of the graph is deuterium ( ${}^2_1\text{H}$ ). What is the binding energy per nucleon of deuterium? **\_1.1 MeV\_**
- b) The point marked to the right of deuterium is helium ( ${}^4_2\text{He}$ ). What is the binding energy per nucleon of helium? **\_7.1 MeV\_**
- c) How much energy is given off *per nucleon* if two deuterium nuclei fuse to form one helium nucleus? **7.1 MeV – 1.1 MeV = 6.0 MeV per nucleon**
- d) How much energy is given off *per helium nucleus* formed?

**Since there are 4 nucleons in  ${}^4_2\text{He}$ , the total energy released is 4 nucleons times 6 MeV/nucleon = 24 MeV.**

## 9.5 Stable and Unstable Nuclei

### 10) Nuclei stability and the binding energy graph

- a) According to the binding energy graph, the nucleus of which element has the most binding energy? **\_iron (Fe)\_**
- b) Why is this nucleus also the most stable?  
**As shown on the graph, iron has the greatest binding energy per nucleon; therefore, iron is the most stable element.**
- c) Which is the least stable heavy element on this graph? Why is it the least stable?  
**Uranium (U) is the least stable because it has the least binding energy of the heavy nuclei.**

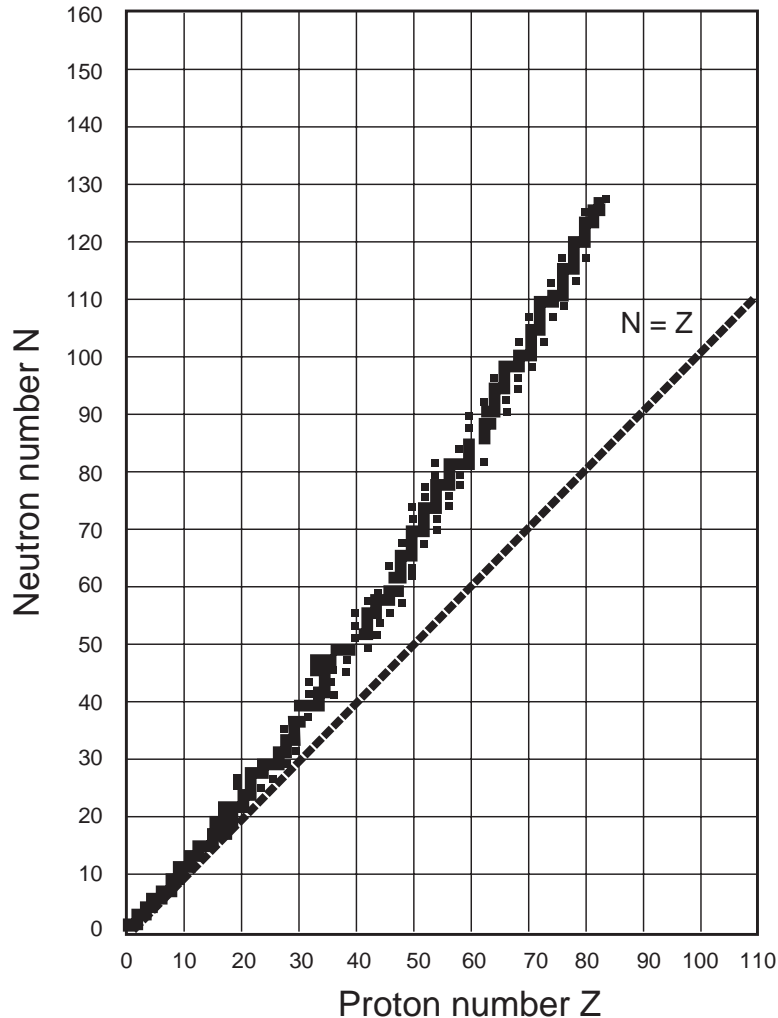
### 11) Graph of Stable Nuclei

The graph below shows the number of neutrons versus the number of protons for stable elements.

- a) Find the slope of the straight line. **\_slope = 1\_**
- b) What does the straight line represent?  
**Nuclei with equal numbers of protons and neutrons.**
- c) What does the group of points that form a curved line show?

**It shows nuclei with more neutrons than protons. These are the isotopes that are stable.**

### Graph of neutron number N versus proton number Z for stable nuclei



#### 12) Determining the stability of nuclei

a) What determines whether an element with 20 or fewer protons is stable?

**Small nuclei with equal numbers of protons and neutrons tend to be stable.**

i) Is the isotope  $^{14}_7\text{N}$  (nitrogen-14) stable? Why or why not?

**$^{14}_7\text{N}$  is stable because it is a small nucleus with equal numbers of protons and neutrons ( 7 protons and 7 neutrons).**

ii) Is the isotope  $^{14}_6\text{C}$  (carbon-14) stable? Why or why not?

**$^{14}_6\text{C}$  is unstable because it is a small nucleus with unequal numbers of protons and neutrons (6 protons and 8 neutrons).**

b) What determines whether an element with many more than 20 protons is stable?

**Large nuclei with more neutrons than protons tend to be stable.**

- c) What will happen to a nucleus with more than 83 protons?

**Nuclei with more than 83 protons are unstable and break into smaller nuclei. These nuclei are radioactive. As they decay into smaller nuclei, they give off radioactive particles, which will be discussed in the next period.**

### 13) The role of the weak nuclear force

- a) A deuterium nucleus can form from two protons. Why would you expect the process of forming a deuterium nucleus to require a large activation energy?

**The two positive protons repel one another. Energy is required to push the protons close enough together so that they are subject to the strong force.**

- b) A deuteron nucleus consists of a neutron and a proton, but the formation process started with two protons. What must have happened to one of the protons? One proton changed into a neutron

What force was involved in this process? the weak force

- c) What can happen to a nucleus with 83 or fewer protons that has too few neutrons?

**The weak nuclear force can change a proton into a neutron.**

- d) What can happen to a nucleus with 83 or fewer protons that has too many neutrons?

**The weak nuclear force can change a neutron into a proton.**

- e) How do the strong and the weak force combine to produce the stable isotopes?

**The weak nuclear force adjusts the relative numbers of neutrons and protons to produce stable nuclei. If a small nucleus contains too many neutrons or protons, the weak nuclear force changes one type of nucleon into the other.**

**The strong nuclear force binding the nucleus together operates over very short distances. Adding more neutral neutrons to large nuclei separates the positive protons, reducing the repulsive electromagnetic force among them.**

### 14) Enlarged graph of Stable Nuclei

The grid on the next page represents an enlargement of a portion of the graph shown in part 12).

- a) The stable isotopes of calcium are Calcium-40 ( ${}^{40}_{20}\text{Ca}$ ), Calcium-42 ( ${}^{42}_{20}\text{Ca}$ ), Calcium-43 ( ${}^{43}_{20}\text{Ca}$ ), Calcium-44 ( ${}^{44}_{20}\text{Ca}$ ), Calcium-46 ( ${}^{46}_{20}\text{Ca}$ ), and Calcium-48 ( ${}^{48}_{20}\text{Ca}$ ). Darken the squares on the grid that represent these stable isotopes.
- b) Mark an X in the square that represents the nucleus of Silicon-40 ( ${}^{40}_{14}\text{Si}$ )

- c) Silicon-40 nuclei last only a few seconds. Why is this nucleus unstable?

**It has too many neutrons for the number of protons.**

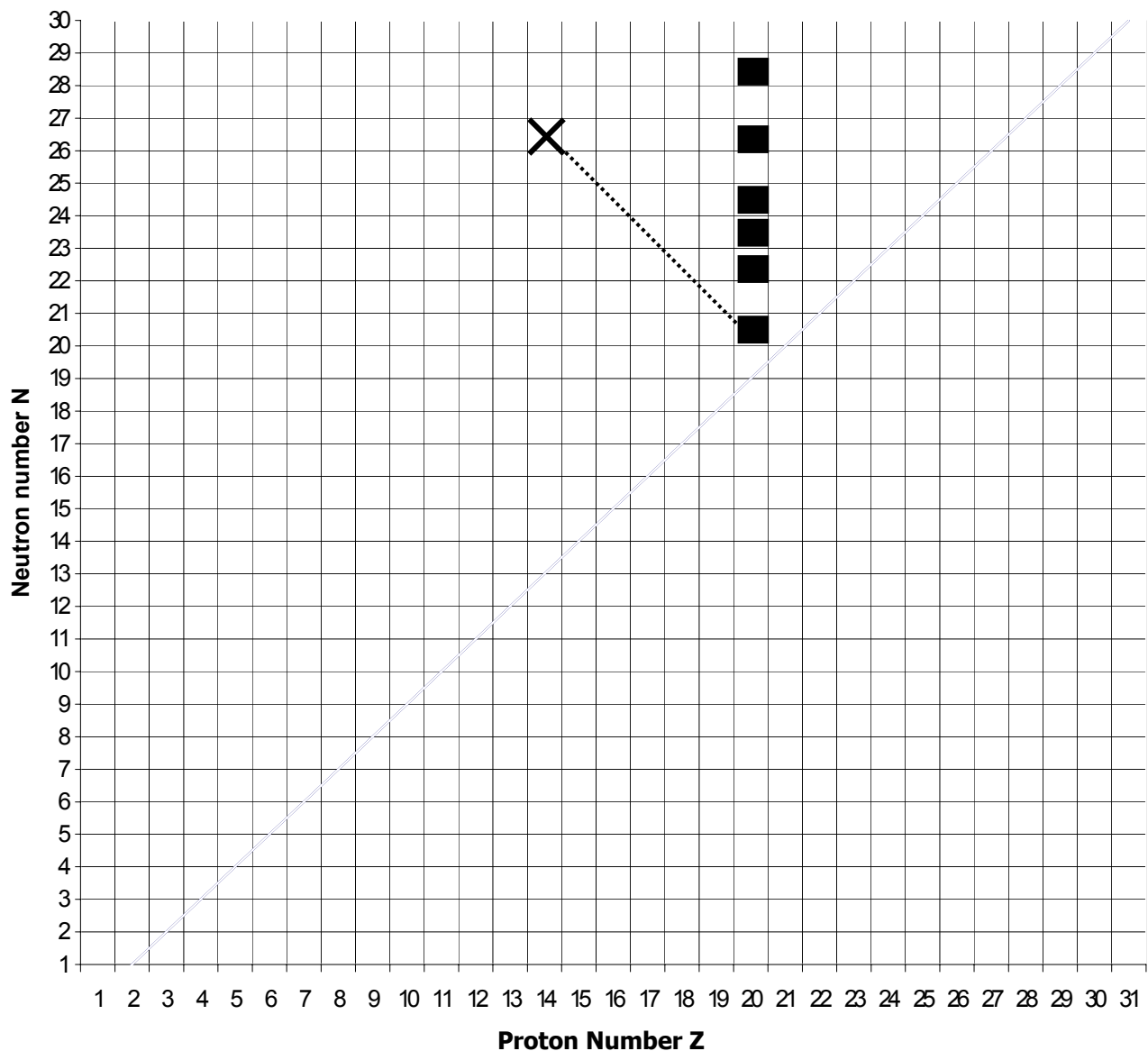
- d) How would this nucleus change as it becomes more stable?

**Neutrons would change into protons.**

- e) How many neutrons must change into protons for this nucleus to become the stable nucleus Calcium-40 ( ${}^{40}_{20}\text{Ca}$ )?

**Six neutrons must become protons.**

- f) Draw a dotted line showing how the box representing Silicon-40 would move as the nucleus turns into a stable nucleus.



**15) Mass of the Light Stick**

- a) Now that the light stick used in part 3) has been radiating energy for approximately an hour, measure the mass of the stick again. Is it possible to measure a change in the mass of the stick with this scale?  
**\_No\_**

Why or why not?

**The light stick represents a chemical reaction. The loss in its mass due to the energy radiated from it is too small to measure.**

- b) Group Discussion Question: When hydrogen and oxygen atoms combine in a fuel cell to form a water molecule, the chemical binding energy given off is approximately 2.5 eV per molecule. When a proton and a neutron combine to form a nucleus of deuterium, the nuclear binding energy given off is  $2.2 \times 10^6$  eV per nucleus.

Why is it that we cannot measure a change in mass in chemical reactions but we can measure a change in mass in nuclear processes? (Hint: approximately how many times greater is the binding energy of a deuterium nucleus than that of a water molecule?)