Introduction to the World of Energy

1.1 Ratios and "per"

How can ratios simplify problem solving?

How are ratios used to find efficiency?

1.2 Exponents and Scientific Notation

Why is scientific notation helpful?

How does it work?

1.3 Energy Sources

How much energy do equal amounts of fuel contain?

1.4 Linear and Exponential Growth

What are the differences between linear and exponential growth rates?

How do graphs illustrate these rates?

Activity 1.1.a: Ratios and "per"

- **Ratios are fractions, such as 60 miles/1hour (60 miles per hour)**
- **Ratios are useful when making comparisons.**

Example:

A truck requires 3 liters of gasoline to travel 15 kilometers. How many kilometers can the truck go on 1 liter of gas?

Solution:

Write the information as a ratio. Simplify the ratio by dividing the numerator by the denominator.

$$\frac{15\text{ km}}{3\text{ liters}} = \frac{5\text{ km}}{\text{liter}}$$

Activity 1.1.c: Ratios and Units

How can ratios help solve problems?

- **Ratios can be used to convert quantities from one unit to another. (For example miles per hour to meters per second.)**
- **The equality 1 hour = 60 min can be written:**

$$\frac{1\ \text{hour}}{60\ \text{min}} \quad \text{or} \quad \frac{60\ \text{min}}{1\ \text{hour}}$$

- **Choose the ratio that allows you to cancel the unwanted units.**

Convert 60 miles/1hour into miles/minute:

$$\frac{60\ \text{miles}}{1\ \cancel{\text{hour}}} \times \frac{1\ \cancel{\text{hour}}}{60\ \text{min}} = \frac{60\ \text{miles}}{60\ \text{min}} = \frac{1\ \text{mile}}{1\ \text{min}}$$

(Example 1.3)

There are 1,609 meters per 1 mile. Use ratios to convert 60 miles per hour into meters per second.

$$\frac{60\ \cancel{\text{miles}}}{1\ \cancel{\text{hour}}} \times \frac{1{,}609\ \text{meters}}{1\ \cancel{\text{mile}}} \times \frac{1\ \cancel{\text{hour}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{sec}} = \frac{27\ \text{meters}}{1\ \text{sec}}$$

Act 1.1.d: Uses of Ratios

Ratios often involve the amount of a quantity per unit of time:

Miles / hour
Heart beats / minute

Other common ratios involve the cost of an item per unit of the item.

Cost of gasoline / gallon
Cost of electricity / kilowatt hour

Ratios allow us to make comparisons more easily.

By looking at the first two rows of data in the table, you cannot easily determine which vehicle gets the best gas mileage.

Type of vehicle	**Minivan**	**School Bus**	**Truck**
Miles driven	218	1,089	37
Gas used	15 gal	85 gal	2.8 gal
Miles per gal			

When you use a ratio to calculate the miles per gallon, the comparison is simple.

Act 1.2: Using Ratios to Find Efficiency

Efficiency is a ratio of the useful energy out of the system per total energy put into the system.

$$\text{Efficiency} = \frac{\text{Useful Energy Out}}{\text{Total Energy In}}$$

Example 1.2

What is the efficiency of an energy conversion that requires 600 joules of energy to produce 200 joules of useful energy?

$$\frac{\text{Useful Energy Out}}{\text{Total Energy In}} = \frac{200\ \cancel{\text{joules}}}{600\ \cancel{\text{joules}}} = 0.33 = 33\ \%$$

- What happens to the other 400 joules of energy?
- Is it possible to get more energy out of a system than you put in?

Rules for Using Numbers with Exponents

1. When **multiplying** numbers with exponents, **add** the exponents

   $10^{A} \times 10^{B} = 10^{(A + B)}$

   $10^{2} \times 10^{-3} = 10^{(2 + (-3))} = 10^{-1} = 1/10 = 0.1$

2. When **dividing** numbers with exponents, **subtract** the exponents

   $10^{A} / 10^{B} = 10^{(A - B)}$

   $10^{2} / 10^{-3} = 10^{(2 - (-3))} = 10^{5} = 100{,}000$

3. When raising numbers with an exponent to a power, multiply the exponents.

   $(10^{A})^{B} = 10^{(A \times B)}$

   $(10^{3})^{2} = 10^{(2 \times 3)} = 10^{6} = 1{,}000{,}000$

4. Any number to the zero power = 1:

   $10^{0} = 1 \quad 237^{0} = 1$

Activity 1.3: Scientific Notation

- **Scientific notation uses the base 10 raised to an exponent.**
- **The exponent shows the number of times that 10 is multiplied by itself.**

$$10^{1} = 10$$

$$10^{2} = 10 \times 10 = 100$$

$$10^{3} = 10 \times 10 \times 10 = 1{,}000$$

$$10^{-1} = 1/10 = 0.1$$

$$10^{-2} = 1/(10 \times 10) = 0.01$$

$$10^{-3} = 1/(10 \times 10 \times 10) = 0.001$$

1) For numbers equal to or greater than one (positive exponents), count the places the decimal point is shifted to the left.

$$2{,}600.0 = 2.6 \times 10^{3}$$

2) For numbers less than one (negative exponents), count the number of places the decimal point is shifted to the right.

$$0.035 = 3.5 \times 10^{-2}$$

Standard Prefixes Denoting Multiples of Ten

Prefix	Symbol	Factor
quad (quadrillion)		10^{15}
tera (trillion)		10^{12}
giga (billion)	G	10^{9}
mega (million)	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

1 kilogram = 10^3 grams = (10 x 10 x 10) grams = 1,000 grams

1 centimeter = 10^{-2} meters = 1/(10 x 10) m = 0.01 meters

Scientific Notation and Calculators

1. To enter a number in scientific notation, press the 10^X key and enter the exponent.

2. If the 10^X symbol is above a key, press [2^{nd} F] before pressing the 10^X key.

   To enter 8×10^{12}, press [8] [X] [10^X] [1] [2]

   To enter 3×10^{-6}, press [3] [X] [10^X] [+ / -] [6]

3. Some calculators use reverse notation. The exponent is entered before the 10^X key is pressed.

To enter 3×10^{-6}, press [3] [X] [6] [+ / -] [10^X] [=]

The TI-25X solar calculators use reverse notation.

4. If your calculator has an EE or EXP key, press that key and then enter the exponent.

To enter 3×10^{-6}, press [3] [EE] or [EXP] and [+ / -] [6]

5. A calculator's [y^X] key does NOT give powers of 10.
   For example, 3.4^8 is NOT the same as 3.4×10^8

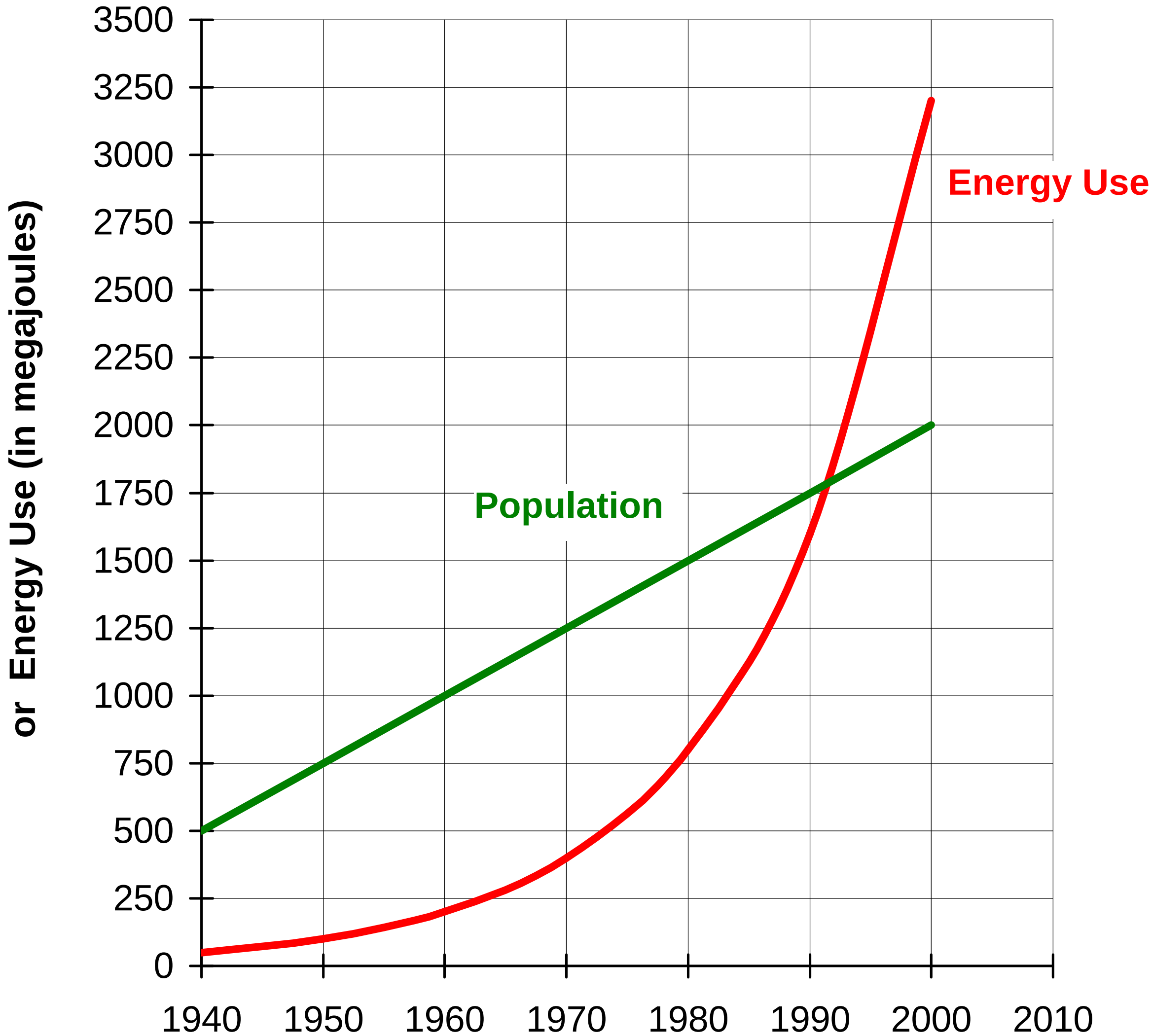
Act. 1.3.d: Energy Content of Fuels

Type of Fuel	Energy in joules/kg of fuel
Coal (bituminous and anthracite)	2.9×10^{7}
Crude Oil	4.3×10^{7}
Gasoline	4.4×10^{7}
Natural Gas	5.5×10^{7}
Wood	1.4×10^{7}
Assorted Garbage and Trash	1.2×10^{7}
Bread	1.0×10^{7}
Butter	3.3×10^{7}
Nuclear fission with Uranium 235	$8 \times 10^{13} =$ $8{,}000{,}000 \times 10^{7}$

Data from *Energy: An Introduction to Physics*
by R.H. Romer, page 583.

Act 1.4: Linear and Exponential Graphs

Fig. 1 Energy Use and Population Growth



Which graph represents exponential growth?

Which represents linear growth?

Linear Growth

- Linear growth is constant. Its graph is a straight line.
- The same amount is added during each time period.
- The amount added is independent of the initial amount.
- The amount added is independent of the number of elapsed time periods.

Linear Population Data

Year	Population	Year	Population
1940	500	1980	1,500
1950	750	1990	1,750
1960	1,000	2000	2,000
1970	1,250	2010	?

Exponential Growth

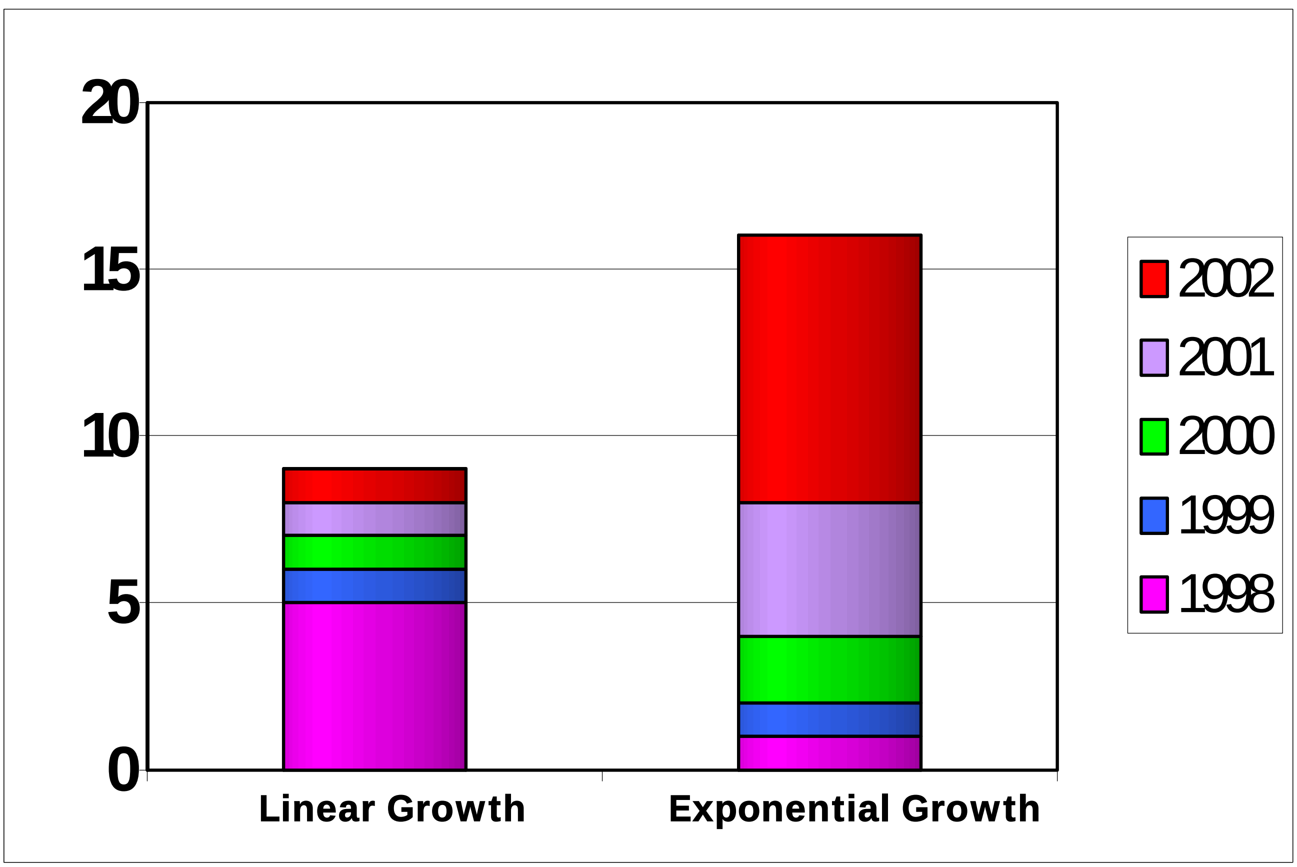
- Exponential growth is not constant. Its graph is an upward curving line.
- The amount added changes with each time period.
- Exponential growth doubles the amount of the quantity during a fixed time period.
- The amount added depends on the initial amount and on the number of time periods.
- The doubling time is the length of time required for the quantity to double.

Exponential Energy Use Data

Year	Energy (in MJ)	Year	Energy (in MJ)
1940	50	1980	800
1950	100	1990	1,600
1960	200	2000	3,200
1970	400	2010	?

Linear and Exponential Growth

Year	Linear increase	Exponential increase
1998	5	1
1999	6	2
2000	7	4
2001	8	8
2002	9	16



Finding the Slope of a Linear Equation

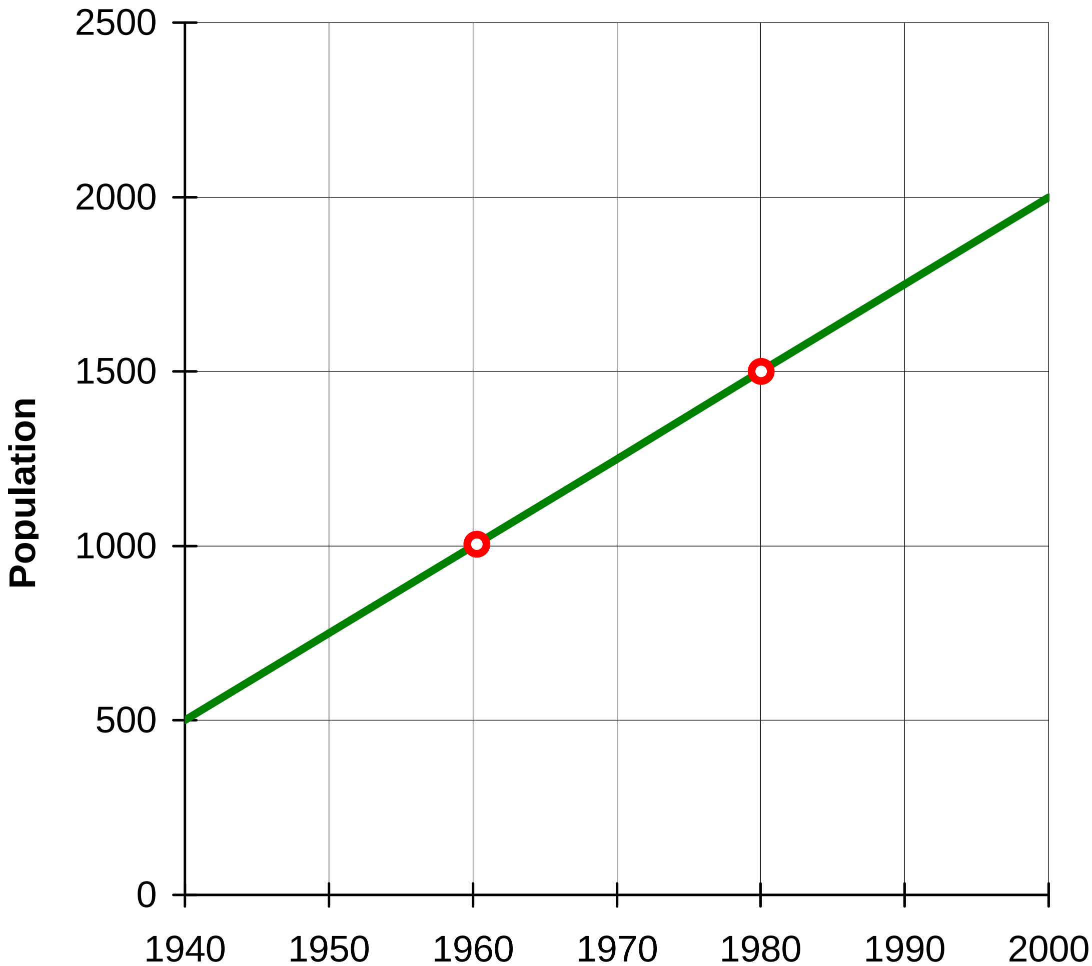
1) Pick any two points on the line.

2) Slope = vertical distance between the points / horizontal distance between the points

3) Using the points circled on the graph,

$$\text{slope} = \frac{1{,}500 - 1{,}000}{1980 - 1960} = \frac{500 \text{ people}}{20 \text{ years}}$$

$$= 25 \text{ people/year}$$



Linear and Exponential Equations

Linear growth is expressed by

$$N = A \times t + B$$

Exponential growth is expressed by

$$N = B \times 2^{t}$$

where N = the amount of the quantity

A = the amount of increase per time period

B = the initial amount

t = the number of time periods elapsed

(We assume there is one doubling per each elapsed time period.)

Act 1.4.f: Group Discussion Question:

If someone gave you $1 and offered to double the amount you have every day, how much would you have on day 7 ?

Day	Amount
1	$1
2	$2
3	$4
4	$8
5	$16
6	$32
7	$64

$$N = B \times 2^{t} = \$1 \times 2^{6} = \$64$$

B = the initial amount = $1

t = the number of time periods elapsed = 6

How much would you have on day 30?

$$N = B \times 2^{t} = \$1 \times 2^{29} = \$\ 536{,}870{,}912$$

Table 1.5: Growth Rates and Doubling Times

Annual Growth Rate (in percent)	Doubling Time (in years)		Annual Growth Rate (in percent)	Doubling Time (in years)
0	Infinite		20	3.8
1	69.7		30	2.6
2	35.0		40	2.1
3	23.4		50	1.7
4	17.7		60	1.5
5	14.2		70	1.3
6	11.9		80	1.2
7	10.2		90	1.1
8	9.0		100	1.0
9	8.0		200	0.6
10	7.3		300	0.5
12	6.1		400	0.4
14	5.3		900	0.3
16	4.7		9900	0.15
18	4.2			

a) If you invest $1,000 at 10% interest compounded annually, how long will it take for your money to double to $2,000?

b) If a stock doubles in value every 2.1 years, what is its rate of growth?

Summary of Period 1

1.2 The concept of *per* is represented by a ratio: one quantity divided by another.

When converting units, use ratios that allow you to cancel the unwanted units.

The efficiency of a system is the ratio of the useful energy out/total energy in

1.3 Powers of 10 simplify calculations with large or small numbers.

When multiplying, add exponents.

When dividing, subtract exponents.

1.4 The energy content of fuels can be expressed as the ratio of the joules of energy per kilogram of fuel.

1.5 Linear growth adds a constant amount of the quantity during each time period.

With exponential growth, the amount added depends on the initial amount and on the number of elapsed time periods.

Summary of Period 1, continued

Exponential growth doubles the amount of the quantity in a fixed time period.

The doubling time is the length of time required for the amount of a quantity to double.

Linear growth is expressed by

$$N = A \times t + B$$

Exponential growth is expressed by

$$N = B \times 2^{t}$$

where N = the amount of the quantity

A = the amount of increase per time period

B = the initial amount

t = the number of time periods elapsed

Period 1 Review Questions

R.1 When using ratios to convert a quantity from one unit to another, how do you decide which value to put in the numerator and which in the denominator of the ratio? Explain your answer with an example.

R.2 State the rules of exponents you used to find the answer to exercise E.3.

R.3 What is another name for:

a) 2×10^{-2} meters ?

b) 6×10^{-6} seconds ?

c) 4×10^{3} grams ?

R.4 Explain how to tell whether a graph line exhibits linear or exponential growth rates. Does every growth rate fit into one of these two types?

R.5 What determines the amount added during each time period to a quantity that is growing linearly?

R.6 What determines the amount added during each time period to a quantity that is growing exponentially?

R.7 What is the doubling time of a quantity? How long will it take a stock, which increases in value at a rate of 10% per year, to double in value?