PHYSICS 828

Home Work Assignment # 1 1/7/2011

Due: Fri., Jan. 14, 2011.

Completed assignments should be placed in the grader N. Ramalingam's mail box in PRB by 5:00 PM.

1. In this problem you will learn about **coherent states** of the quantum harmonic oscillator and explore some of their properties. These states are of great importance in quantum optics, lasers and in many areas of condensed matter physics. A *coherent state* is defined by

$$|z\rangle = C_z \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}} |n\rangle,$$

where z is any complex number, and $|n\rangle$ is the eigenstate of the number operator $N = a^\dagger a$ with eigenvalue n, and C_z is a normalization constant that you will determine below.

(a) Show that $|z\rangle$ is a (right) eigenstate, i.e., an eigenket, of the destruction operator a and find the corresponding eigenvalue.

(b) Show that the normalization is $C_z = \exp\left(-|z|^2/2\right)$

(c) Can you find a (right) eigenstate, or eigenket, of the creation operator $a^\dagger$? If so, find it. If not, explain why not.

(d) Find $\langle z|w\rangle$ for complex z and w. Are the coherent states orthogonal?

(e) Show that the set of all coherent states $\{|z\rangle\}$ is complete basis set by demonstrating the resolution of the identity:

$$\frac{1}{\pi} \int d^2z |z\rangle\langle z| = \widehat{\mathbf{1}}.$$

where $d^2z = dx\, dy$ with $z = x + iy$.

[If you are looking for a challenge (that is optional and will not be graded): Show that the set of all coherent states is overcomplete, i.e., not all $|z\rangle$ are linearly independent.]

(f) Find the mean number $\langle z|N|z\rangle$ and the fluctuation $\langle z|N^2|z\rangle$.

(g) Calculate $\langle z_0|X|z_0\rangle$, $\langle z_0|P|z_0\rangle$ where $z_0 = x_0+iy_0$. Find the uncertainties δX and δP in the coherent state $|z_0\rangle$. What is the product $(\delta X)(\delta P)$?

(h) Find the time evolution of a coherent state: $|\psi_z(t)\rangle = \exp(-iHt/\hbar)\,|\psi_z(0)\rangle$, where H is the harmonic oscillator Hamiltonian, and the initial state $|\psi_z(0)\rangle = |z_0\rangle$ is a coherent state with $z_0 = \rho_0 \exp(i\theta_0)$.
Show that $|\psi_z(t)\rangle$ is also a coherent state but with a time-dependent "z".

(i) Find the time evolution of the expectation values of $\langle X\rangle(t) = \langle\psi_z(t)|X|\psi_z(t)\rangle$, $\langle P\rangle(t)$ and $\langle H\rangle(t)$. Compare your results with that for a *classical* harmonic oscillator.

2. Shankar Ex. 10.2.1 (p. 259)

3. Shankar Ex. 10.2.2 (p. 259-260)

4. Shankar Ex. 10.2.3 (p. 260)

5. Consider the states of a system of two spin-1/2 particles, where we use the notation

$$|\sigma_1\sigma_2\rangle = |\sigma_1\rangle \otimes |\sigma_2\rangle$$

with each $\sigma_i =\uparrow$ or $\downarrow$. Show that

$$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$$

is an *entangled* state, but

$$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$$

is *not* entangled.

6. Read Shankar Ex. 10.1.2 (p. 251-252) to make sure you understand the question and the answer he asks you to prove. You do *not* need to hand in the solution.