Lecture notes 1

Standard physics vs. new physics

The black hole information paradox has challenged our fundamental beliefs about spacetime and quantum theory. Which belief will have to change to resolve the paradox?

There are two kinds of changes that one can have:

(i) Our fundamental principles of physics remain unchanged. But a somewhat unexpected phenomenon new phenomenon is discovered in the theory, and this resolves the puzzle.

(ii) When black holes form then we trigger 'new physics'. This new physics could represent changes in fundamental principles, or could take the form of new interactions that are not contained in our current knowledge of particle physics of quantum gravity.

We have seen that the fuzzball resolution of the information paradox falls in category (i). The new phenomenon was that bound state is string theory did not have a fixed size $\sim l_p$. Instead the size grew with the number of quanta N in the state, in such a way that the size was always of order the horizon radius. This invalidated the tradition picture of the hole, and resolved the paradox.

In this section we will briefly recall some alternative proposals for resolving the information paradox. These will for the most part fall in category which fall in category (ii); i.e., they postulate some new physics which becomes relevant when black holes are formed.

1.1 The final state boundary condition

All the laws of nature that we know today have a common underlying form. We have to specify a choice of *initial conditions*, and then the laws of physics determine the future evolution of the system. In classical mechanics the initial conditions can be a particle's position and velocity at some $t = t_i$, in quantum mechanics it could be a wavefunction $\psi(x, t_i)$, and in field theory it could be a wavefunctional $\psi[\phi(x)]$ at t_i.

The information paradox arises when we study such an evolution in a black hole geometry; the vacuum region on an initial slice evolves to a state containing entangled pairs. Horowitz and Maldacena proposed that we modify the rules of physics in a basic way: at a black hole singularity, we impose a 'final state boundary condition'. They suggest that as our spatial slice evolves towards a singularity, the state on it is not determined by the initial state $\psi_i\rangle$ on the slice but by the requirement that the state near the black hole singularity take a definite value $|\psi_{BH}\rangle$. By choosing a fixed state at the singularity we ensure that no information actually falls into the singularity, and as a result the information gets forcibly encoded in the radiation at infinity.

To see the essence of this model, let us recall that we have three kinds of matter on our spacelike slice:

(a) The initial matter that falls in to make the black hole. Let us call this matter A. We model it as a bit that can take the values $0, 1$.

(b) The Hawking particles that escape to infinity. Let us call this set of particles B. They are also modelled by a bit with values $0, 1$.

(c) The Hawking particles that fall into the hole; we call these C, and they are also bits with values $0, 1$.

The Hawking pairs are entangled as

$$\frac{1}{\sqrt{2}}\left(|0\rangle_b|0\rangle_c + |1\rangle_b|1\rangle_c\right) \tag{1.1}$$

Suppose the infalling matter A is a bit in the state $\alpha|0\rangle_a + \beta|1\rangle_a$. Then the overall state on the slice is

$$\begin{aligned}
|\Psi\rangle &= (\alpha|0\rangle_a + \beta|1\rangle_a) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle_b|0\rangle_c + |1\rangle_b|1\rangle_c\right) \\
&= \left(\frac{1}{\sqrt{2}}(\alpha|0\rangle_a + \beta|1\rangle_a)|0\rangle_c\right) \otimes |0\rangle_b + \left(\frac{1}{\sqrt{2}}(\alpha|0\rangle_a + \beta|1\rangle_a)|1\rangle_c\right) \otimes |1\rangle_b \\
&\equiv |\chi_1\rangle \otimes |0\rangle_b + |\chi_2\rangle \otimes |1\rangle_b
\end{aligned} \tag{1.2}$$

where we have rewritten the state in a form that will be useful below. . At this stage we have the usual problem: the information of A is not in the radiation, and the radiation B is entangled with the quanta C in the hole. Now suppose we require that the state at the singularity has the form

$$|\psi\rangle_{BH} = \frac{1}{\sqrt{2}}\left(|0\rangle_a|0\rangle_c + |1\rangle_a|1\rangle_c\right) \tag{1.3}$$

We postulate that the state outside the hole is given by the projection

$$\begin{aligned}
|\psi\rangle_B &= {}_{BH}\langle\psi|\Psi\rangle \\
&= {}_{BH}\langle\psi|\chi_1\rangle|0\rangle_b + {}_{BH}\langle\psi|\chi_2\rangle|1\rangle_b \\
&= \frac{\alpha}{2}|0\rangle_b + \frac{\beta}{2}|1\rangle_b = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(\alpha|0\rangle_a + \beta|1\rangle_a)\right)
\end{aligned} \tag{1.4}$$

We see the state $|\psi\rangle_B$ is not correctly normalized, but this is expected if we make a projection. But apart from this normalization, the state B agrees with the state of the initial matter A, so the information that fell in the hole has come out. Further, the black hole has been left in a unique state $|\psi\rangle_{BH}$, so it is not entangled with anything outside.

This approach solves the problem 'by construction'; we have defined the evolution so that the information would be transferred to the radiation. But the price we have paid is that we have lost causality. Information must move faster than the speed of light, and in a nonlocal way, so our theory has 'quantum teleportation' across macroscopic spacelike intervals.

Given this price, what is the motivation to require a 'final state boundary condition'? The issue of initial conditions arises in a fundamental way when we study the Universe. As we follow our cosmology back to the big bang, we are faced with the question: what sets the initial conditions to start off the evolution? It would be very interesting if this initial condition was somehow unique; if the initial condition were not unique, then there would always be features of the Universe that cannot be derived even with a complete theory of all fundamental laws.

Hartle and Hawking had proposed a mechanism which would select a special initial condition for the Universe. Suppose we rotate time to Euclidean signature: $t \to -i\tau$. Then the quantum evolution of states changes from $\sim Exp[-iEt]$ to $\sim Exp[-E\tau]$. In a normal field theory, we can imagine carrying out this evolution for a large time Δtau. Then the evolution selects out the state with the lowest value of E – the vacuum $|0\rangle$. Thus we can make $|0\rangle$ a natural initial condition at $t = t_0$ if we assume that at times $t < t_0$ the evolution was Euclidean, and persisted for an infinitely long time.

With gravity, energy is not naturally positive definite, since gravitational potential energy is negative. Thus it is not clear what the 'lowest energy state' would be. But with general relativity, we are allowed a choice of topologies for our spacetime, which allows a slightly modified form of the above argument. Suppose we wish to fix the initial condition of the Universe at $t = t_0$. Then for times $t < t_0$ we rotate time to Euclidean signature $t \to -i\tau$, so that spacetime is a 4-dimensional Euclidean manifold. We can now choose this manifold to smoothly end as we go towards the past, as shown in fig.**??**. We perform a path integral over all shapes of this Euclidean manifold, holding its boundary at $\tau = -it_0$ fixed. This generates a natural state at $\tau = -it_0$, which is then the special initial state at $t = t_0$ for the future evolution in Lorentzian signature.

If we can have a unique initial condition at the big bang, then perhaps we should also have a unique final condition at a 'big crunch'; i.e. at a place where the Universe collapses in the future to a point. But such a condition at the crunch would be a 'final state boundary condition'. Returning to our black hole, we see that the black hole singularity is somewhat similar to a big crunch, with the difference that only a part of the spacelike slice in the black hole meets the singularity. One is therefore led to suggest a final state boundary condition at the black hole singularity, as in the proposal of Horowitz and Maldacena.

The difficulty with this proposal is that in we have not found any evidence for final state boundary conditions in string theory. Even though this is a very complicated theory of quantum gravity, it seems to respect the traditional structure of physical laws: initial conditions can be given for string fields, and the future evolution is determined, in a manner that respects locality and causality. One may argue that we have not studied string fields in detail in the domain where we make black holes. But if we have a fundamental change to a physical principle like the freedom of initial condition, one might expect that there would be *some* indication of this change, perhaps through very small corrections to processes that do not involve black holes. We have not found such indirect evidence so far, and so this proposal seems to invoke physics that has not yet been found in a well defined theory of quantum gravity.

1.2 Nonlocal interactions

An essential feature of the information paradox is the causal structure created by the horizon. Inside the horizon, light cones point inwards, so that all timelike or null trajectories must move towards smaller r. Thus nothing can move from inside the horizon to the outside without violating causality. This traps the information of the initial matter inside the hole.

Giddings has postulated that there may exist nonlocal interactions in the theory which transport information acausally across the horizon. Suppose we decompose the degrees of freedom of the theory into three sets:

$$H = H_{bh} \otimes H_{near} \otimes H_{far} \tag{1.5}$$

where H_{bh} is the Hilbert space inside the hole, H_{near} is the Hilbert space of degrees of freedom outside the horizon but still near the hole (say upto $r = 10M$), and H_{far} is the Hilbert space far from the hole. We can now postulate a new interaction that transfers quanta from inside the hole to the outside, of the form

$$\hat{H}_{trans} \sim \frac{1}{r_h}\hat{a}^\dagger_{near}\mathcal{N}\hat{a}_{bh} \ + \ \frac{1}{r_h}\hat{a}^\dagger_{bh}\mathcal{N}^\dagger\hat{a}_{near} \tag{1.6}$$

where N is a matrix which can take a mode inside the horizon and convert it to a mode outside the hole.

This postulated interaction is ‘new physics’, so let us see what constraints we would like to put on it:

(i) This interaction should be triggered only when we have a black hole. The horizon radius r_h appears as a coefficient in the interaction terms.

(ii) We would still like to retain out classical intuition that an observer falling into the hole sees nothing unusual at the horizon. Thus the matrix $\mathcal{N}$ is chosen to affect only modes with wavelength $\lambda \sim T^{-1}$, where T is the temperature of the hole. Thus we modify the Hawking radiation in a way that in can carry information out of the hole, but we do not affect the infall of particles which would typically have $E >> T$.

Let us compare this approach with what we have learnt from the fuzzball scenario:

(a) This approach requires a violation of locality and causality, and invokes interactions which have not been found yet in string theory. In the fuzzball scenario, on the other hand, we just use the known properties of string theory, which is local and causal.

(b) One consequence of the interaction (1.6) is that the rate of emission from the hole becomes more than the rate given by Hawking's computation. This is because the Hawking rate arises from the evolution of the vacuum. If we add any interaction that creates extra excitations around the horizon, then the energy carried by these excitations is positive, and leads to an additional positive flow of energy from the hole. By contrast, we have seen that the radiation rate from the few nonextremal fuzzballs that have been constructed matches exactly onto the radiation rate expected for Hawking radiation from the hole.

Bibliography