Thermodynamics

The goal of thermodynamics is to understand how heat can be converted to work


Main lesson:
Not all the heat energy can be converted to mechanical energy
This is because heat energy comes with disorder (entropy), and overall disorder cannot decrease

Temperature


Random directions of velocity

Higher temperature means higher velocities $v$

Add up energy of all molecules: Internal Energy of gas $U$

Mechanical energy: all atoms move in the same direction
$\frac{1}{2} M v^{2}$

## Statistical mechanics

For one atom
$\langle E\rangle=\left\langle\frac{1}{2} m v_{x}^{2}\right\rangle+\left\langle\frac{1}{2} m v_{y}^{2}\right\rangle+\left\langle\frac{1}{2} m v_{z}^{2}\right\rangle=\frac{1}{2} k T+\frac{1}{2} k T+\frac{1}{2} k T=\frac{3}{2} k T$

Ideal gas: No Potential energy from attraction between atoms

$$
\langle U\rangle=\frac{3}{2} N k T
$$



Pressure

Pressure is caused because atoms bounce off the wall

$$
\Delta p_{x}=2 p_{x}
$$

$$
\Delta t=\frac{2 L_{x}}{v_{x}}
$$

$$
F=\frac{\Delta p_{x}}{\Delta t}=\frac{2 m v_{x}^{2}}{2 L_{x}}=\frac{m v_{x}^{2}}{L_{x}}
$$

$$
\begin{aligned}
& F=\frac{\Delta p_{x}}{\Delta t}=\frac{2 m v_{x}^{2}}{2 L_{x}}=\frac{m v_{x}^{2}}{L_{x}} \\
& \left\langle m v_{x}^{2}\right\rangle=2 \times \frac{1}{2} k T=k T \\
& F=\frac{k T}{L_{x}}
\end{aligned}
$$

## Pressure

$$
P=\frac{F}{A}=\frac{k T}{L_{x}} \frac{1}{L_{y} L_{z}}=\frac{k T}{V}
$$

Many particles $\quad P=\frac{N k T}{V}$

$$
P V=N k T
$$


$A=L_{y} L_{z}$

Volume $V=L_{x} L_{y} L_{z}$

Work


Work done BY the gas $\Delta W=F \Delta L_{x}$

We can write this as $\quad \Delta W=(P A) \Delta L_{x}=P \Delta V$
This is useful because the body could have a generic shape


Internal energy of gas decreases $\quad U \rightarrow U-\Delta W$

$$
d W=P d V
$$



Gas expands, work is done BY the gas
Work done is Area under curve


Gas is pushed in, work is done ON the gas Work done is negative of Area under curve


By convention, we use POSITIVE sign for work done BY the gas

Getting work from Heat
Gas expands, work is done BY the gas



Volume in increases
Internal energy decreases ... this means Temperature decreases
Pressure decreases

But how do we get the engine back to its initial state?


We will have to put in exactly as much work as we got out ... so this does not help

The heat engine

Add heat, so pressure goes back up


## Two relations


(1) Net work done equal Area inside cycle
(2) Net work done equals net heat added (since engine returns to original state)

37. A constant amount of an ideal gas undergoes the cyclic process $A B C A$ in the $P V$ diagram shown above. The path $B C$ is isothermal. The work done by the gas during one complete cycle, beginning and ending at $A$, is most nearly
(A) 600 kJ
(B) 300 kJ
(C) 0
(D) -300 kJ
(E) -600 kJ

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28. A sample of nitrogen gas undergoes the cyclic thermodynamic process shown above. Which of the following gives the net heat transferred to the system in one complete cycle $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$ ?
(A) -80 J
(B) -40 J
(C) 40 J
(D) 80 J
(E) 180 J

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## Volume in increases

Temperature decreases (atoms slower)
Pressure decreases (atoms slower and also more distance to travel)


To keep temperature constant, add heat
Pressure still drops, since volume rises
To keep pressure constant, need to add even more heat


## Terminology

Isothermal: constant temperature
Isobaric: constant pressure
Adiabatic: no heat added or removed

29. For an ideal gas, consider the three thermodynamic processes-labeled 1,2 , and 3 -shown in the $P V$ diagram above. Each process has the same initial state and the same final volume. One process is adiabatic, one is isobaric, and one is isothermal. Which of the following correctly identifies the three processes?

|  | Adiabatic | Isobaric | Isothermal |
| :---: | :---: | :---: | :---: |
| (A) | 1 | 2 | 3 |
| (B) | 2 | 1 | 3 |
| (C) | 2 | 3 | 1 |
| (D) | 3 | 1 | 2 |
| (E) | 3 | 2 | 1 |


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| :--- | :---: | :---: | :---: | :---: |
| (Asothermal |  |  |  |  |
| (A) | 1 | 2 | 3 |  |
| (B) | 2 | 1 | 3 |  |
| (C) | 2 | 3 | 1 |  |
| (D) | 3 | 1 | 2 |  |
| (E) | 3 | 2 | 1 |  |

6. An ideal monatomic gas expands quasi-statically to twice its volume. If the process is isothermal, the work done by the gas is $W_{i}$. If the process is adiabatic, the work done by the gas is $W_{a}$. Which of the following is true?
(A) $W_{i}=W_{a}$
(B) $0=W_{i}<W_{a}$
(C) $0<W_{i}<W_{a}$
(D) $0=W_{a}<W_{i}$
(E) $0<W_{a}<W_{i}$
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To keep temperature constant, add heat


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8. For which of the following thermodynamic processes is the increase in the internal energy of an ideal gas equal to the heat added to the gas?
(A) Constant temperature
(B) Constant volume
(C) Constant pressure
(D) Adiabatic
(E) Cyclic

## GREPracticeBook

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(A) Constant temperature
(B) Constant volume
(C) Constant pressure
(D) Adiabatic
(E) Cyclic


Increase in internal energy equals heat added minus Work done by gas

## Entropy




Does anything stay the SAME?

Why do the atoms slow down?

## Volume in increases

## Temperature decreases (atoms slower)

Pressure decreases (atoms slower and also more distance to travel)


# Stationary wall 




Moving wall


So atoms slow down when we expand the box ....



Disorder in position $\quad \Delta x=L$
Disorder in velocity $\Delta v=2 v$



The Area of the two regions is the same $\longrightarrow$ Entropy

## Three notational modifications


(B) Consider Log of Area instead of Area

$$
\ln A=\ln (\Delta x \Delta p)
$$

(C) Multiply by $k$

$$
S=k \ln A=k \ln (\Delta x \Delta p)
$$

Note: If there are $N$ atoms then multiply entropy by $N$


$$
S=N k \ln A=N k \ln (\Delta x \Delta p)
$$

36. Consider the quasi-static adiabatic expansion
of an ideal gas from an initial state $i$ to a final state $f$. Which of the following statements is NOT true?
(A) No heat flows into or out of the gas.
(B) The entropy of state $i$ equals the entropy of state $f$.
(C) The change of internal energy of the gas is
$-\int P d V$.
(D) The mechanical work done by the gas is

$$
\int P d V
$$

(E) The temperature of the gas remains constant.
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(C) The change of internal energy of the gas is $-\int P d V$.
(D) The mechanical work done by the gas is

$$
\int P d V
$$

(E) The temperature of the gas remains constant.






If entropy does not change in such processes, then how DOES it change?

Two ways: (A) Add heat
(B) Change things quickly (irreversible process)

## (A) Add heat



momentum

$\Delta Q \propto \Delta S$

$$
\Delta Q=T \Delta S
$$

(Definition of temperature in thermodynamics)

91. The diagram above shows a Carnot cycle for an ideal air conditioner, which is to cool a house on a hot summer day. The air conditioner absorbs heat at the lower temperature inside and pumps it to the environment at the higher temperature outside. Which of the following gives the ratio of the heat $Q_{b c}$ absorbed in the house (i.e., between points $b$ and $c$ on the cycle) to the work done during the cycle?
(A) 0
(B) 0.033
(C) 0.97
(D) 1.0
(E) 30 .


## GREPracticeBook

Total work done $=$ Total heat absorbed

$$
\Delta Q=T \Delta S
$$

91. The diagram above shows a Carnot cycle for an $Q_{\text {emitted }}=305 \Delta S$ ideal air conditioner, which is to cool a house on a hot summer day. The air conditioner absorbs heat at the lower temperature inside and pumps it to the environment at the higher temperature outside. Which of the following gives the ratio of the heat $Q_{b c}$ absorbed in the house (i.e., between points $b$ and $c$ on the cycle) to the work done during the cycle?
(A) 0
(B) 0.033
$Q_{\text {absorbed }}=295 \Delta S$
(C) 0.97
(D) 1.0
(E) 30 .
92. A monatomic ideal gas changes from an initial state $\left(P_{i}, V_{i}, T_{i}, n_{i}\right)$ to a final state $\left(P_{f}, V_{f}\right.$, $T_{f}, n_{f}$ ), where $P_{i}<P_{f}, V_{i}=V_{f}, T_{i}<T_{f}$ and $n_{i}=n_{f}$. Which of the following gives the change in entropy of the gas?
(A) $\frac{3}{2} n R \ln \left(\frac{T_{f}}{T_{i}}\right)$
(B) $\frac{3}{2} n R \ln \left(\frac{T_{i}}{T_{f}}\right)$
(C) $\frac{5}{2} n R \ln \left(\frac{T_{f}}{T_{i}}\right)$
(D) $\frac{5}{2} n R \ln \left(\frac{T_{i}}{T_{f}}\right)$
(E) 0

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58. A monatomic ideal gas changes from an initial state $\left(P_{i}, V_{i}, T_{i}, n_{i}\right)$ to a final state $\left(P_{f}, V_{f}\right.$, $T_{f}, n_{f}$ ), where $P_{i}<P_{f}, V_{i}=V_{f}, T_{i}<T_{f}$ and $n_{i}=n_{f}$. Which of the following gives the change in entropy of the gas?
(A) $\frac{3}{2} n R \ln \left(\frac{T_{f}}{T_{i}}\right)$
(B) $\frac{3}{2} n R \ln \left(\frac{T_{i}}{T_{f}}\right)$
(C) $\frac{5}{2} n R \ln \left(\frac{T_{f}}{T_{i}}\right)$

(D) $\frac{5}{2} n R \ln \left(\frac{T_{i}}{T_{f}}\right)$
(E) 0

$$
\begin{aligned}
S_{f}-S_{i} & =\frac{3}{2} n R \int_{T_{i}}^{T_{f}} \frac{d T}{T} \\
& =\frac{3}{2} n R \ln \frac{T_{f}}{T_{i}}
\end{aligned}
$$

$$
\Delta Q=T \Delta S
$$

$$
\Delta S=\frac{\Delta Q}{T}
$$

No work $\Delta Q=\Delta U$
$U=\frac{3}{2} N k T=\frac{3}{2} n R T$
$\Delta U=\frac{3}{2} n R \Delta T$
$\Delta S=\frac{3}{2} n R \frac{\Delta T}{T}$

## Irreversibility

How do we increase entropy?

Two ways: (A) Add heat
(B) Change things quickly (irreversible process)


What is we move the piston at half the speed?

In each bounce, half as much slowdown
But there will be twice as many bounces ...
So total slowdown will be the same ...

So we don't have to specify the speed of the piston ...

But what happens if we move the piston VERY fast?


Atoms don't touch the piston, so they don't slow down !
momentum

momentum


Entropy increases !
47. A sealed and thermally insulated container of total volume $V$ is divided into two equal volumes by an impermeable wall. The left half of the container is initially occupied by $n$ moles of an ideal gas at temperature $T$. Which of the following gives the change in entropy of the system when the wall is suddenly removed and the gas expands to fill the entire volume?
(A) $2 n R \ln 2$
(B) $n R \ln 2$
(C) $\frac{1}{2} n R \ln 2$
(D) $-n R \ln 2$
(E) $-2 n R \ln 2$
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(A) $2 n R \ln 2$

$$
L \rightarrow 2 L \quad v \rightarrow v
$$

(B) $n R \ln 2$
(C) $\frac{1}{2} n R \ln 2$
(D) $-n R \ln 2$
(E) $-2 n R \ln 2$
$A \rightarrow 2 A$
momentum

$S=k \ln A \rightarrow k \ln (2 A)=k \ln A+k \ln 2$
$\Delta S_{\text {total }}=N k \ln 2=n R \ln 2$


Can we reverse this process ? NO!
If we try to move piston back, it WILL hit the atoms and speed them up ...

Slow process are called QUASISTATIC. They are REVERSIBLE. There is NO net increase in entropy

Sudden changes are IRREVERSIBLE. There IS a net increase in entropy (disorder)

Entropy can increase but never decrease
7. Which of the following is true about any system that undergoes a reversible thermodynamic process?
(A) There are no changes in the internal energy of the system.
(B) The temperature of the system remains constant during the process.
(C) The entropy of the system and its environment remains unchanged.
(D) The entropy of the system and its environment must increase.
(E) The net work done by the system is zero.
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