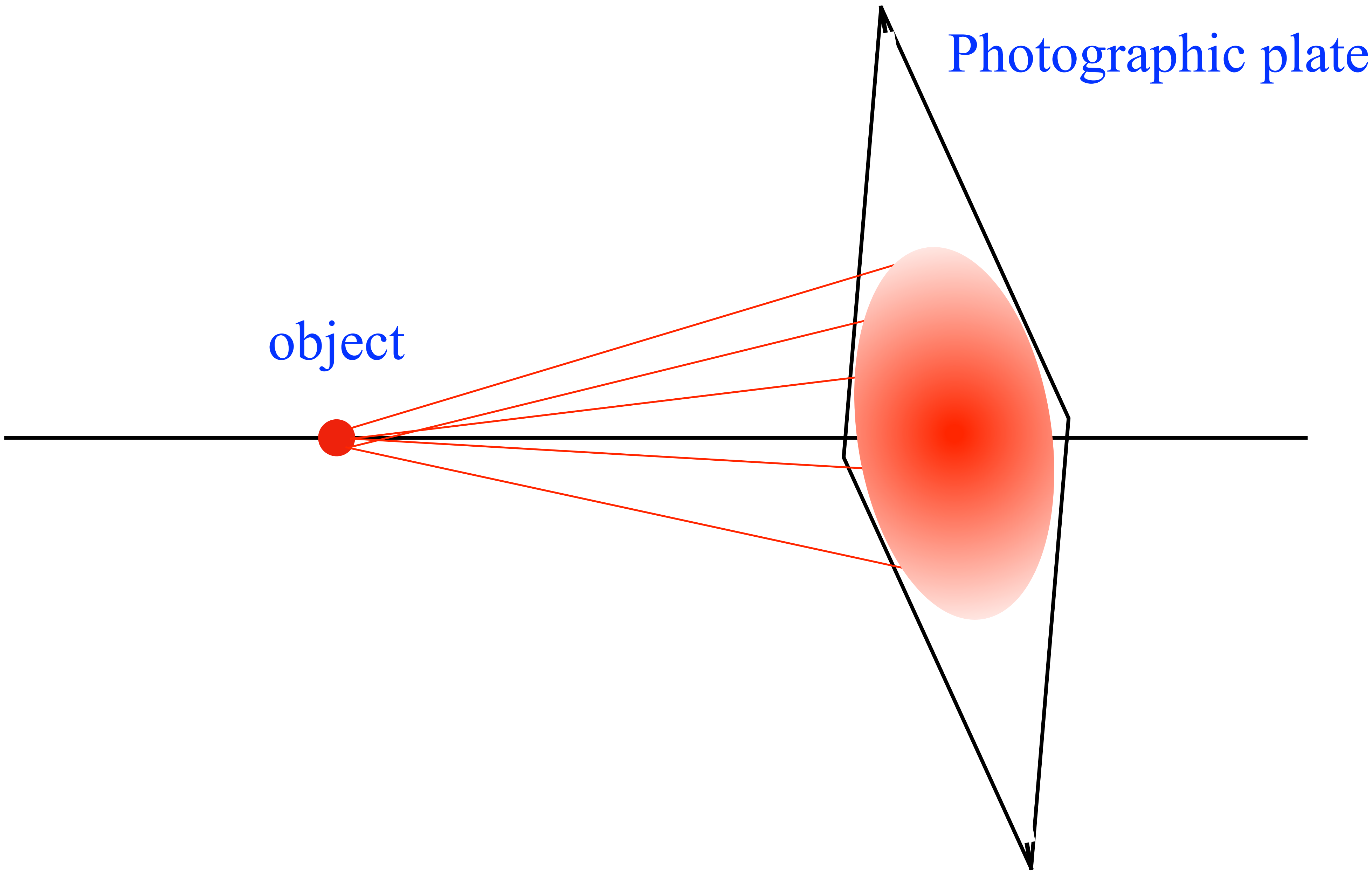
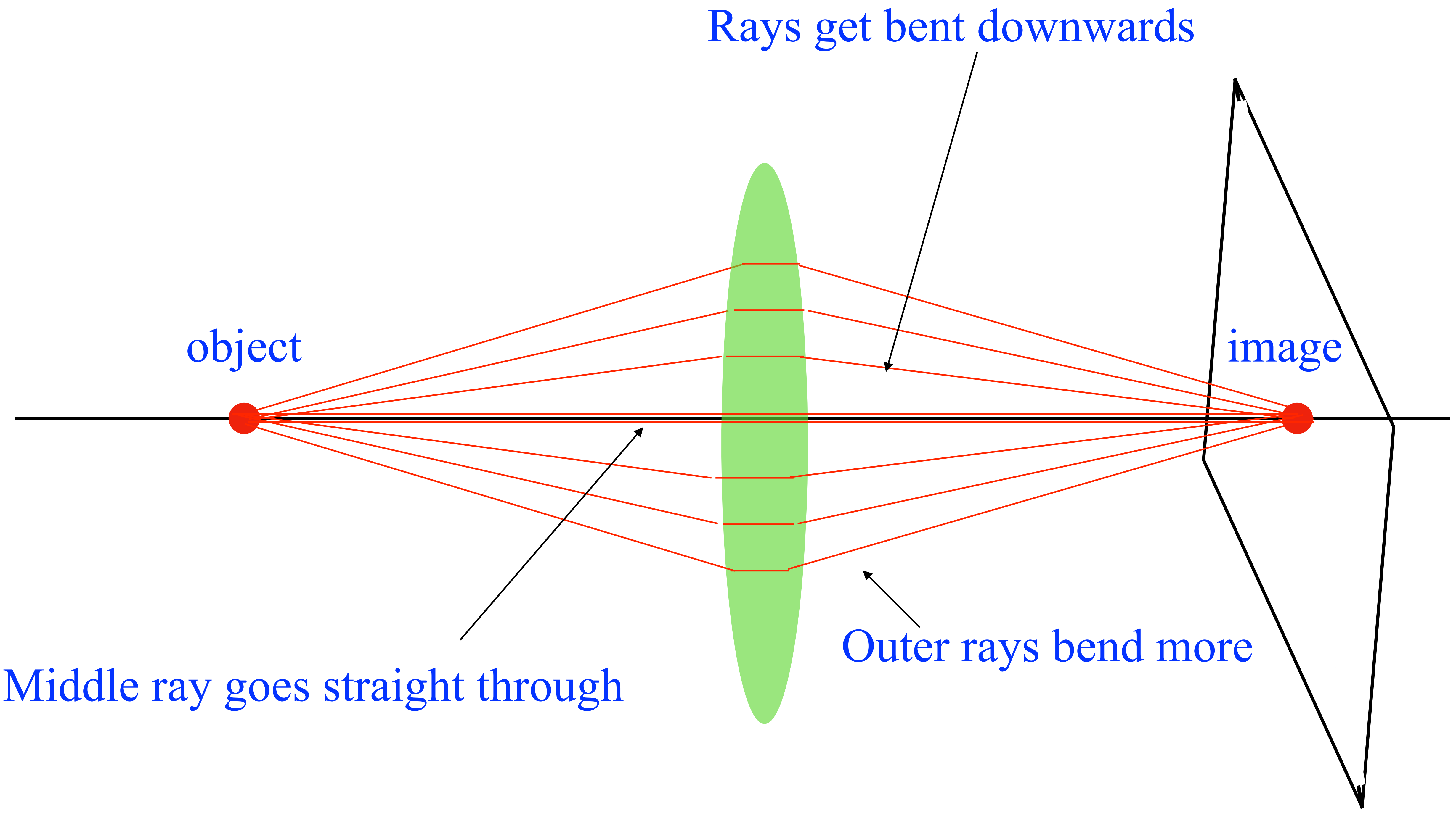
Optics

Start with a point source of light



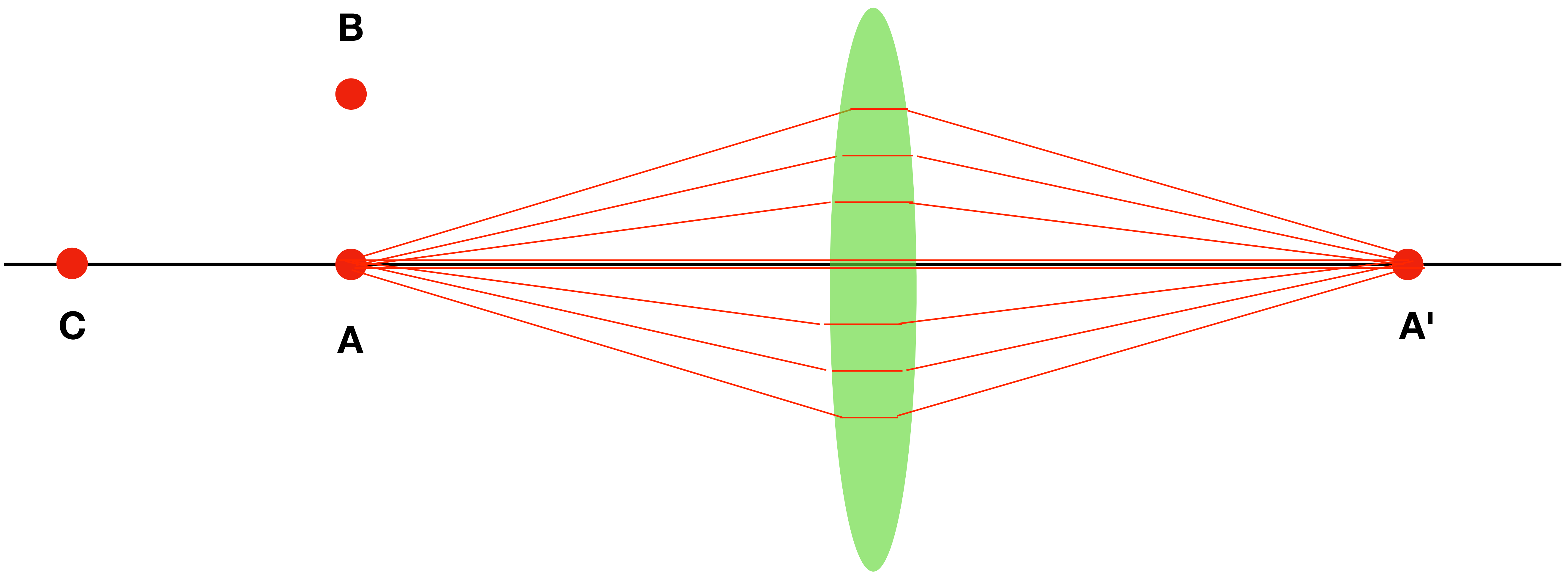
We do not get a sharp picture of the source; we just get a diffuse blur

The lens



If we arrange the shape of the lens just right, the rays will focus to a point

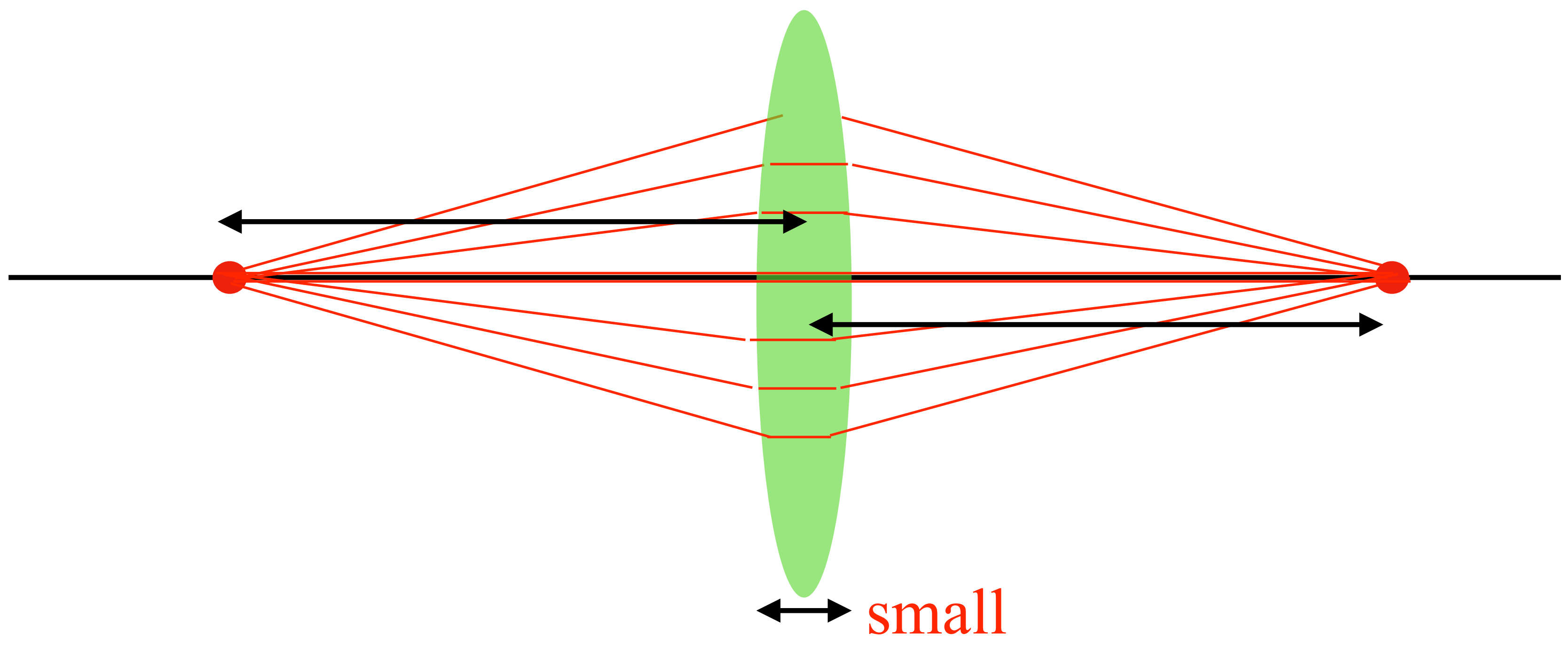
Question: If we make the lens so that an object at A focuses, then will objects at B, C, ... focus somewhere as well?



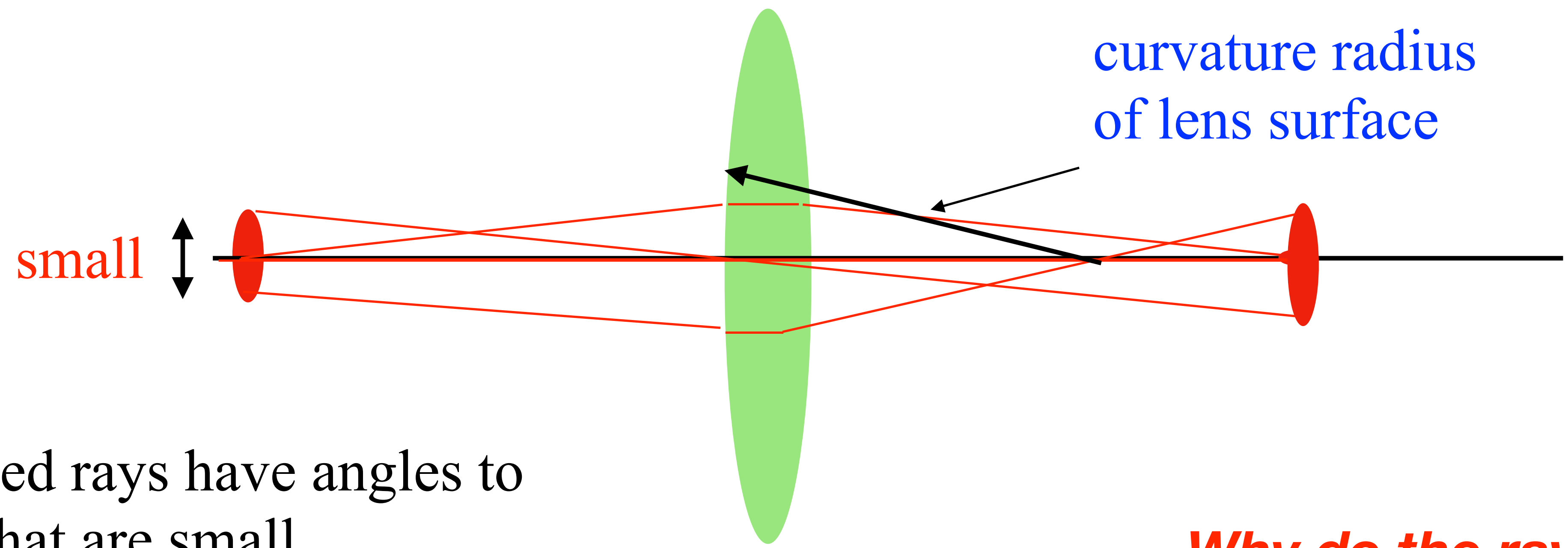
Answer: No, they will not ... at best we can manage a good approximation ....

The approximation

(a) The lens in thin compared to the distances of the object and image



(b) The objects are small compared to the curvature radius of the lens surfaces

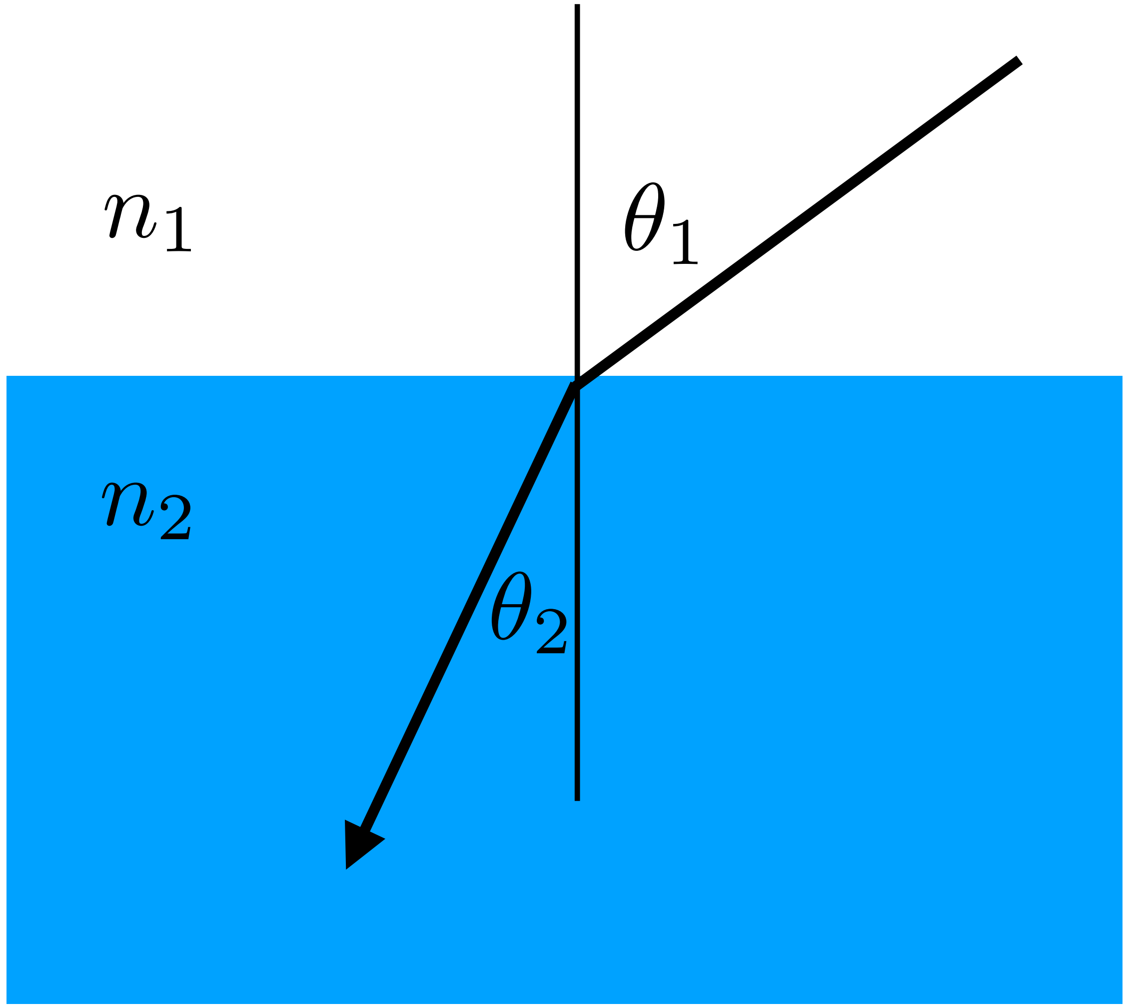


All the concerned rays have angles to the horizontal that are small

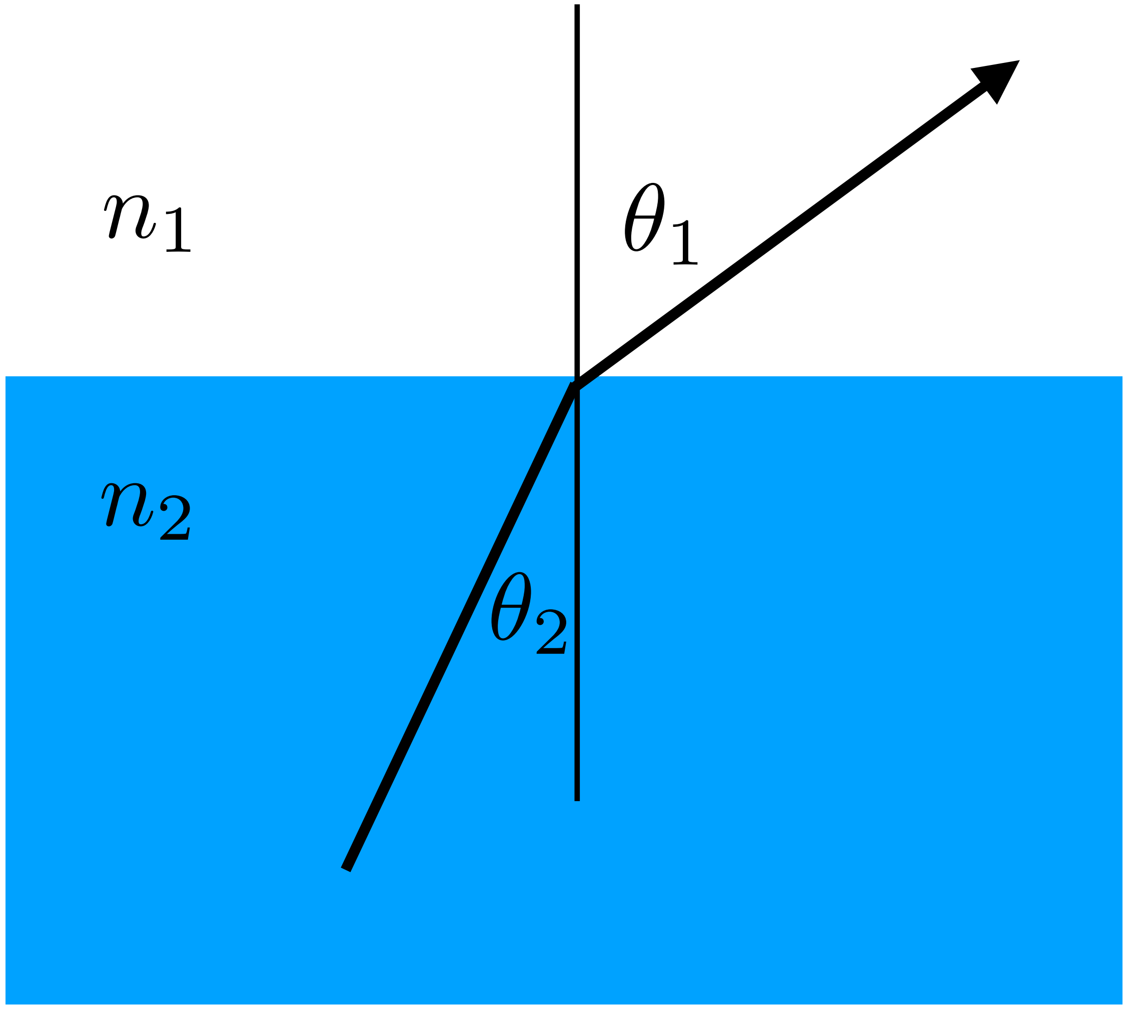
Why do the rays bend ?

Snell's law

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$



The ray bends closer to the normal when it enters a denser medium



The path of light rays is always reversible

The ray bends away from the normal when it enters a rarer medium

56. A light source is at the bottom of a pool of water (the index of refraction of water is 1.33). At what minimum angle of incidence will a ray be totally reflected at the surface?

(A) 0°
(B) 25°
(C) 50°
(D) 75°
(E) 90°

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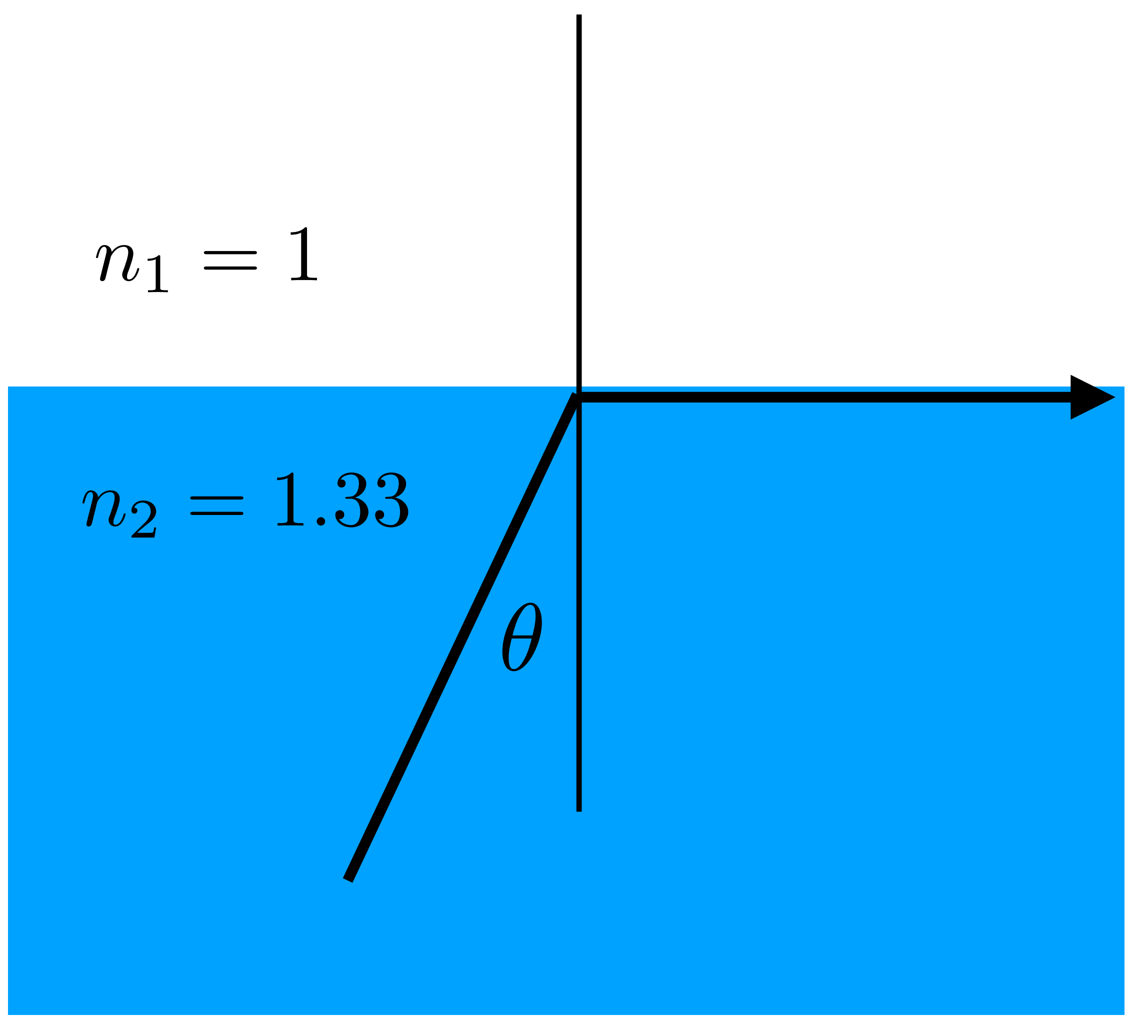
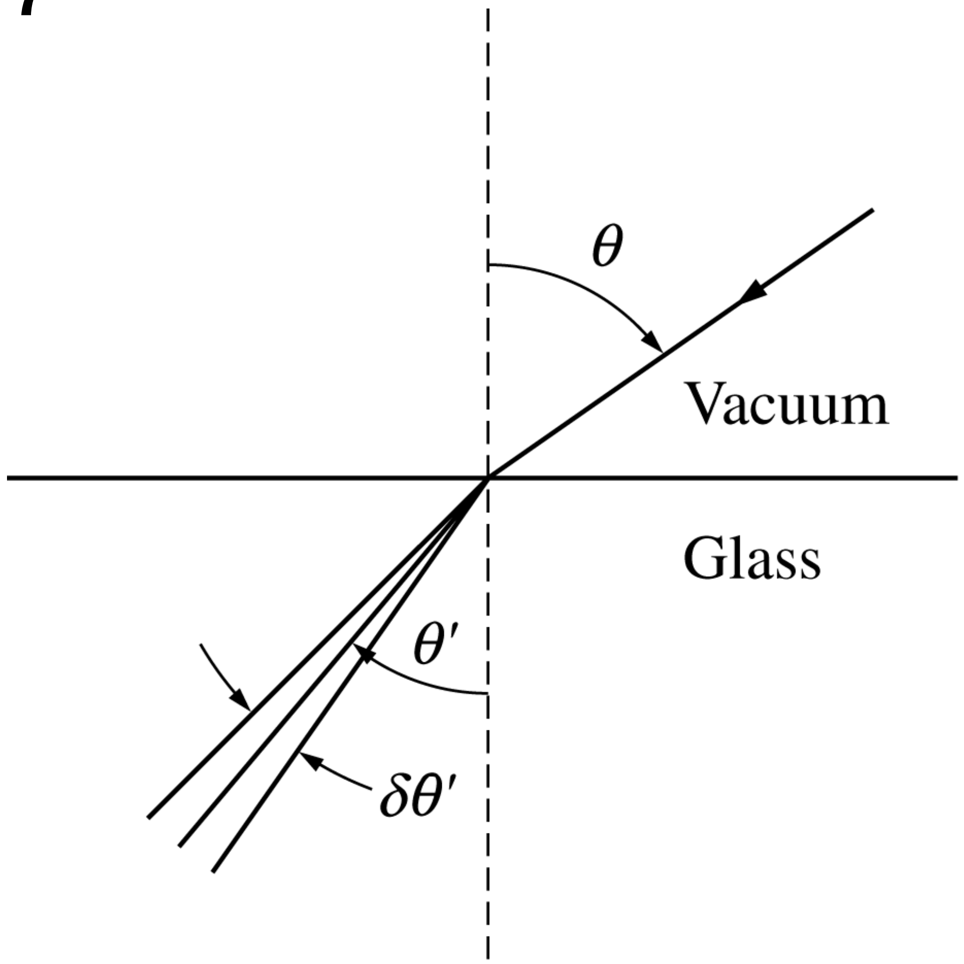


$$\frac{\sin\theta}{\sin 90^o} = \frac{1}{1.33}$$

$$\sin\theta \approx 0.75$$

GR0177



97. A beam of light has a small wavelength spread $\delta\lambda$ about a central wavelength λ. The beam travels in vacuum until it enters a glass plate at an angle θ relative to the normal to the plate, as shown in the figure above. The index of refraction of the glass is given by $n(\lambda)$. The angular spread $\delta\theta'$ of the refracted beam is given by

(A) $\delta\theta' = \left| \frac{1}{n}\,\delta\lambda \right|$

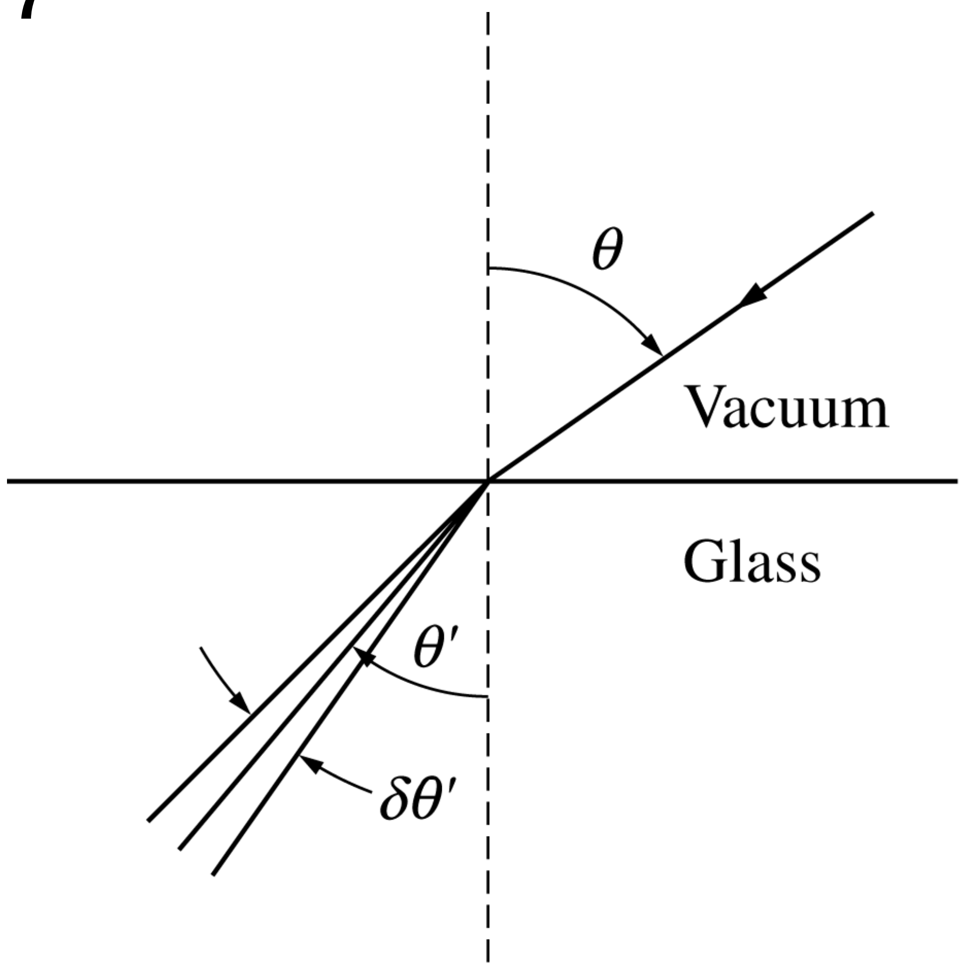
(B) $\delta\theta' = \left| \frac{dn(\lambda)}{d\lambda}\,\delta\lambda \right|$

(C) $\delta\theta' = \left| \frac{1}{\lambda}\,\frac{d\lambda}{dn}\,\delta\lambda \right|$

(D) $\delta\theta' = \left| \frac{\sin\theta}{\sin\theta'}\,\frac{\delta\lambda}{\lambda} \right|$

(E) $\delta\theta' = \left| \frac{\tan\theta'}{n}\,\frac{dn(\lambda)}{d\lambda}\,\delta\lambda \right|$

GR0177



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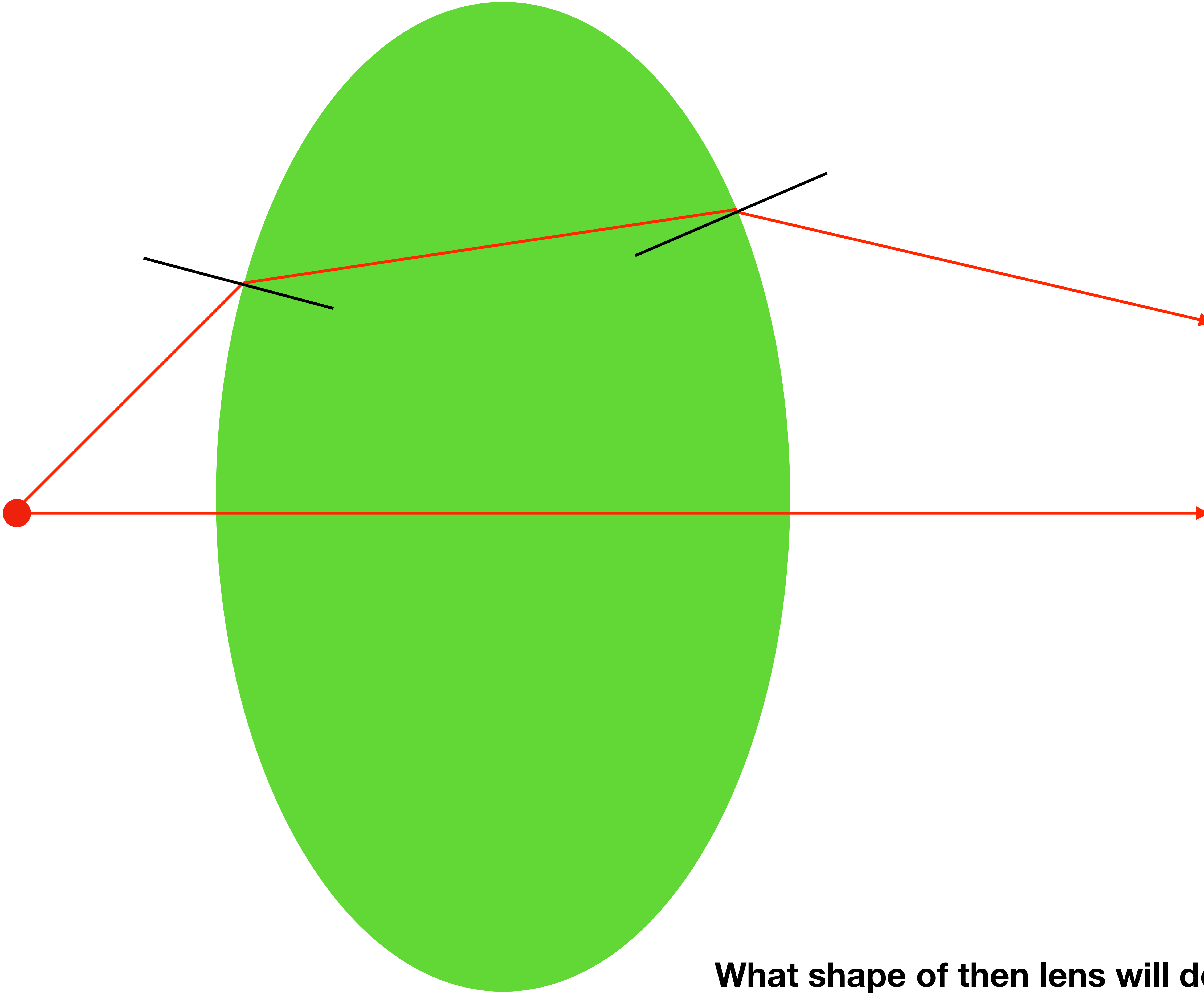
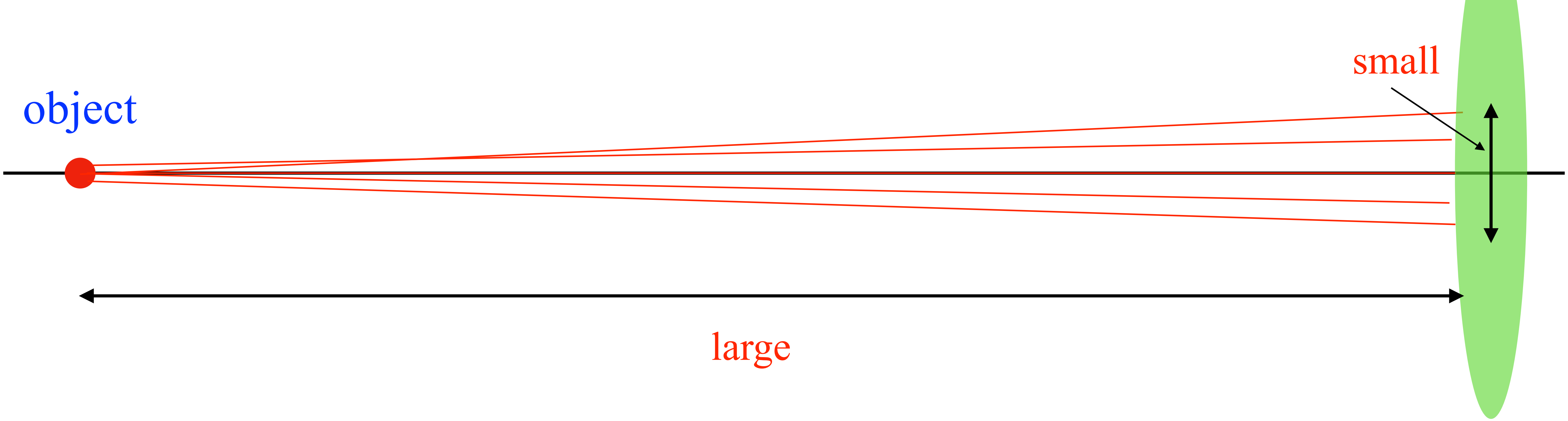
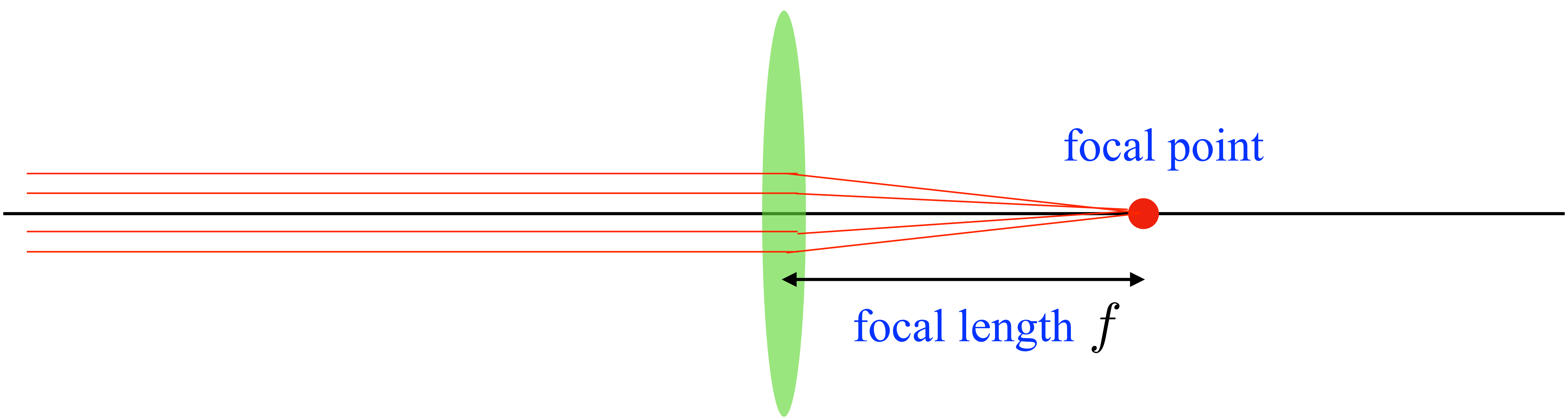


What shape of then lens will do this job ?

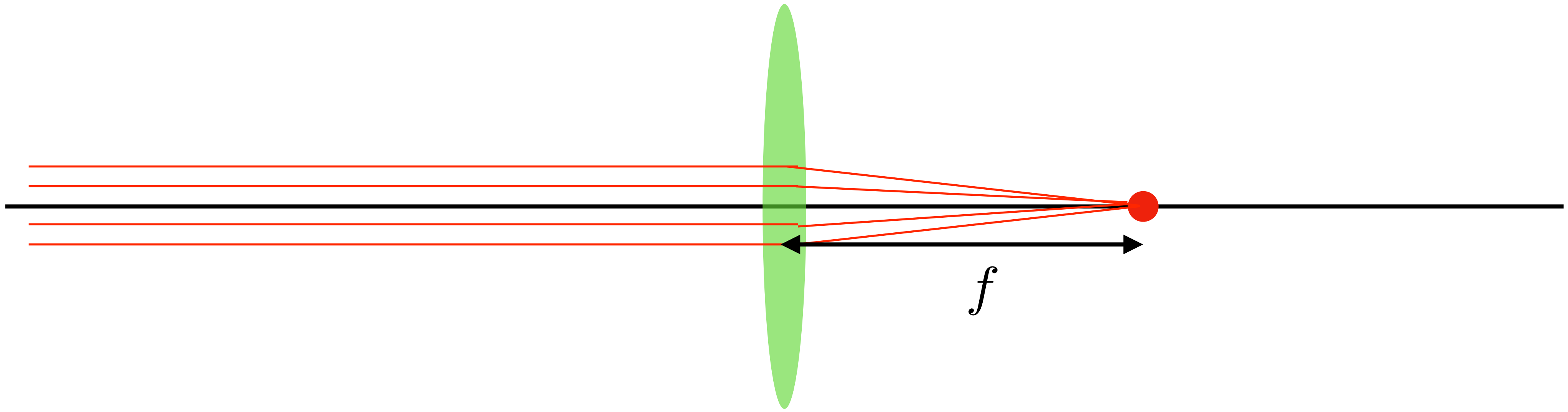
Where do the rays focus?



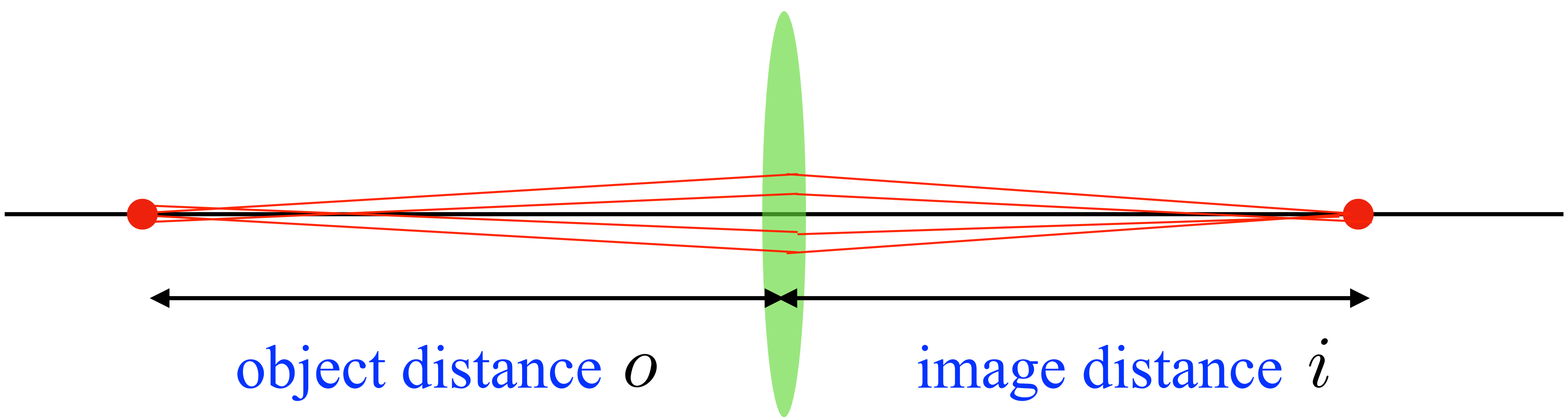
If the object if very far away, and we are looking at a small transverse region near the lens, the the rays look almost parallel



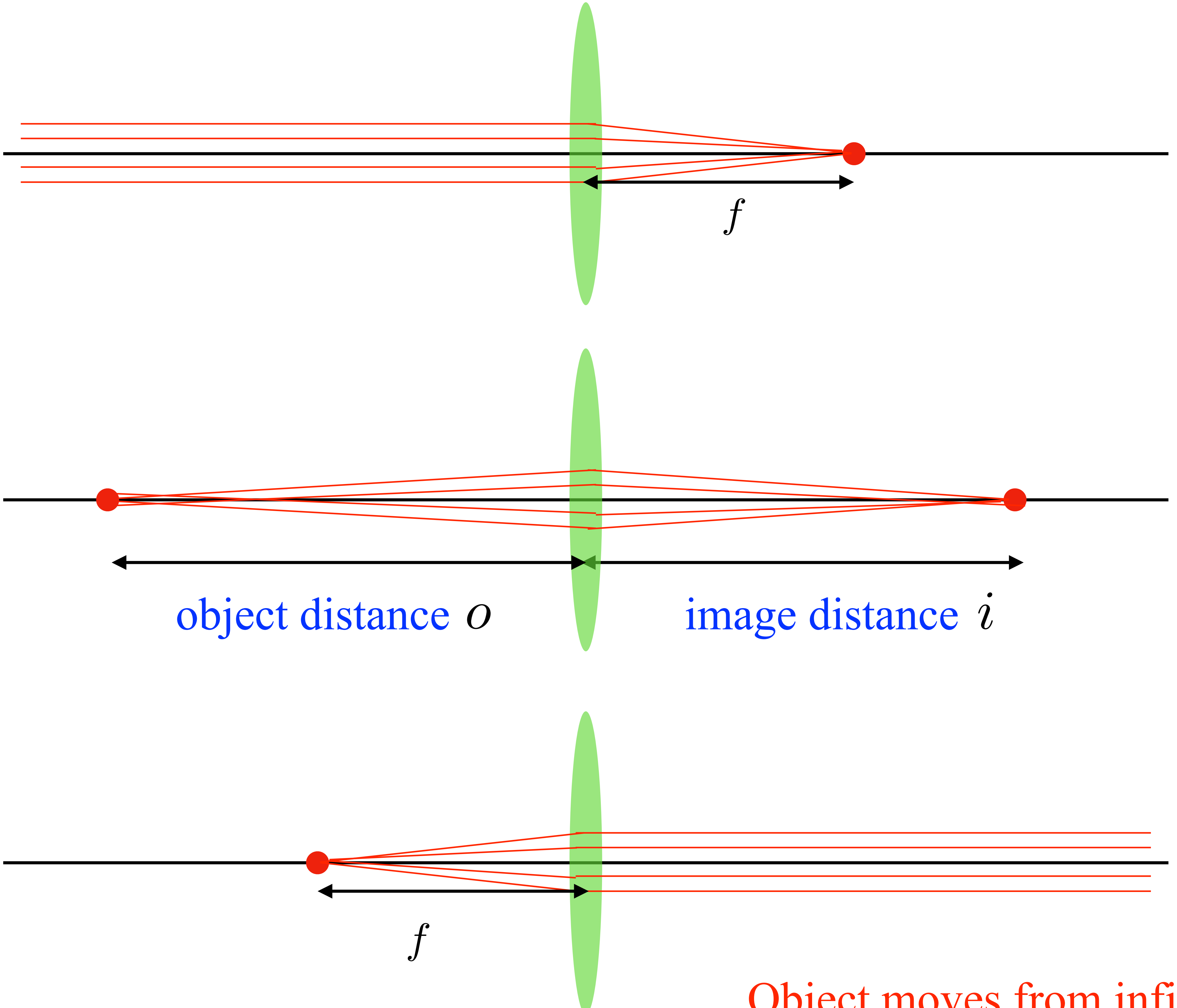
Rays from infinity focus at the focal point



If the object comes closer, then it is harder to focus the rays, so the image forms further away



$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$



$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

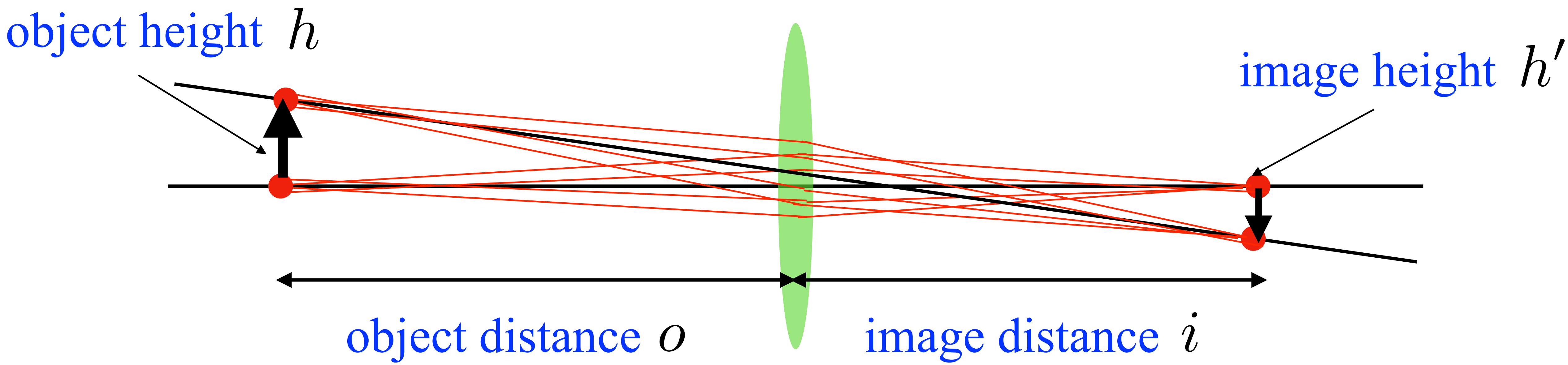
Object moves from infinity to focal point

Image moves from focal point to infinity

Magnification

How do we locate the image of a point off the axis ?

We rotate the rays



(a) Size is given by $\dfrac{\text{image height } h'}{\text{object height } h} = \dfrac{\text{image distance } i}{\text{object distance } o}$

(b) The image is inverted

We write $$\frac{h'}{h} = -\frac{i}{o}$$

28E: The sun subtends an angle of 0.5 degrees.

You produce an image of the sun on a screen, using a convex lens of focal length 20 cm.

What is the diameter of the image ?

HRW

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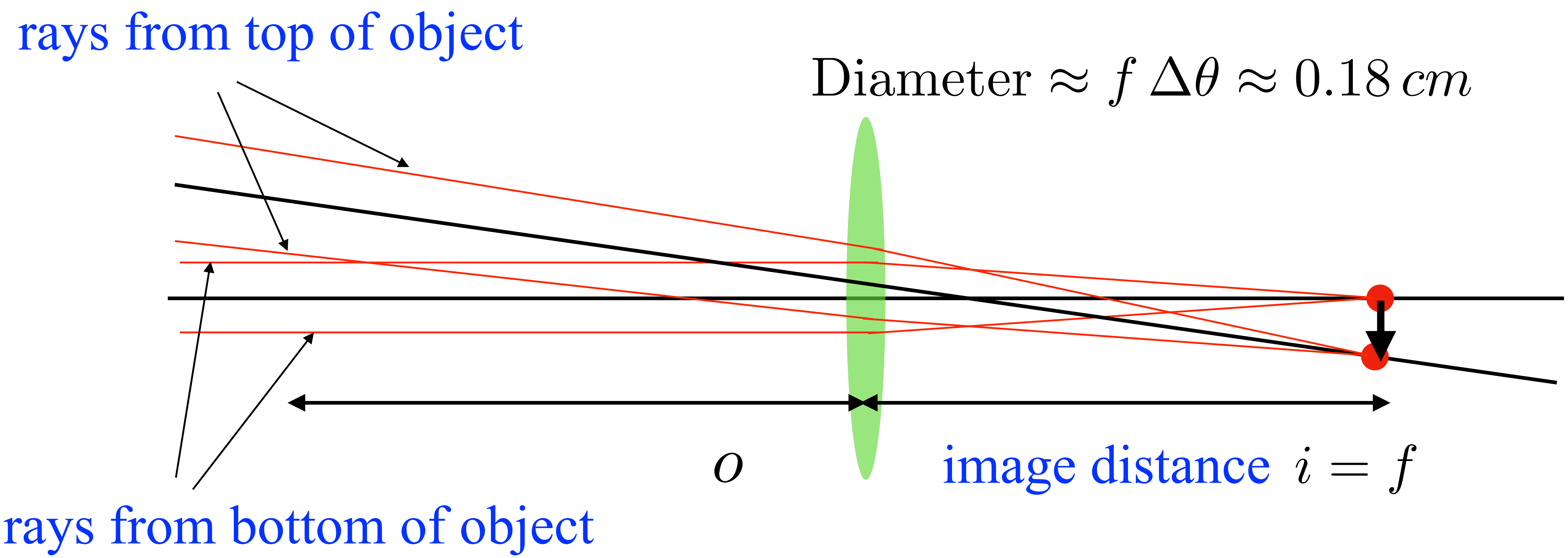
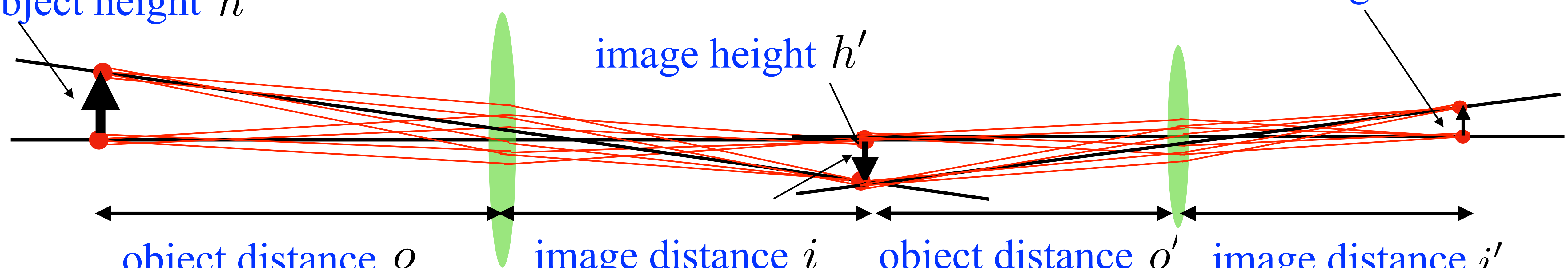
What is the diameter of the image ?

$$\Delta\theta \approx 0.5^0 \approx \frac{1}{2}\frac{1}{57}\,\text{radians}$$

$$\text{Diameter} \approx f\,\Delta\theta \approx 0.18\,cm$$



Two lenses



$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f_1} \qquad \frac{1}{o'} + \frac{1}{i'} = \frac{1}{f_2}$$

Magnification

$$M = \left(\frac{h''}{h}\right) = \left(\frac{h'}{h}\right)\left(\frac{h''}{h'}\right)$$

$$= \left(-\frac{i}{o}\right)\left(-\frac{i'}{o'}\right) = \left(\frac{i}{o}\right)\left(\frac{i'}{o'}\right)$$

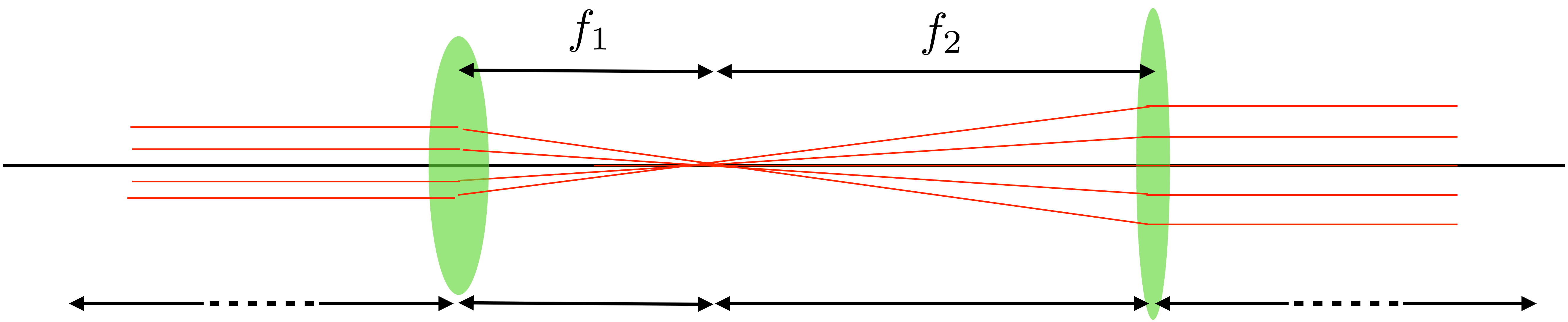
Final image is upright

HRW

A beam expander is made of two lenses as shown.

What is the ratio of the intensities of the emergent beam to the incident beam, if

$f_1 = 10\,cm, \quad f_2 = 20\,cm$ **?**



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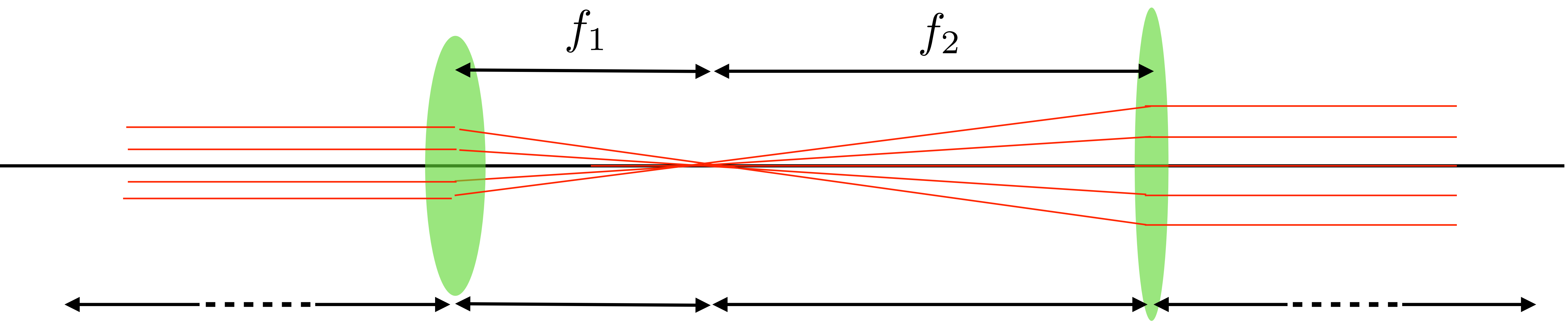

$$\frac{I_f}{I_i} = \left(\frac{f_1}{f_2}\right)^2 = \frac{1}{4}$$

58. A collimated laser beam emerging from a commercial HeNe laser has a diameter of about 1 millimeter. In order to convert this beam into a well-collimated beam of diameter 10 millimeters, two convex lenses are to be used. The first lens is of focal length 1.5 centimeters and is to be mounted at the output of the laser. What is the focal length, f, of the second lens and how far from the first lens should it be placed?

	f	Distance
(A)	4.5 cm	6.0 cm
(B)	10 cm	10 cm
(C)	10 cm	11.5 cm
(D)	15 cm	15 cm
(E)	15 cm	16.5 cm

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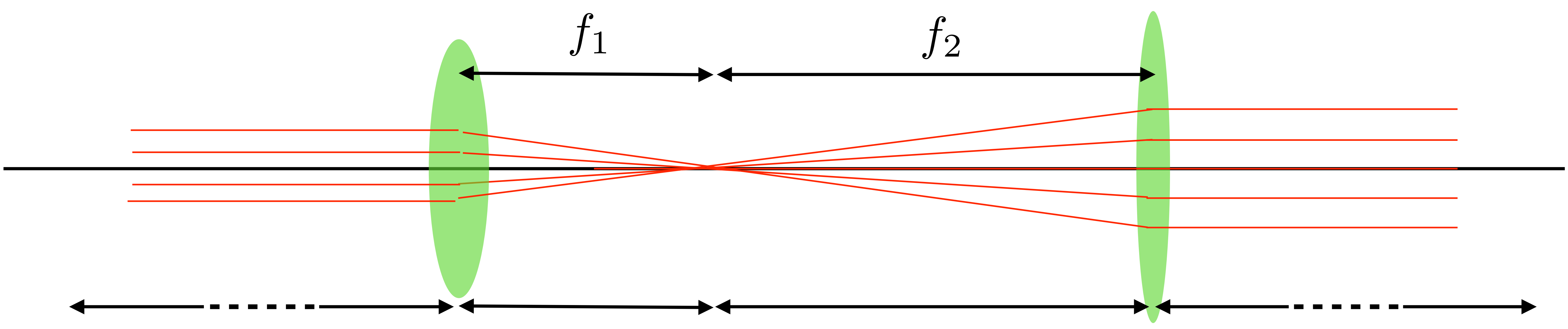
$$\frac{f_2}{f_1} = 10$$

$$f_2 = 15\,cm$$

$$f_1 + f_2 = 16.5\,cm$$



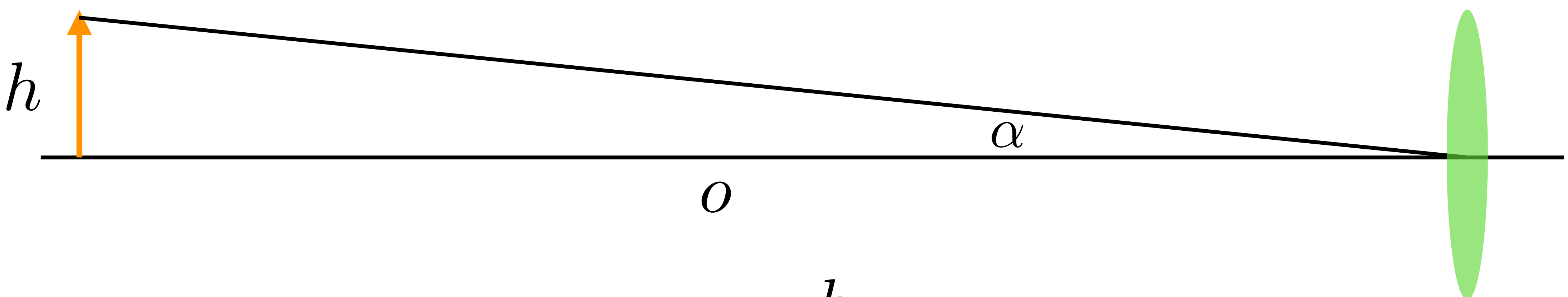
The telescope

Angular magnification

The magnification relation is

$$\frac{h'}{h} = -\frac{i}{o} \quad \Rightarrow \quad h' = -\frac{h\,i}{o}$$

An object like the sun is very far away. So o is very large. But the sun is very big, so h is also very large

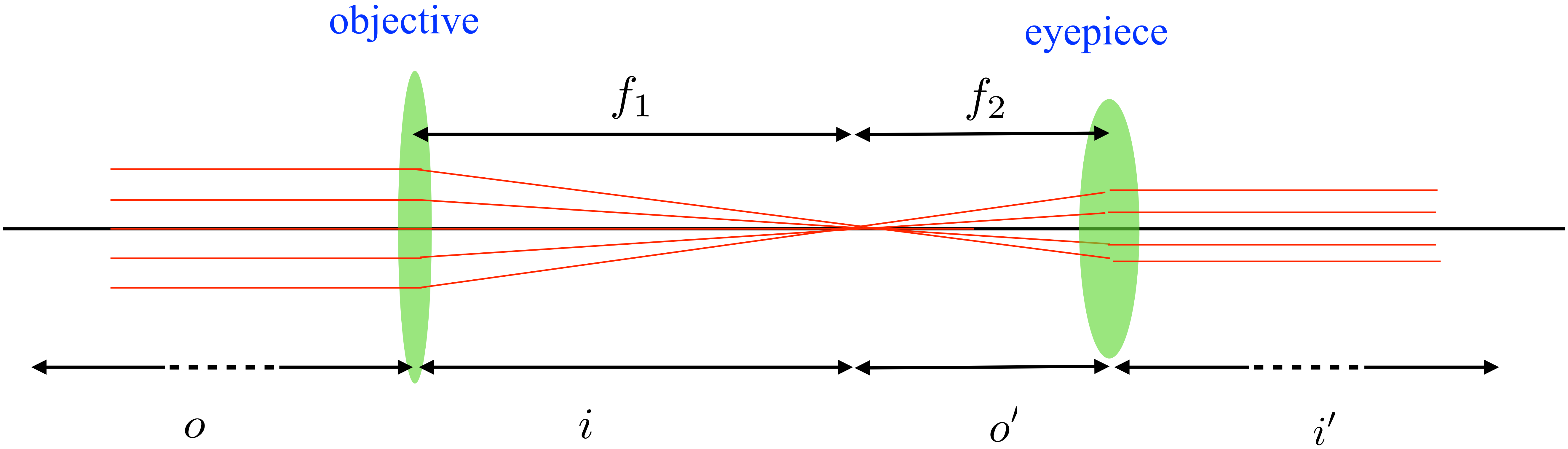


In this situation we look at the ratio $\frac{h}{o}$

Since all angles are assumed small, we have $\tan\alpha \approx \sin\alpha \approx \alpha$

Far away objects will be described by α, rather than h, and we talk about angular magnification rather than magnification

The telescope



Magnification $$\frac{h''}{h} = \left(\frac{i}{o}\right)\left(\frac{i'}{o'}\right)$$

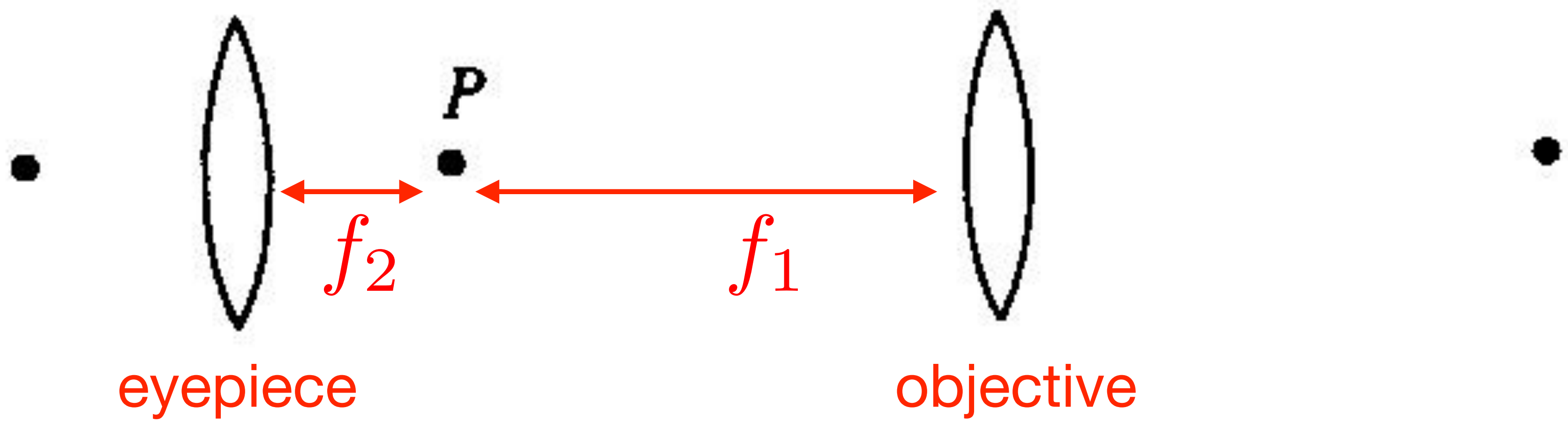
Angular sizes $$\alpha_i = \frac{h}{o} \qquad \alpha_f = \frac{h''}{i'}$$

We have $$i = f_1 \qquad o' = f_2$$

Angular magnification of telescope $$\frac{\alpha_f}{\alpha_i} = \frac{i}{o'} = \frac{f_1}{f_2}$$

22 A simple telescope consists of two convex lenses, the objective and the eyepiece, which have a common focal point P, as shown in the figure above. If the focal length of the objective is 1.0 meter and the angular magnification of the telescope is 10, what is the optical path length between objective and eyepiece?

(A) 0.1 m
(B) 0.9 m
(C) 1.0 m
(D) 1.1 m
(E) 10 m



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(A) 0.1 m
(B) 0.9 m
(C) 1.0 m
(D) 1.1 m
(E) 10 m

$$\text{Angular magnification} = \frac{f_1}{f_2} = 10$$

$$f_1 = 1$$

$$f_2 = .1$$

$$\text{Total length} = 1 + .1 = 1.1$$

32. A refracting telescope consists of two converging lenses separated by 100 cm. The eye-piece lens has a focal length of 20 cm. The angular magnification of the telescope is

(A) 4
(B) 5
(C) 6
(D) 20
(E) 100

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(A) 4
(B) 5
(C) 6
(D) 20
(E) 100

$$f_1 + f_2 = 100$$

$$f_2 = 20$$

$$f_1 = 80$$

$$\text{Angular magnification} = \frac{f_1}{f_2} = 4$$

47E: If the angular magnification of a telescope is 36 and the diameter of the objective is 75 mm, what is the minimum diameter of the eyepiece required to collect all the light from the objective ?

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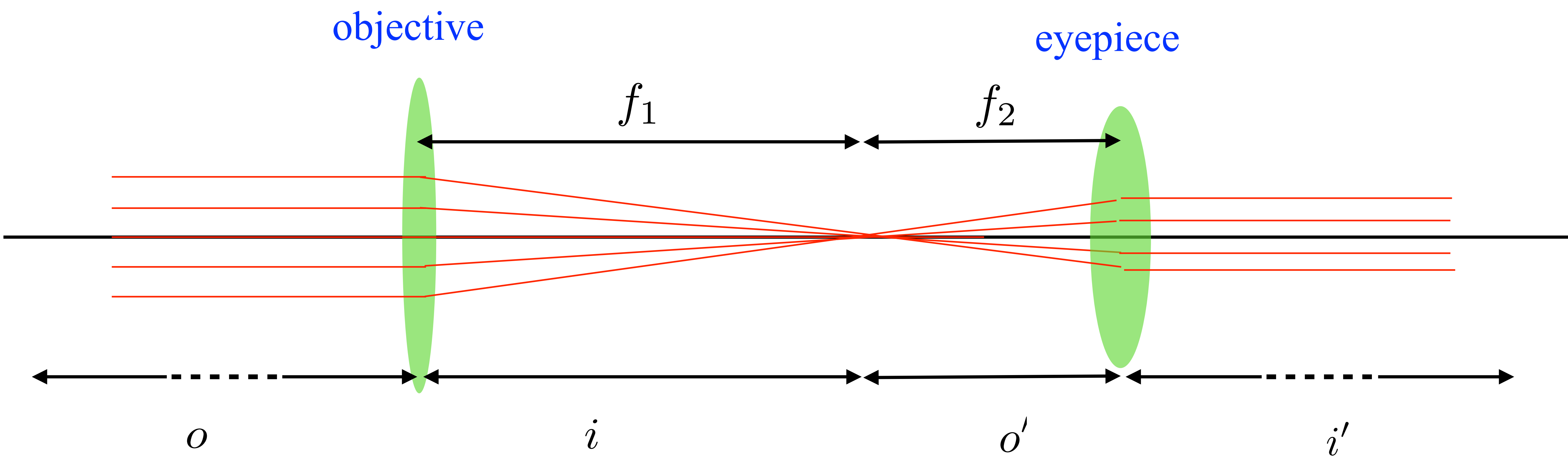


Angular magnification of telescope

$$\frac{\alpha_f}{\alpha_i} = \frac{i}{o'} = \frac{f_1}{f_2}$$

$$\frac{f_1}{f_2} = 36$$

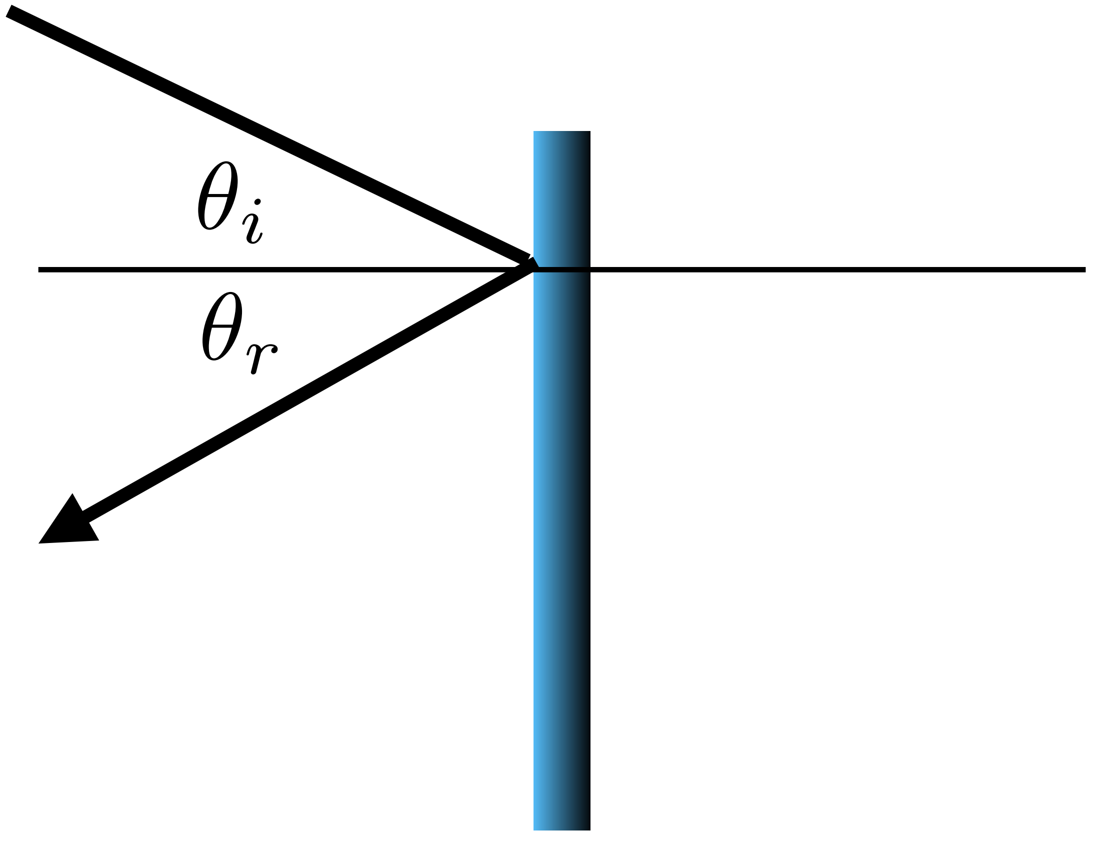
$$d_{objective} = \frac{1}{36} \times 75 \approx 2.1\, mm$$

Mirrors

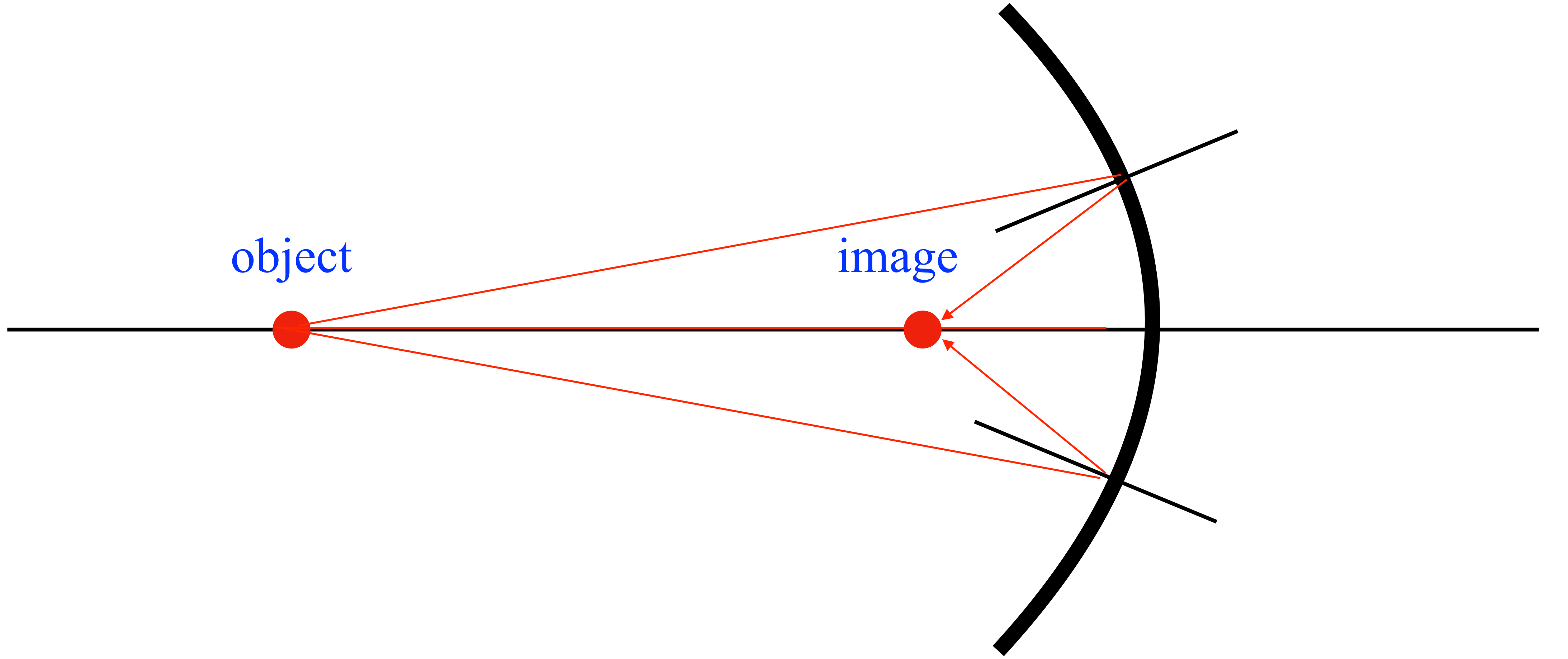
Basic law of reflection:

Angle of incidence equals angle of reflection

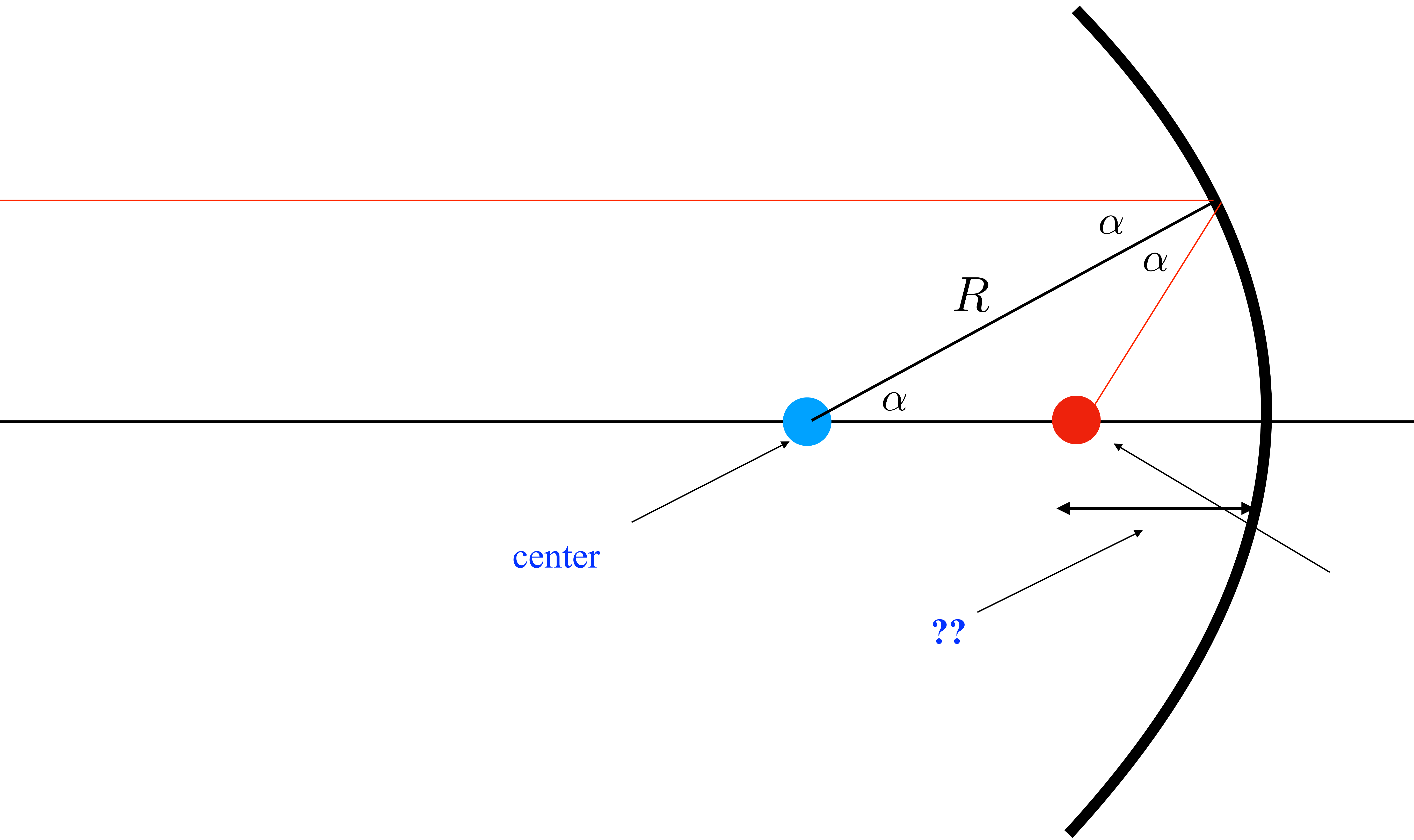
$$\theta_r = \theta_i$$



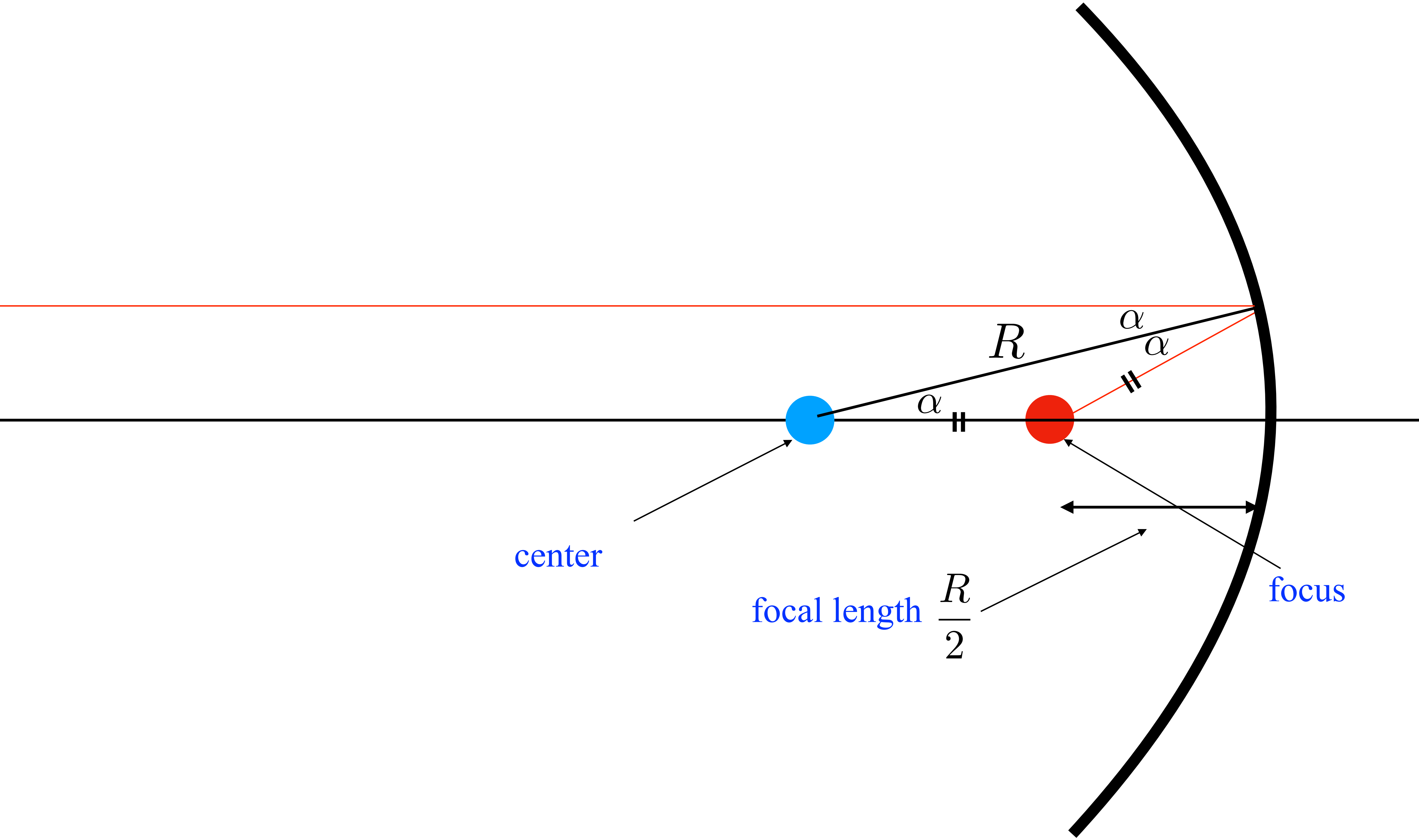
We can use this to focus:

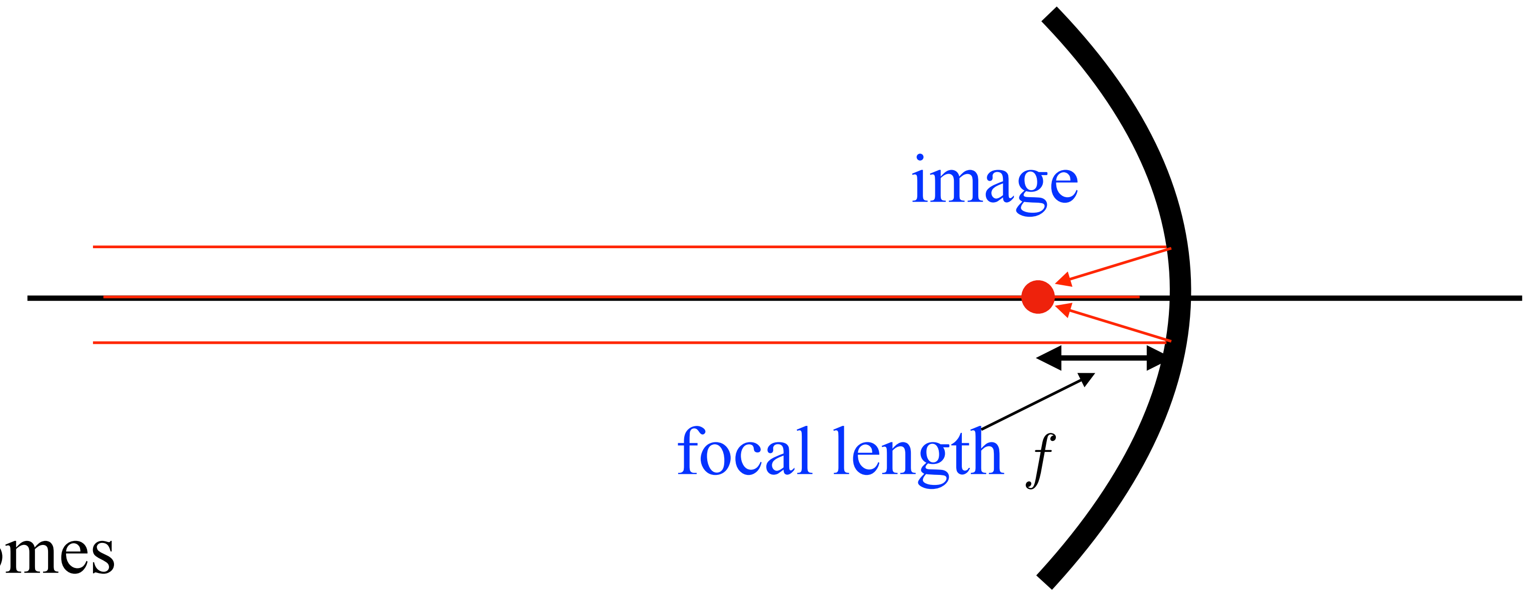


First we consider parallel rays, which correspond to a source that is infinitely far away



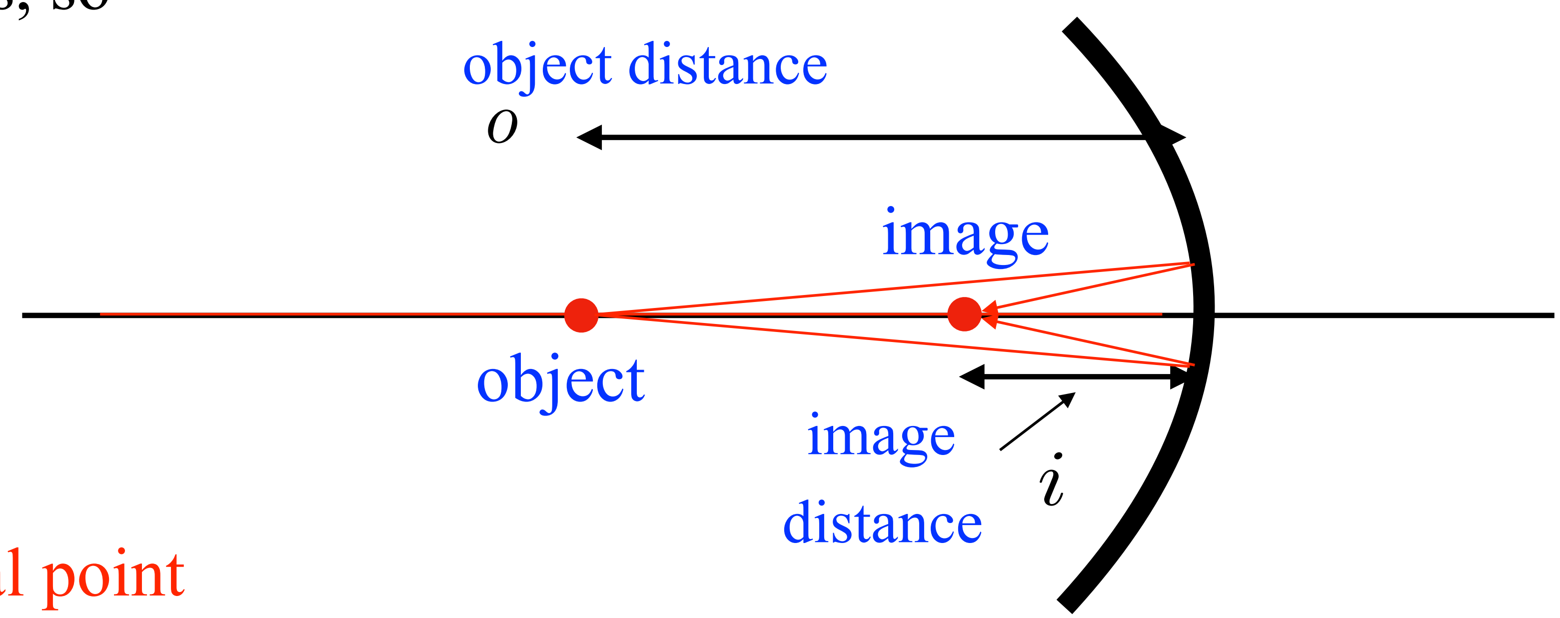
If we take the limit of small angles, then all the parallel rays do pass through a point ...



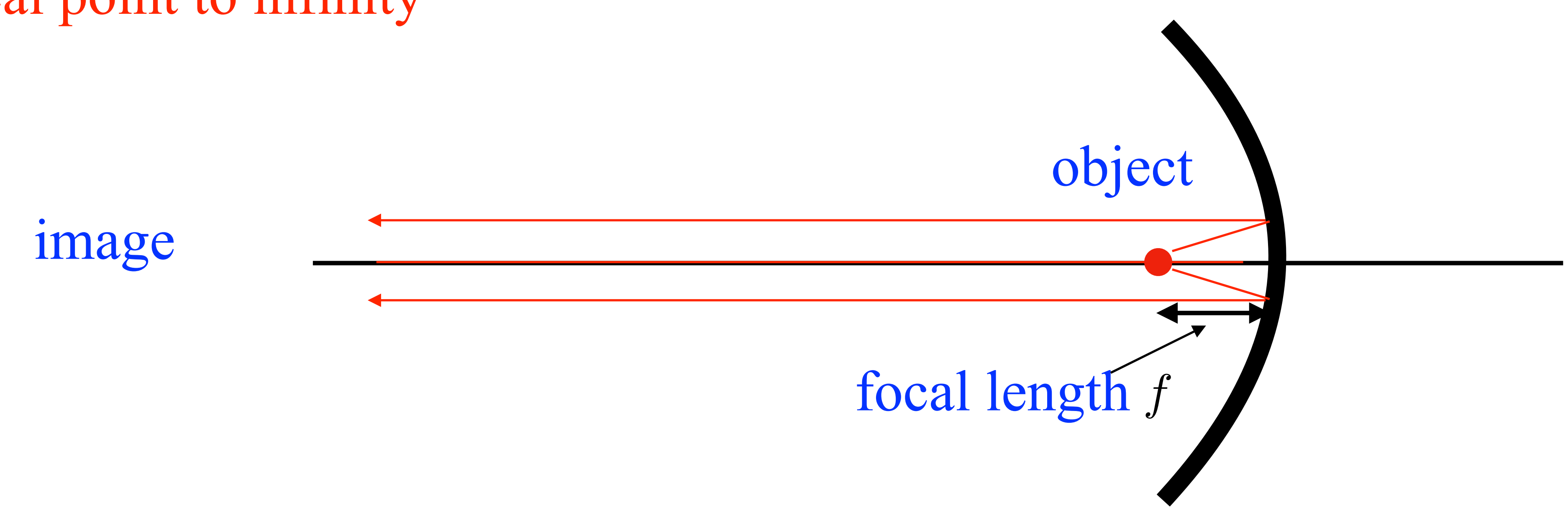


As the object comes closer, it becomes more difficult to converge the rays, so the image forms further away

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$



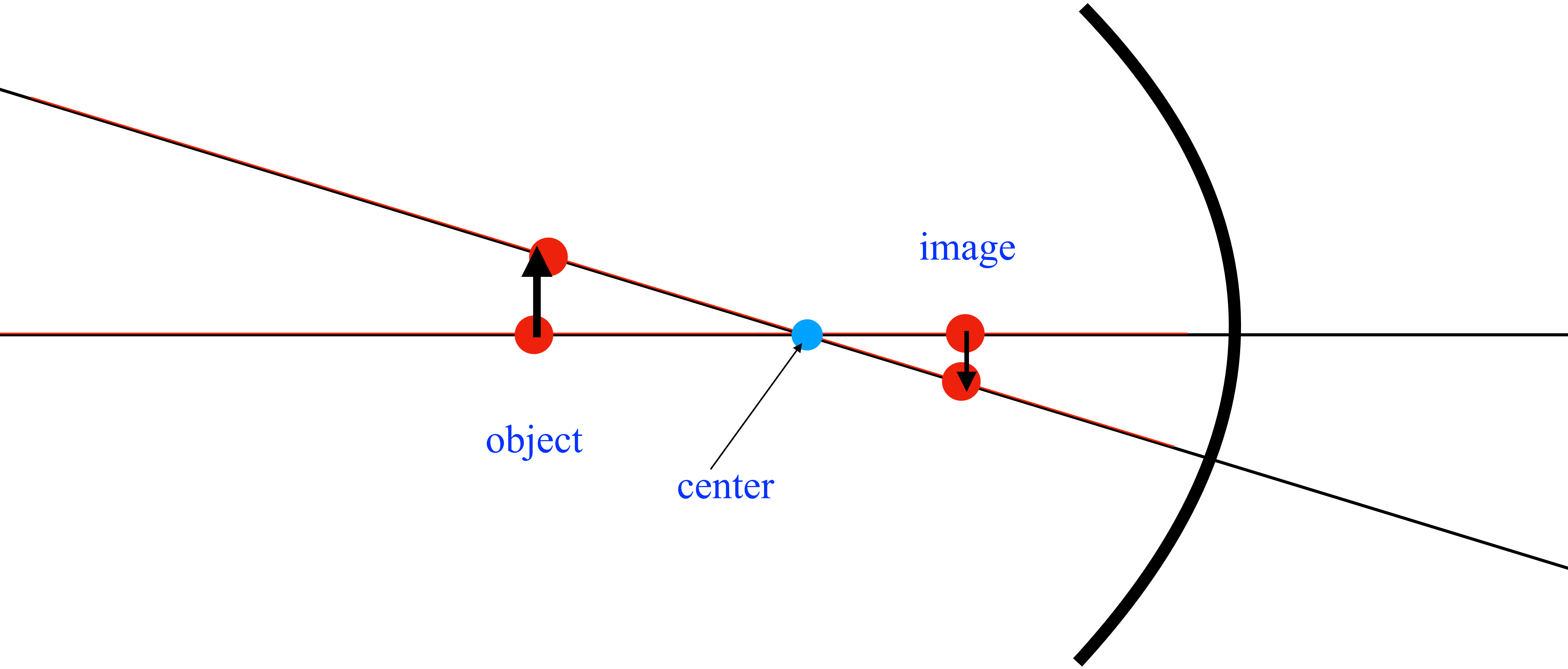
Object moves from infinity to focal point
Image moves from focal point to infinity

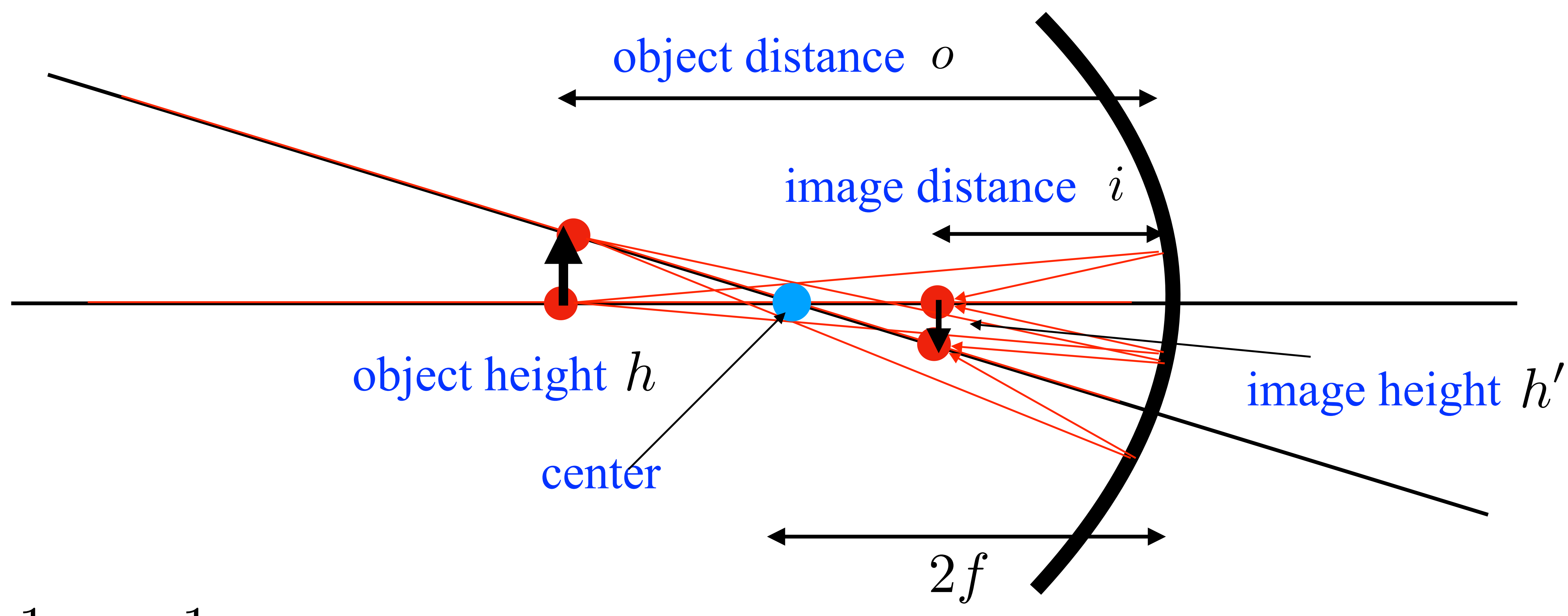


Magnification

Each point of object lies on an 'axis' through the center

The image of each point will lie on this axis



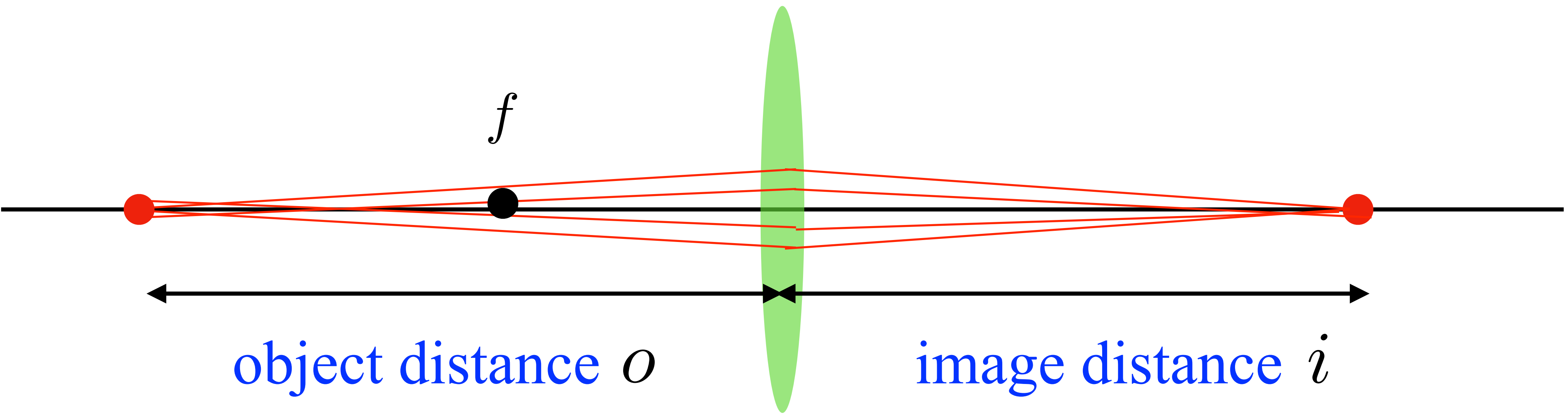


$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

Size is given by $$\frac{h'}{h} = \frac{2f - i}{2f - o} = -\frac{i}{o}$$

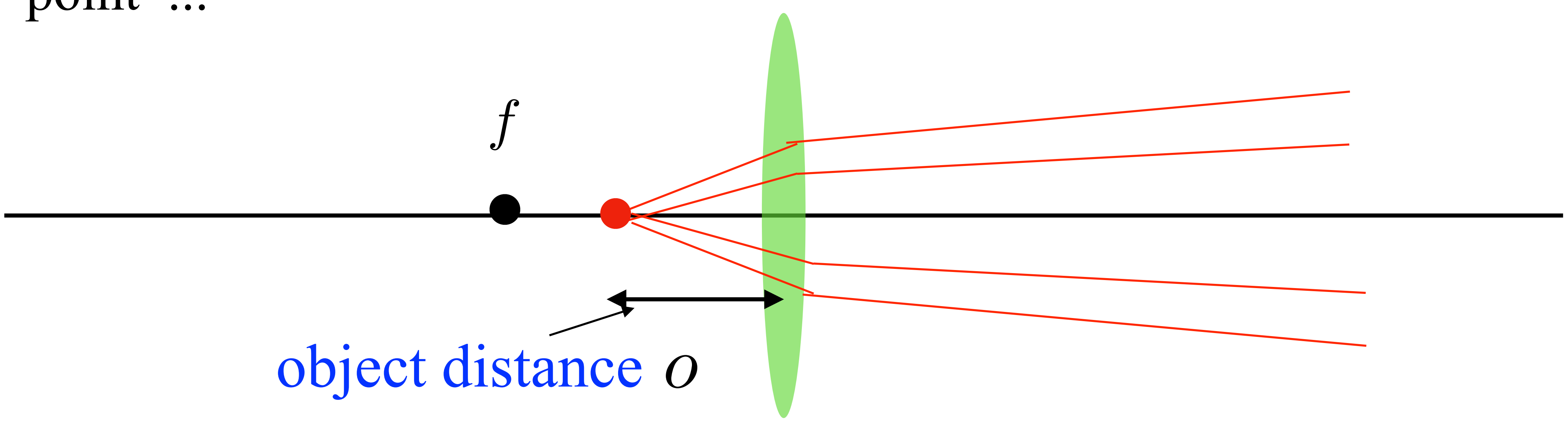
The image is inverted

Virtual images

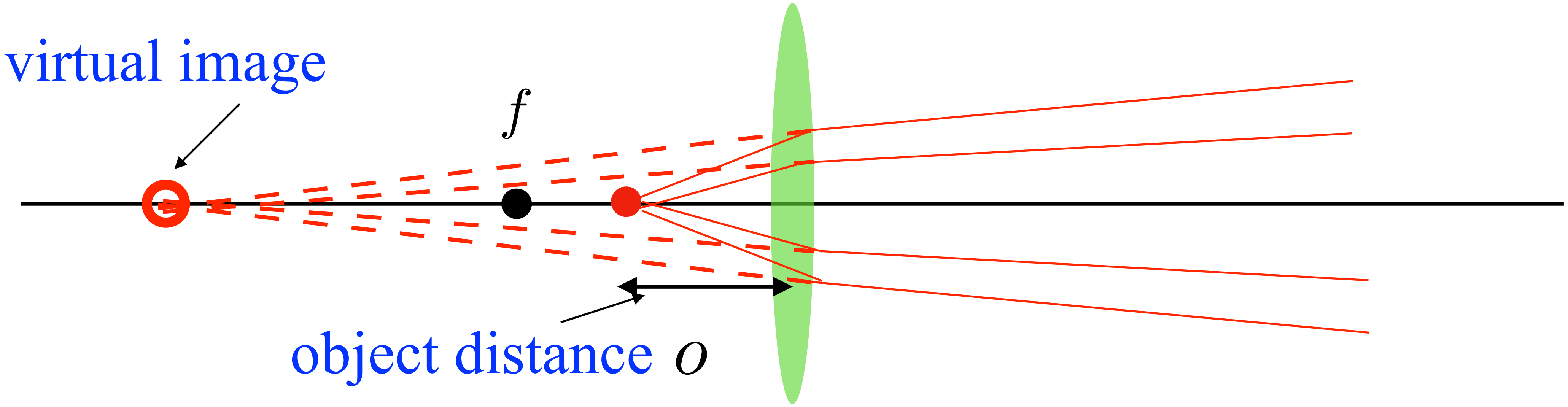


$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

If we place the object closer than the focal point, then the rays are unable to converge to a point ...

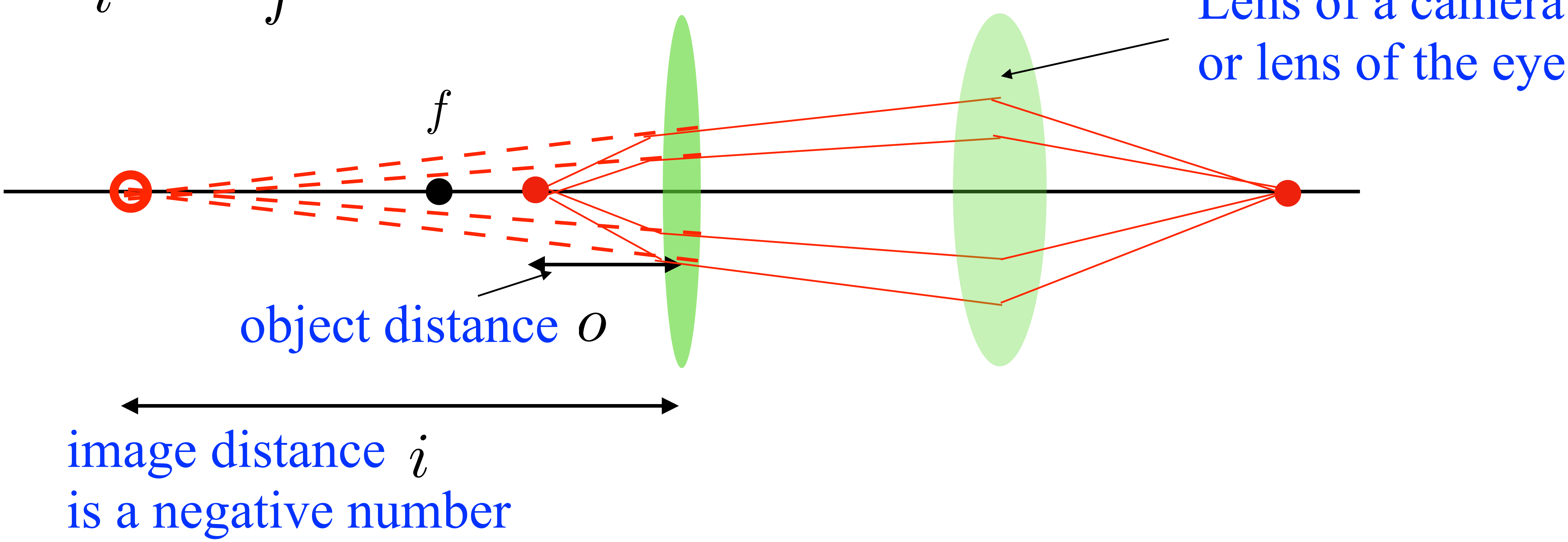


But all is not lost, as long as the rays still behave as if they came from a point, since we can use some other lens to make them converge ....

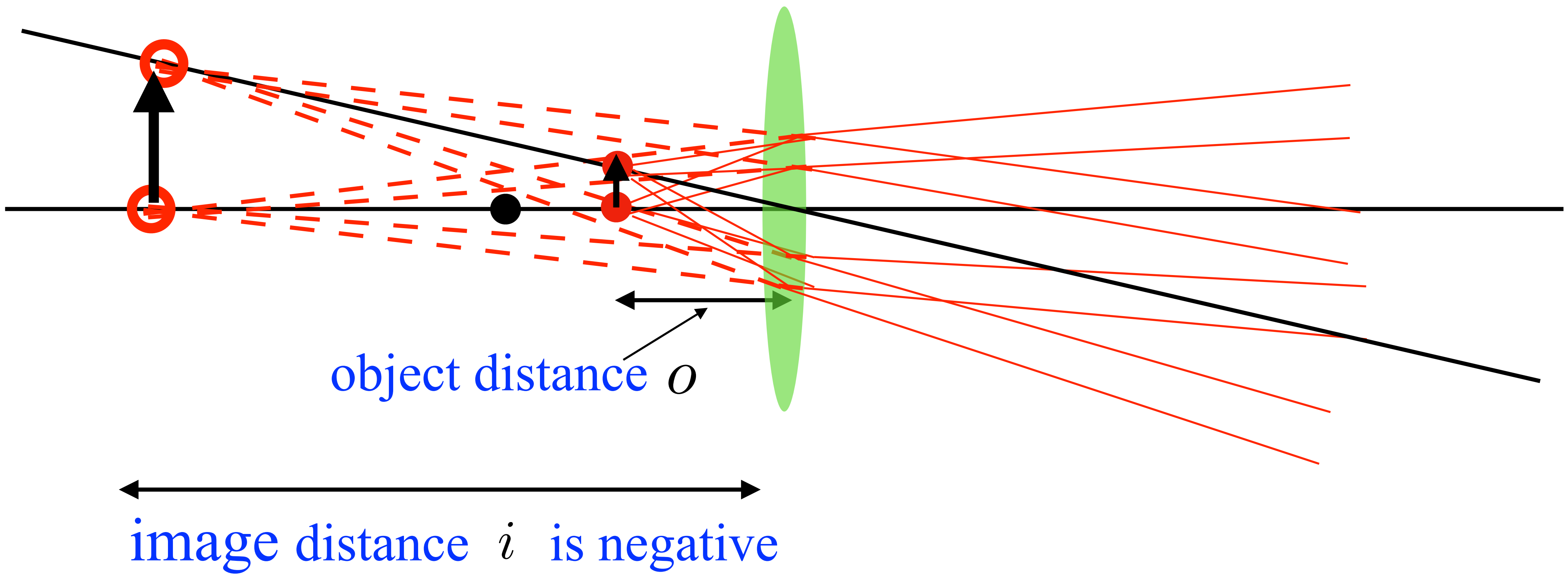


We can now use another lens to finally focus the rays to a point ....

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$



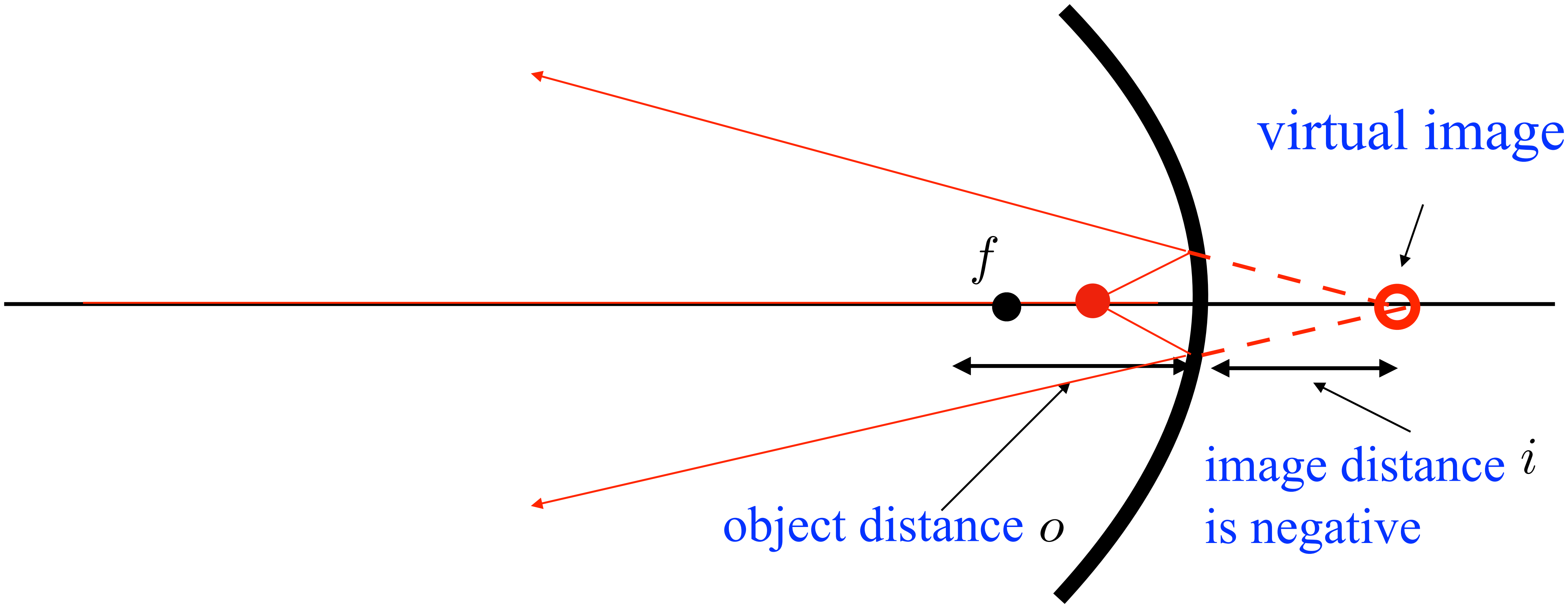
Magnification



Magnification $$\frac{h'}{h} = -\frac{i}{o}$$ is positive

So image is upright

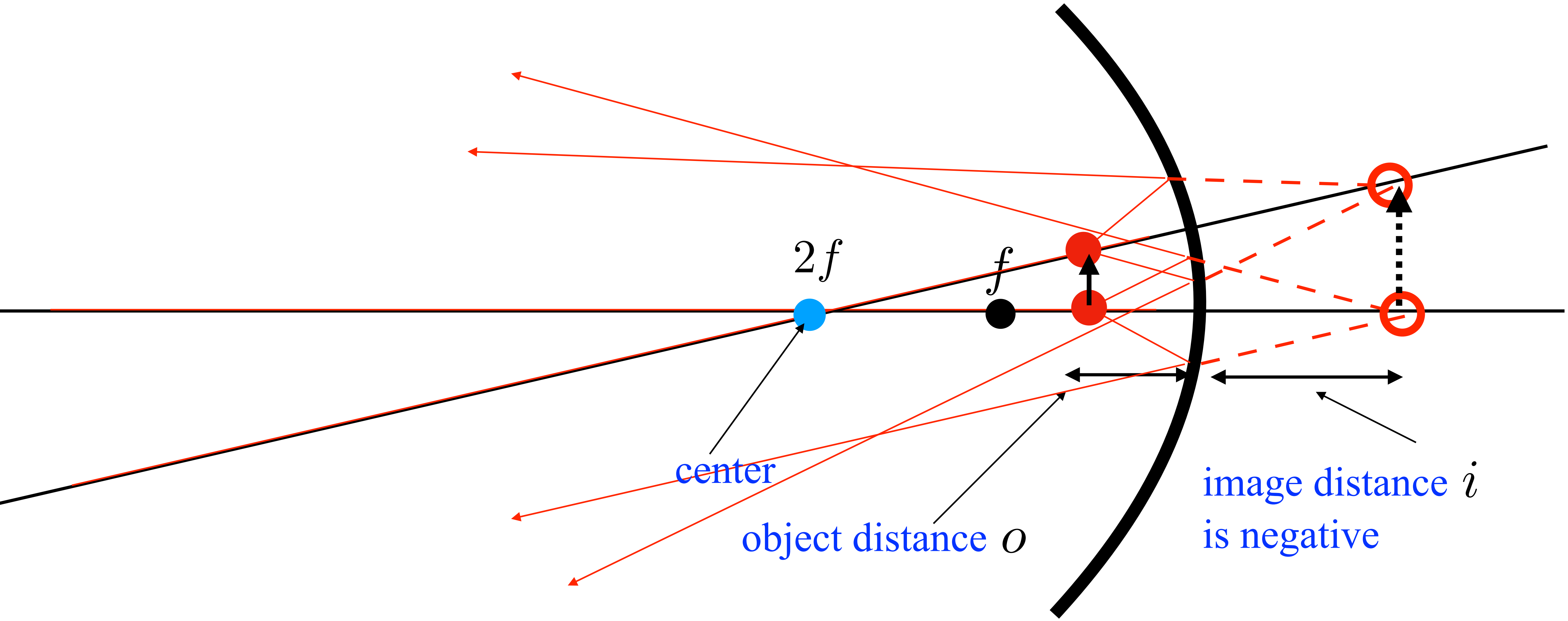
The same thing happens for a concave mirror



$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

i is a negative number

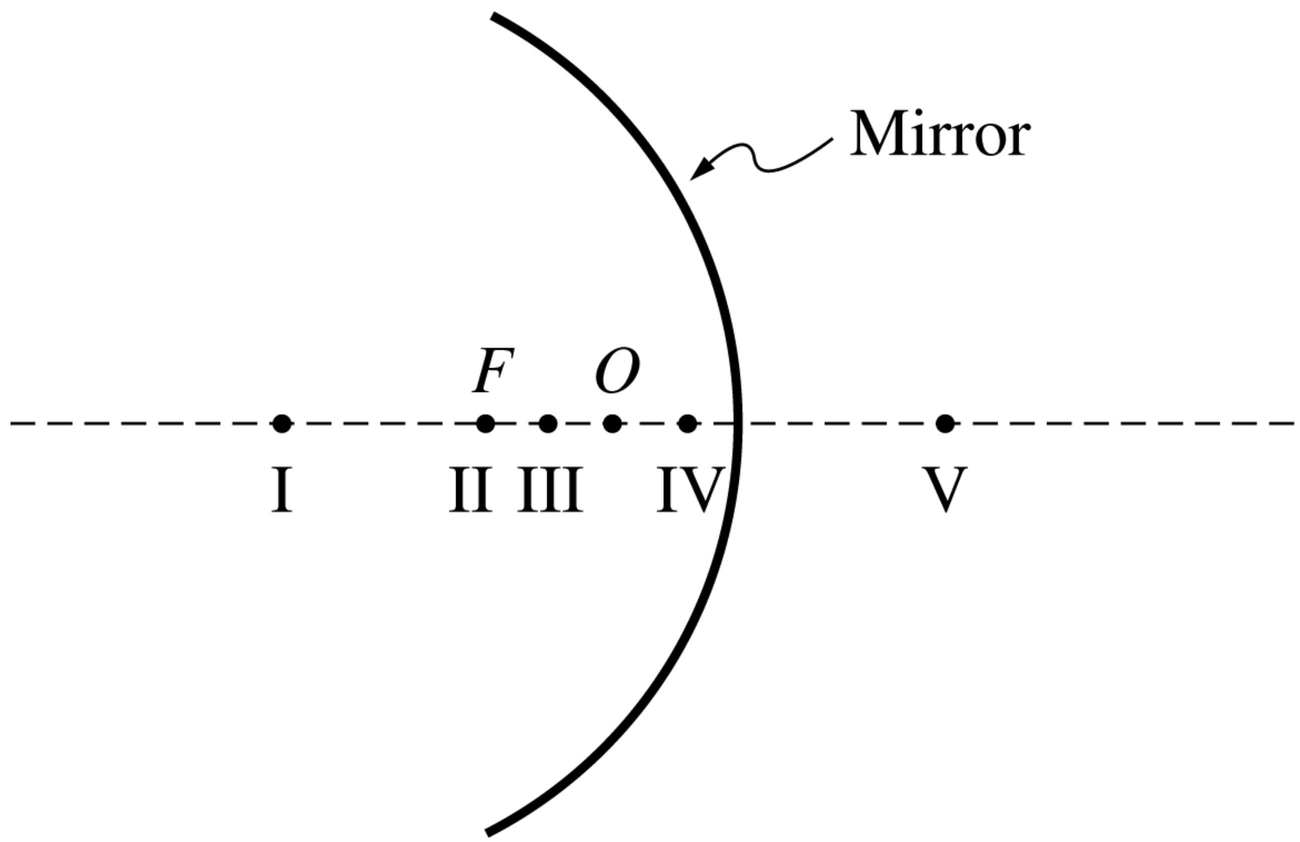
We can rotate about the center



$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

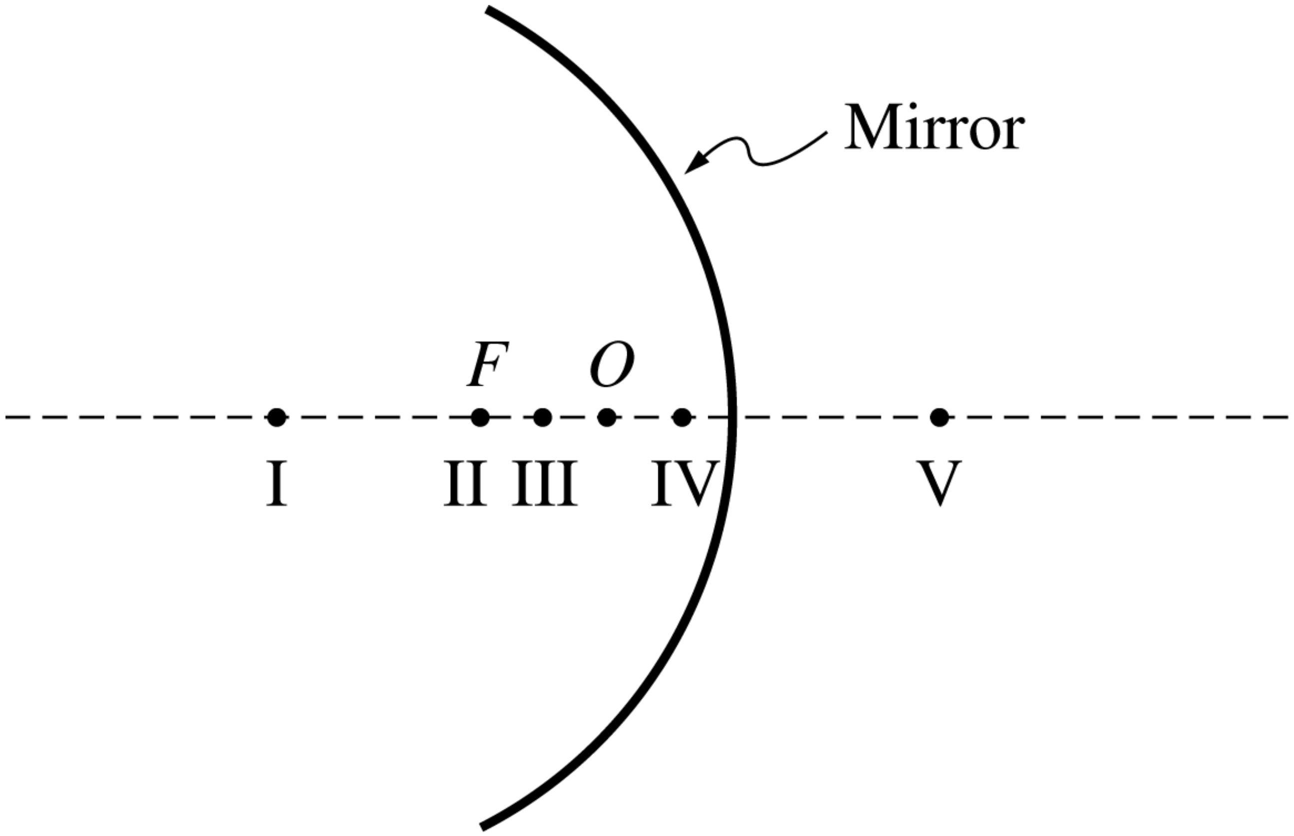
Magnification $-\frac{i}{o}$ is a positive number

So the virtual image is upright



12. A spherical, concave mirror is shown in the figure above. The focal point F and the location of the object O are indicated. At what point will the image be located?

(A) I
(B) II
(C) III
(D) IV
(E) V



12. A spherical, concave mirror is shown in the figure above. The focal point F and the location of the object O are indicated. At what point will the image be located?

(A) I
(B) II
(C) III
(D) IV
(E) V

15E: A shaving mirror has a radius of curvature of 35 cm. It is positioned so that the (upright) image of a man's face is 2.5 times the size of the face.

How far is the mirror from the face?

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How far is the mirror from the face?

$$f = \frac{35}{2}$$

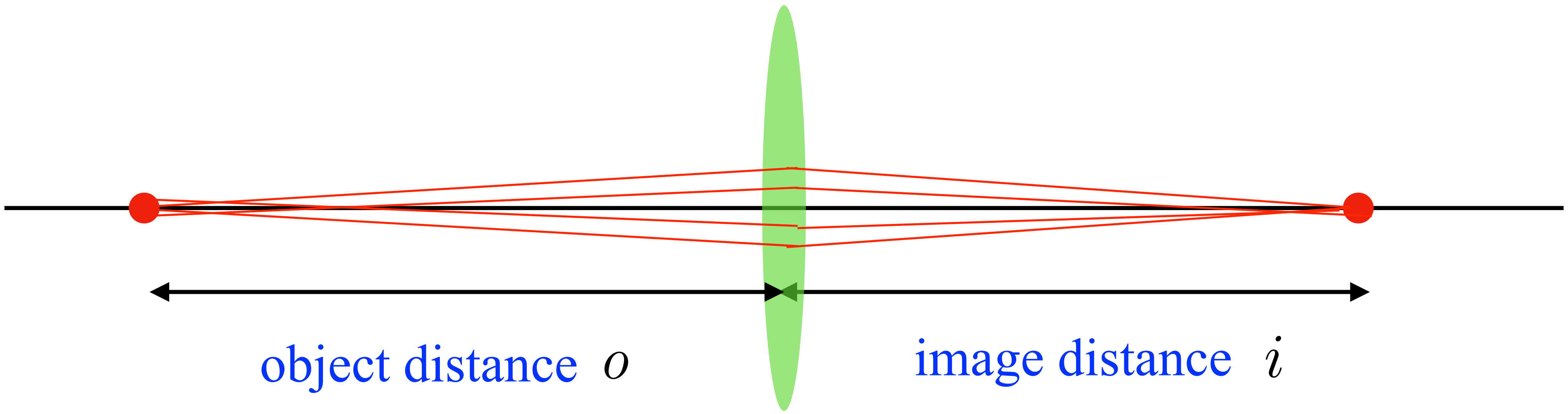
$$\frac{i}{o} = -2.5$$

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

$$o = 10.5\,cm$$

Sign changes

The standard configurations with all numbers positive and images real are



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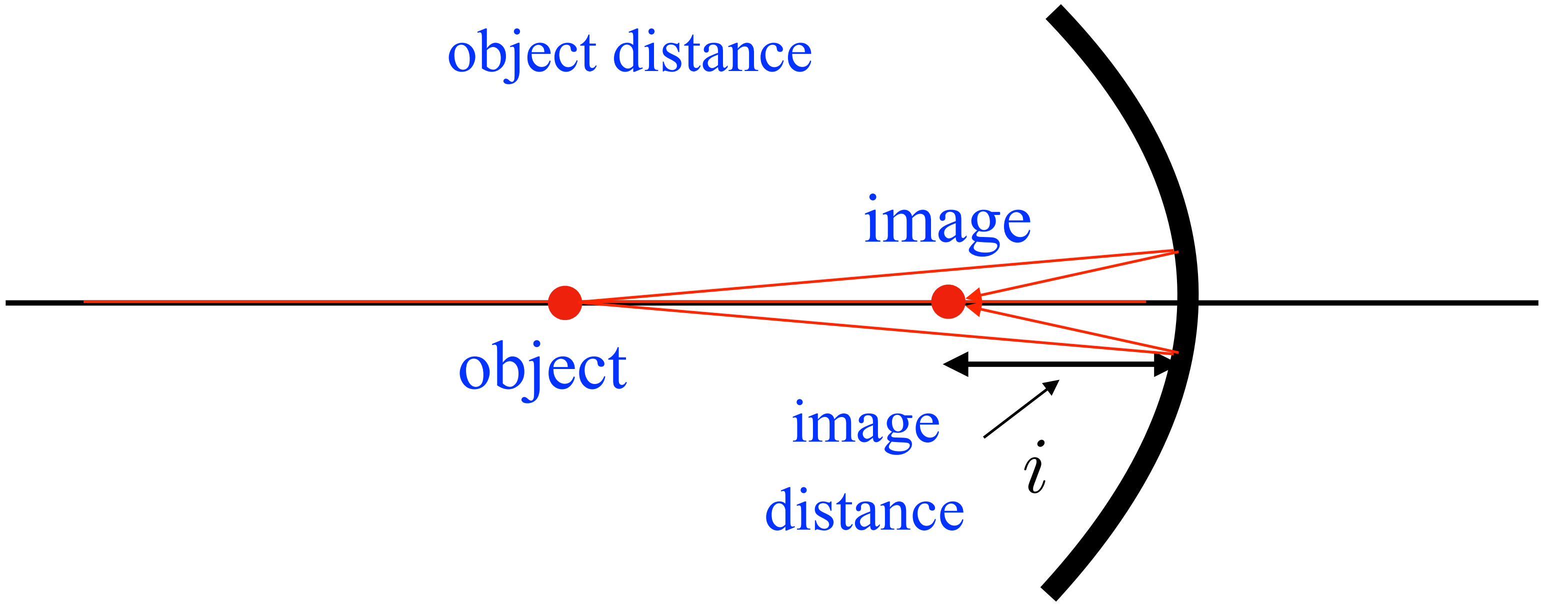


$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

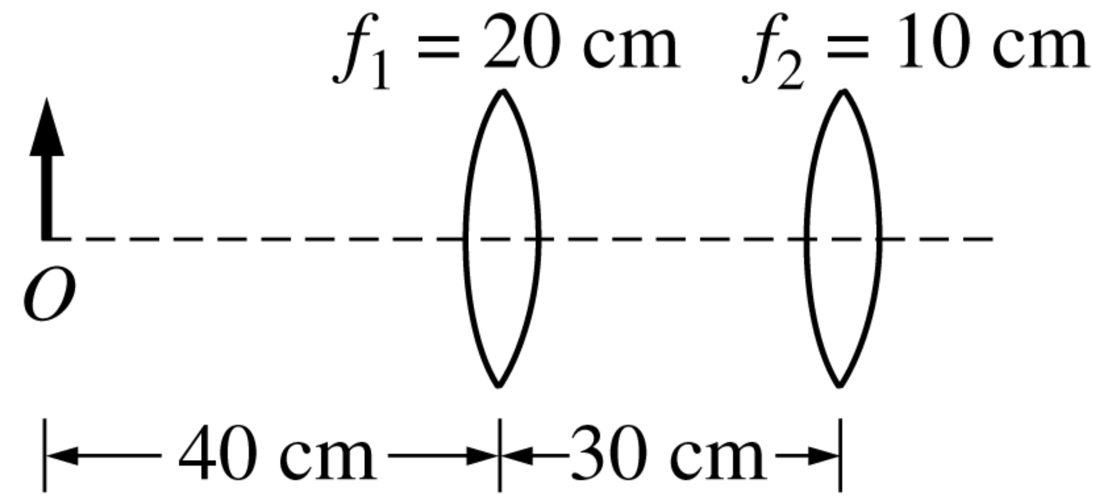
Any changes bring in a negative sign, but the formulae remain the same

(1) Convex lens has positive f , concave lens has negative f

(2) Concave mirror has positive f, convex mirror has negative f

(3) An object on the correct side has positive o ,
an object on the wrong side has negative o

(4) An image on the correct side has positive i ,
an object on the wrong side has negative i



11. An object is located 40 centimeters from the first of two thin converging lenses of focal lengths 20 centimeters and 10 centimeters, respectively, as shown in the figure above. The lenses are separated by 30 centimeters. The final image formed by the two-lens system is located

(A) 5.0 cm to the right of the second lens
(B) 13.3 cm to the right of the second lens
(C) infinitely far to the right of the second lens
(D) 13.3 cm to the left of the second lens
(E) 100 cm to the left of the second lens

$i_2 = 5$

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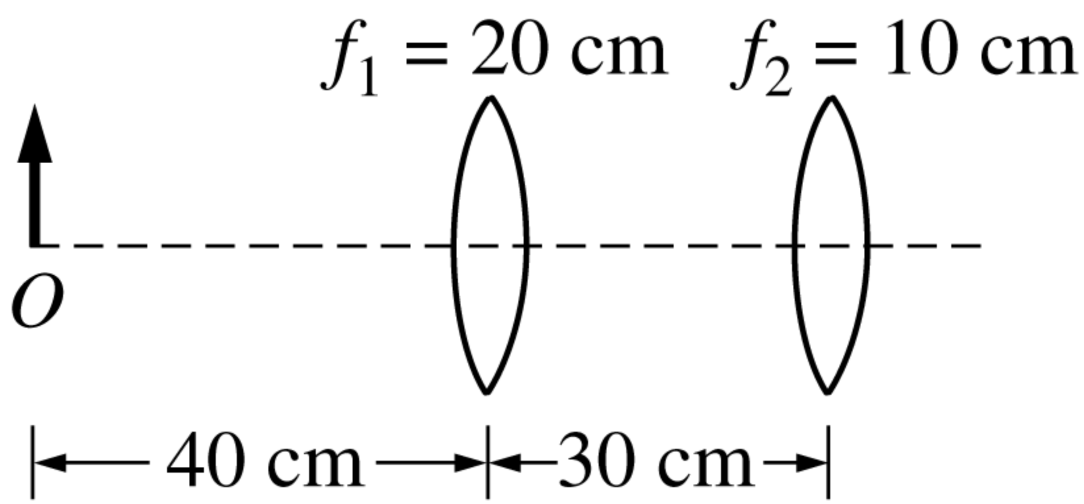

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first image

$$\frac{1}{i_1} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40}$$

$i_1 = 40$

to right of first lens

This is $o_2 = -10$

from the second lens

second image

$$\frac{1}{i_2} = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$$

$i_2 = 5$

Resolving power

13. Two stars are separated by an angle of 3×10^{-5} radians. What is the diameter of the smallest telescope that can resolve the two stars using visible light ($\lambda \cong 600$ nanometers) ? (Ignore any effects due to Earth's atmosphere.)

(A) 1 mm
(B) 2.5 cm
(C) 10 cm
(D) 2.5 m
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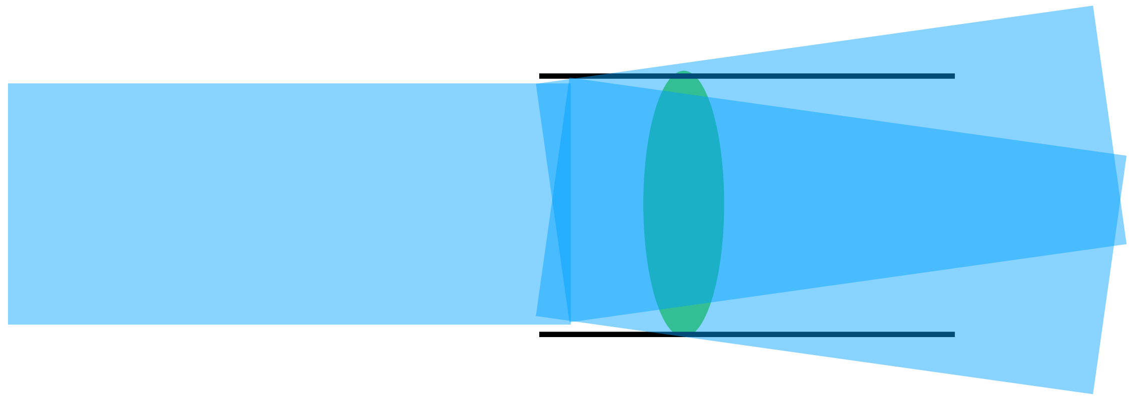

$$\Delta\theta \sim \frac{\lambda}{d}$$

$$\Delta\theta \sim \frac{\lambda}{d} \sim 3 \times 10^{-5}$$

$$d \sim 2\,cm$$