The Physics GRE

Sample test put out by ETS
https://www.ets.org/s/gre/pdf/practice_book_physics.pdf

OSU physics website has lots of tips, and 4 additional tests
http://www.physics.ohio-state.edu/undergrad/ugs_gre.php

Solutions to these tests are available online in some places (but not all explanations are good) ...

Study guides and supporting material can be borrowed for use in the PRB .

The oscilloscope
17. The outputs of two electrical oscillators are compared on an oscilloscope screen. The oscilloscope spot is initially at the center of the screen. Oscillator $Y$ is connected to the vertical terminals of the oscilloscope and oscillator $X$ to the horizontal terminals. Which of the following patterns could appear on the oscilloscope screen, if the frequency of oscillator $Y$ is twice that of oscillator $X$ ?

(B)

(C)

(D)

(E)


A steady current or a slowly changing potential can be measured by a Voltmeter


A steady current or a slowly changing current can be measured by an Ammeter


But the voltage may be oscillating at 10 Khz ... that is 10,000 oscillations per second.

How should we measure voltages that change quickly?

## There are two ways of using an oscilloscope



The X direction is a uniform rate sweep, so it marks time $t$

Put the voltage V to be measured on the Y terminals

Comparing two different voltage sources:
Put first voltage on the X terminals
Put second voltage on Y terminals
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What is the ratio of the frequency of vertical oscillations to the frequency of horizontal oscillations ?


Example of an analog oscilloscope Lissajous figure, showing a harmonic relationship of 1 horizontal oscillation cycle to 3 vertical oscillation cycles.

The Capacitor

25. Two real capacitors of equal capacitance ( $C_{1}=C_{2}$ ) are shown in the figure above. Initially, while the switch $S$ is open, one of the capacitors is uncharged and the other carries charge $Q_{0}$. The energy stored in the charged capacitor is $U_{0}$. Sometime after the switch is closed, the capacitors $C_{1}$ and $C_{2}$ carry charges $Q_{1}$ and $Q_{2}$, respectively; the voltages across the capacitors are $V_{1}$ and $V_{2}$; and the energies stored in the capacitors are $U_{1}$ and $U_{2}$. Which of the following statements is INCORRECT?
(A) $Q_{0}=\frac{1}{2}\left(Q_{1}+Q_{2}\right)$
(B) $Q_{1}=Q_{2}$
(C) $V_{1}=V_{2}$
(D) $U_{1}=U_{2}$
(E) $U_{0}=U_{1}+U_{2}$



The battery pushes electrons out from its negative terminal.

In effect this creates a flow of positive charge out of the positive terminal

But after a very short time electrons pile up near the end of the wire.

They repel each other, pushing back against the pressure from the battery

Can we do anything to make the current flow go on longer?

Suppose we expand the end of the wire so that it has a large area ...

Then we can flow out more charge before the 'pushback' stops the current flow


If we could place some positive charges near the negative plate, that would attract the electrons, and reduce the tendency of the electrons to push back ....


But where would we get such charges from?

The capacitor

Store a lot of charge !

## Large area $A$



## Defining capacitance



Voltage is like a 'pressure'
More Voltage allows storage of more charge
$Q \propto V$

$$
Q=C V
$$

Units:

1 Farad


At the start capacitor plates accept charges with no pushback

So C can be replaced with a wire

$$
I=\frac{V}{R}
$$



With some charge, the capacitor acts like a 'reverse battery'

So C can be replaced with a wire

$$
I=\frac{V-V^{\prime}}{R}
$$



The charge on the capacitor approaches $\quad Q=C V$

Then there is no net voltage in the circuit, so $\quad I=0$


If you quickly vary the voltage, then the capacitor never charges very much
There is hardly any 'pushback voltage'


So in this situation C can be replaced with a wire

## GR9677



Figure 1


Figure 2

1. The capacitor shown in Figure 1 above is charged by connecting switch $S$ to contact $a$. If switch $S$ is thrown to contact $b$ at time $t=0$, which of the curves in Figure 2 above represents the magnitude of the current through the resistor $R$ as a function of time?
(A) $A$
(B) $B$
(C) $C$
(D) $D$
(E) $E$

## GR9677



Figure 1


Figure 2

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## Energy stored in a capacitor

Start wth no charge on plates; the energy is zero
The battery moves charge from the negative plate to the positive plate
Suppose the charges on the plates have reached $\quad Q^{\prime}$ and $-Q^{\prime}$
Then the potential difference is $V=\frac{Q^{\prime}}{C}$
Move an additional charge $d Q^{\prime}$
This needs an energy $d E=V d Q^{\prime}=\frac{Q^{\prime}}{C} d Q^{\prime}$
Thus the total stored energy is


$$
E=\int_{Q^{\prime}=0}^{Q} \frac{Q^{\prime}}{C} d Q^{\prime}=\frac{Q^{2}}{2 C}
$$

Note that $E=\frac{Q^{2}}{2 C}=\frac{1}{2} C V^{2}=\frac{1}{2} Q V$

Charging/discharging a capacitor

## Charging



$$
\frac{d Q(t)}{d t}=I(t)
$$

The capacitor acts like a 'reverse battery'
Solution


## Discharging



$$
V(t)=I(t) R
$$

$$
\frac{Q(t)}{C}=I(t) R
$$

$$
\frac{1}{C} \frac{d Q(t)}{d t}=\frac{d I(t)}{d t} R
$$

$$
\frac{d I(t)}{d t}=\frac{1}{R C} I(t)
$$



Solution

$$
\begin{aligned}
& I(t)=I_{0} e^{-\frac{t}{R C}} \\
& I_{0}=\frac{V_{0}}{R}=\frac{Q_{0}}{R C} \\
& Q(T)=Q_{0} e^{-\frac{t}{R C}}
\end{aligned}
$$


11. The capacitor in the circuit shown above is initially charged. After closing the switch, how much time elapses until one-half of the capacitor's initial stored energy is dissipated?
(A) $R C$
(B) $\frac{R C}{2}$
(C) $\frac{R C}{4}$
(D) $2 R C \ln (2)$
(E) $\frac{R C \ln (2)}{2}$

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GR9277
36. The circuit shown above is used to measure the size of the capacitance $C$. The $y$-coordinate of the spot on the oscilloscope screen is proportional to the potential difference across $R$, and the $x$-coordinate of the spot is swept at a constant speed $s$. The switch is closed and then opened. One can then calculate $C$ from the shape and the size of the curve on the screen plus a knowledge of which of the following?
(A) $V_{0}$ and $R$
(B) $s$ and $R$
(C) $s$ and $V_{0}$
(D) $R$ and $R^{\prime}$
(E) The sensitivity of the oscilloscope

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(B) $s$ and $R$
(C) $s$ and $V_{0}$
(D) $R$ and $R^{\prime}$

## Inductors

An inductor is just a coil


There is no resistance
So if a steady current is passing, there is no effect of the inductor ... it is just a piece of ordinary wire with no resistance

But if we try to change the current, the inductor does not want to allow that to happen, and it resists the change ....

When current passes through the coil, there is a magnetic field generated


But a magnetic field costs energy, so we cannot immediately go from (a) to

What can stop the current from increasing? The coil develops a reverse voltage, which opposes the increase of current ...


$$
\frac{d I}{d t}>0
$$

How much is this reverse voltage ?

$$
\begin{gathered}
V \propto \frac{d I}{d t} \\
V=L \frac{d I}{d t}
\end{gathered}
$$

Current through an inductor cannot change quickly: the inductor will oppose the change

If we make a slow change in the current, the inductor has no effect: it can be ignored
39. Which two of the following circuits are high-pass filters?

III.

(A) I and II
(B) I and III
(C) I and IV
(D) II and III
(E) II and IV
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45. The circuits below consist of two-element combinations of capacitors, diodes, and resistors. $V_{\text {in }}$ represents an ac-voltage with variable frequency. It is desired to build a circuit for which $V_{\text {out }} \approx V_{\text {in }}$ at high frequencies and $V_{\text {out }} \approx 0$ at low frequencies. Which of the following circuits will perform this task?
(A)

(B)

(C)

(D)

(E)


## High frequency

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(A)

(B)

(C)

(D)

(E)


## Low frequency

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(A)

(B)

(C)

(D)

(E)


Charging/discharging an inductor

## Charging



The inductor acts like a 'reverse battery'

$$
\begin{aligned}
& V-V(t)=I(t) R \\
& V-L \frac{d I(t)}{d t}=I(t) R \\
& \frac{d I(t)}{d t}=\frac{V}{L}-\frac{R}{L} I(t)
\end{aligned}
$$

Solution

$$
I(t)=I_{0}\left(1-e^{-\frac{R}{L} t}\right)
$$

$$
I_{0}=\frac{V}{R}
$$

## Discharging

Current cannot suddenly stop, since the energy in the magnetic field cannot vanish suddenly ...

So now the inductor has to act like a 'forward battery'

$I(t)$
$V(t)=I(t) R$
$L \frac{d I(t)}{d t}=-I(t) R$
$\frac{d I(t)}{d t}=-\frac{R}{I} I(t)$


## Solution

$I(t)=I_{0} e^{-\frac{R}{L} t}$

40. In the circuit shown above, the switch $S$ is closed at $t=0$. Which of the following best represents the voltage across the inductor, as seen on an oscilloscope?
(A) Voltage

(B) Voltage

(C) Voltage

(D) Voltage

(E) Voltage


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(C)

(D) Voltage

(E) Voltage


## Oscillating currents

An oscillating current or voltage source

$$
\begin{aligned}
& I=I_{0} \cos (\omega t) \\
& V=V_{0} \cos (\omega t)
\end{aligned}
$$



Average current is zero



Root mean square current (RMS current)

$$
I_{r m s}=\frac{1}{\sqrt{2}} I_{0}
$$

$\longrightarrow$ Heat dissipated $I^{2} R=\frac{V^{2}}{R}=V I$


Heat dissipated
$\left\langle I^{2}\right\rangle R=I_{r m s}^{2} R$

40. A series AC circuit with impedance $Z$ consists of resistor $R$, inductor $L$, and capacitor $C$, as shown above. The ideal emf source has a sinusoidal output given by $\boldsymbol{\mathcal { E }}=\boldsymbol{\mathcal { E }}_{\max } \sin \omega t$, and the current is given by $I=I_{\max } \sin (\omega t-\phi)$. What is the average power dissipated in the circuit? ( $I_{r m s}$ is the root-mean-square current.)
(A) $I_{r m s}^{2} R$
(B) $\frac{1}{2} I_{r m s}^{2} R$
(C) $\frac{1}{2} I_{r m s}^{2} Z$
(D) $\frac{1}{2} I_{r m s}^{2} R \cos \phi$
(E) $\frac{1}{2} I_{r m s}^{2} Z \cos \phi$

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(E) $\frac{1}{2} I_{r m s}^{2} Z \cos \phi$

## RLC circuits


(GRO177)
38. An AC circuit consists of the elements shown above, with $R=10,000$ ohms, $L=25$ millihenries, and $C$ an adjustable capacitance. The AC voltage generator supplies a signal with an amplitude of 40 volts and angular frequency of 1,000 radians per second. For what value of $C$ is the amplitude of the current maximized?
(A) 4 nF
(B) 40 nF
(C) $4 \mu \mathrm{~F}$
(D) $40 \mu \mathrm{~F}$
(E) $400 \mu \mathrm{~F}$


We write $\quad I=\operatorname{Re}\left[I_{0} e^{i \omega t}\right]$
and just write the complex current from now on. $\quad I=I_{0} e^{i \omega t}$

$$
\begin{aligned}
& \frac{d I}{d t}=i \omega I \\
& V=(i L \omega) I
\end{aligned}
$$

Compare to $\quad V=I R$ $\square$


Current $I$ means that $I$ coulombs flow into the plate each second

$$
\frac{d Q}{d t}=I
$$

$$
Q=C V \quad \Rightarrow \quad C \frac{d V}{d t}=I
$$

$$
V=V_{0} e^{i \omega t}
$$

$\rightarrow$

$$
C(i \omega V)=I
$$

$$
V=I\left(\frac{1}{i C \omega}\right)
$$

$$
\text { Reactance } \frac{1}{i C \omega}
$$



$$
R_{e f f}=R+\frac{1}{i C \omega}+i L \omega
$$

$$
I=\frac{V}{R_{e f f}}
$$

The current is maximal when the effective resistance is minimal ...

$$
\begin{aligned}
& |I|=\frac{|V|}{\left|R_{e f f}\right|} \\
& R_{e f f}=R+i\left(-\frac{1}{C \omega}+L \omega\right) \\
& \left|R_{e f f}\right|^{2}=R^{2}+\left(-\frac{1}{C \omega}+L \omega\right)^{2}
\end{aligned}
$$

If $R, C, L$ are fixed, and we can only vary $\omega$, then $\left|R_{e f f}\right|$ is minimal when

$$
\omega=\frac{1}{\sqrt{L C}}
$$

At this frequency we say that the circuit is in RESONANCE


GR9277
81. In the $R L C$ circuit shown above, the applied voltage is

$$
\varepsilon(t)=\varepsilon_{\mathrm{m}} \cos \omega t
$$

For a constant $\varepsilon_{\mathrm{m}}$, at what angular frequency $\omega$ does the current have its maximum steady-state amplitude after the transients have died out?
(A) $\frac{1}{R C}$
(B) $\frac{2 L}{R}$
(C) $\frac{1}{\sqrt{L C}}$
(D) $\sqrt{\frac{1}{L C}-\left(\frac{R}{2 L}\right)^{2}}$
(E) $\sqrt{\left(\frac{1}{R C}\right)^{2}-\left(\frac{L}{R}\right)^{2}}$


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# Challenge question 


(A)

(B)

94. In the circuit shown above, $R_{2}=3 R_{1}$ and the battery of emf $\mathcal{E}$ has negligible internal resistance. The resistance of the diode when it allows current to pass through it is also negligible. At time $t=0$, the switch $S$ is closed and the currents and voltages are allowed to reach their asymptotic values. Then at time $t_{1}$, the switch is opened. Which of the following curves most nearly represents the potential at point $A$ as a function of time $t$ ?
(C)

(D)

(E)


