Name: ____________________

Physics 7502 Quantum Mechanics Spring 2019

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Quiz 1

Given: Mon, Jan 14, 2019 *Time: 20 minutes*

Problem 1: The radial equation for the Hydrogen atom can be reduced to

$$H''(\rho) + (\frac{2l+2}{\rho} - 1)H'(\rho) + [\frac{(\lambda - 1 - l)}{\rho}]H(\rho) = 0 \tag{1}$$

where

$$\lambda = \sqrt{\frac{2m}{(-E)}}\frac{k}{2\hbar} \tag{2}$$

(i) Solve this equation by the series method, thus obtaining the energy levels E_n. (5 points)

(ii) Using your result, write down $H(\rho)$ for the case $n = 3, l = 0$. (3 points)

(iii) What is the degeneracy of the level $n = 3$? (2 points)

Solution: : (i) We write

$$H = \sum_{q\geq 0} a_q \rho^q \tag{3}$$

$$H'' + (\frac{2l+2}{\rho} - 1)H' + [\frac{(\lambda - 1 - l)}{\rho}]H = 0 \tag{4}$$

$$q(q-1)a_q\rho^{q-2} + (\frac{2l+2}{\rho} - 1)qa_q\rho^{q-1} + [\frac{(\lambda - 1 - l)}{\rho}]a_q\rho^q = 0 \tag{5}$$

We look at the coefficient of ρ^{q-1}

$$(q+1)qa_{q+1} + (2l+2)(q+1)a_{q+1} - qa_q + [(\lambda - 1 - l)]a_q = 0 \tag{6}$$

$$(q+1)(q+2l+2)a_{q+1} = (q - \lambda + 1 + l)a_q \tag{7}$$

$$a_{q+1} = \frac{(q - \lambda + 1 + l)}{(q+1)(q+2l+2)}a_q \tag{8}$$

Thus we get

$$\lambda = l + 1 + q_{max}, \quad q_{max} = 0, 1, 2, \ldots \tag{9}$$

We define

$$n = l + 1 + q_{max} \tag{10}$$

Then

$$\sqrt{\frac{2m}{(-E)}}\frac{k}{2\hbar} = n \tag{11}$$

$$\frac{2m}{(-E)}\frac{k^2}{4\hbar^2} = n^2 \tag{12}$$

$$E = -\frac{mk^2}{2\hbar^2 n^2} = -\frac{mZ^2e^4}{2(4\pi\epsilon_0)^2\hbar^2}\frac{1}{n^2} \tag{13}$$

(ii) We have $\lambda = n = 3$ and $l = 0$. Thus

$$a_{q+1} = \frac{(q - \lambda + 1 + l)}{(q+1)(q+2l+2)}a_q = \frac{(q-3+1)}{(q+1)(q+2)}a_q = \frac{q-2}{(q+1)(q+2)}a_q \tag{14}$$

We set $a_0 = 1$. Then

$$a_1 = \frac{(-2)}{2} = -1 \tag{15}$$

$$a_2 = \frac{(-1)}{(2)(3)}(-1) = \frac{1}{6} \tag{16}$$

$$a_3 = 0 \tag{17}$$

Thus

$$H = 1 - \rho + \frac{1}{6}\rho^2 \tag{18}$$

(iii) We have

$$n = 3, \quad l = 0, 1, 2, \quad \rightarrow \quad 1 + 3 + 5 = 9 \tag{19}$$

Thus the degeneracy is 9.