Toward a comprehensive picture of student understanding of force, velocity, and acceleration

Rebecca Rosenblatt, Eleanor C. Sayre and Andrew F. Heckler

Department of Physics, The Ohio State University, Columbus, Ohio 43201

Abstract. Students' difficulties with conceptual questions about force, velocity, and acceleration have been well documented. However, there has been no single systematic study of student understanding of all paired relations among the concepts of force, velocity, and acceleration. For example, a student who believes an object with a net force on it must be moving might not believe an accelerating object must be moving. In this paper, we describe the development of a test to build a more comprehensive picture of student understanding. We describe modifications to increase the validity of the test by reducing false positives and unwanted inconsistencies. We also report preliminary data suggesting that there are definite patterns in student understanding of the various relations between force, velocity, and acceleration. For example, there are a higher number of students reporting that force and velocity are directionally related then that acceleration and velocity are directionally related.

Keywords: test development, mechanics, physics education research
PACS: 01.50.Kw, 01.40.Fk

INTRODUCTION

Student difficulties with conceptual questions about the relationships among force, velocity, and acceleration have been extensively documented. For example, many students believe that if an object is moving, something must be pushing it in that direction [1, 2, 3, 4]. Students also commonly believe that if something is pushing a moving object, the object has to be going in the direction of the push [3, 5]. Other researchers have found similar mistakes in students' understanding of the relationships between velocity and acceleration [6, 7]. However, there has been no systematic study of students' understanding of all six possible paired, directional relations among the concepts of force, velocity, and acceleration. For example, in a landmark paper Viennot (1979) studied student understanding of the conditional relation "given the velocity of an object, what is the (net) force on the object?" Yet there were no questions probing the condition "given a net force on an object, what is its velocity?", or any questions regarding the relations between velocity and acceleration or acceleration and force [4].

For one dimensional motion there are three ways for any vector, such as force, velocity, or acceleration, to be related to any other vector: the vectors can be aligned, $\uparrow\uparrow$, opposite to one another, $\uparrow\downarrow$, or one of them can be zero, $\uparrow 0$. Using acceleration and velocity to illustrate the seven possible combinations of these relationships yields: $\vec{a} \uparrow\uparrow \vec{v}$ ($\vec{v}$ must be parallel to $\vec{a}$); $\vec{a} \uparrow\downarrow \vec{v}$ ($\vec{v}$ must be antiparallel to $\vec{a}$); $\vec{a} \uparrow 0\ \vec{v}$ ($\vec{v}$ must be zero); $\vec{a}(\uparrow\uparrow, \uparrow 0\ )\vec{v}$ ($\vec{v}$ can be zero or parallel to $\vec{a}$); $\vec{a}(\uparrow\uparrow, \uparrow\downarrow)\vec{v}$ ($\vec{v}$ can be parallel or antiparallel to $\vec{a}$); $\vec{a}(\uparrow\downarrow, \uparrow 0)\vec{v}$ ($\vec{v}$ can be zero or antiparallel to $\vec{a}$); $\vec{a}(\uparrow\uparrow, \uparrow\downarrow, \uparrow 0)\vec{v}$ ($\vec{v}$ can be zero or parallel or antiparallel to $\vec{a}$).

In this paper, we describe the initial stages of a study designed to build a more comprehensive picture of student understanding of all possible relations between force, velocity, and acceleration through the development and implementation of a simple multiple-choice test. We then compare three different forms of a test to gain insight into the validity of the test items, and report preliminary results from what we conclude is the most valid version of the test.

TEST DEVELOPMENT

Unlike tests in previous studies [1, 2, 3, 4, 5], the tests in this study consist mostly of text without diagrams. The three instruments vary in answer choice form and in story contexts of the questions (see Table 1). Data were collected via students' responses on the multiple choice tests, and a small subset were interviewed via a think-aloud protocol while they were completing the tests. All students were in the calculus based introductory mechanics course for physicists and engineers at The Ohio State University. Test 1 was administered as part of an in-class quiz graded for participation only. Tests 2 and 3 were administered as part of a required special assignment graded for participation only. In each case, the tests were proctored and reports from the proctors anecdotally indicated that students made reasonable efforts to answer the questions. Each test was administered to a separate group of students in different, traditionally taught lecture sections.

Test 1

This test is a simple multiple-choice, single-response test where each item has four choices. Students' answers could be practically categorized into two categories: either the answer was correct or the answer was consistent with a common misconception such as velocity is in the direction of net force. However, there are several drawbacks to the test

TABLE 1. Example of answer choices for the different tests for an $\vec{a} \rightarrow \vec{v}$ question. The prompt is "A car is on a hill and the direction of its acceleration is uphill. Which statement best describes the motion of the car?"

Test 1	Test 2 (Always, Sometimes, Never)	Test 3	Test 3 model
a) It is moving uphill	(A) (S) (N): The car is moving uphill	a) it is moving uphill	$\vec{a} \uparrow\uparrow \vec{v}$
b) It is moving downhill	(A) (S) (N): The car is moving downhill	b) it is moving downhill	$\vec{a} \uparrow\downarrow \vec{v}$
c) It is not moving	(A) (S) (N): The car is not moving	c) it is not moving	$\vec{a} \uparrow 0 \vec{v}$
d) Not enough information		d) both a and b are possible	$\vec{a}(\uparrow\uparrow, \uparrow\downarrow)\vec{v}$
		e) both a and c are possible	$\vec{a}(\uparrow\uparrow, \uparrow 0\ )\vec{v}$
		f) a, b, and c are possible	$\vec{a}(\uparrow\uparrow, \uparrow\downarrow, \uparrow 0)\vec{v}$

as designed. First, the correct answer is often "not enough information," and this choice may be ambiguous enough to incorrectly categorize students as correctly understanding the relationship in question when they only partially understand it. Another problem is its limited ability to capture a student's individual model.

Only four of the possible models are available answers on Test 1. Any students holding the other three models are grouped into one of the four available models making the data less meaningful. For example, see Table 1, a student who believes that velocity can be either against or in the direction of acceleration but not zero, i.e. model $\vec{a}(\uparrow\uparrow, \uparrow\downarrow)\vec{v}$, would probably say that there is not enough information to answer the example question. We would therefore have no way to distinguish these students from the students who believe that velocity can be either in the direction of the acceleration or zero but not opposite, i.e. model $\vec{a}(\uparrow\uparrow, \uparrow 0\ )\vec{v}$, or from students who are correct and think velocity can be in either direction or zero, i.e. model $\vec{a}(\uparrow\uparrow, \uparrow\downarrow, \uparrow 0)\vec{v}$.

Another drawback is that the answer choice "not enough information" is a unique kind of answer choice, and students may or may not choose it for reasons that are unrelated to the physics content of the problem. Informal interviews gave anecdotal evidence that this may have occurred.

Test 2 and Test 3

In response to these issues a second test version was developed. This test consisted of similar questions but the answer choices were "always" (A), "sometimes" (S), or "never" (N), for each option $\uparrow\uparrow$, $\uparrow\downarrow$, or $\uparrow 0$, see Table 1. Students were given two simple practice questions (with answers provided) to familiarize themselves with the novel format, and we found in think-aloud interviews that they did not have difficulty in understanding this format.

All combinations of answers are available for the student to choose in this format, which reduces the chance of grouping students with different models together and allows more accurate categorization of students into distinct conceptual models. For example, only students who said sometimes for all options were labeled correct, and only students who responded that given a force the velocity was always in the same direction and never opposite to the force and never zero were labeled as holding the common misconception, $\vec{F} \uparrow\uparrow \vec{v}$.

However, interviews revealed that while students often would answer in a (formally) logically consistent manner within a given question (e.g. "if it is always moving right, then it is never moving left or at rest"), on other questions the same student would answer in logically inconsistent ways (e.g. "it is *always* moving right, but *sometimes* it could be going left"). On average, questions were answered illogically 9% of the time. In all, only 29 out of the 72 students answered all Test 2 questions logically. While the lack of self consistency is interesting and could provide some insight into student answering and reasoning habits, we designed Test 3 to include most possible (and logically consistent) answers, resulting in a multiple choice format with up to 7 mutually exclusive choices (see Table 1).

In addition to the development of the answer choice format, there was development of the text of the posed questions. The most significant change in question format involved the term "net force." Interviews revealed that students often misinterpret the term "net force." For example, if a question reads "*The net force is to the right. Which way is the velocity*?" a student might respond by saying that the velocity could be left or right as long as there is another force, in addition to the net force, pushing parallel to the velocity. Consequently, we revised test questions to reduce the use of "net force," replacing it with longer phrases such as, "there may be several forces but the forces to the left are greater." Interviews showed that students did not have difficulties understanding this question format, and there was no evidence that they interpreted the question differently than what was intended.

PERFORMANCE DIFFERENCES BETWEEN TESTS

In order to compare the three test formats, Table 2 presents the response percentages for three questions in each test. For these examples, the question posed is the same, and the only difference is in the answer choices. There are several points to consider from these results. First, it appears as though students taking Test 1 chose the correct answer more often than those taking Tests 2 or 3 ($p < 0.01$). The differences in the answering patterns between Tests 2 and 3 are smaller, though a chi-squared test for independence produces $p < 0.01$ for the $\vec{F} \rightarrow \vec{v}$ question. Overall, the scores for students

TABLE 2. Comparison of student responses for equivalent questions in three different test forms. Each student took only one test form. Correct answers are designated with a *.

$\vec{F} \rightarrow \vec{a}$ A group of workers is pushing on a car on a level driveway and there is a net force on the car toward the street. What can you say about the acceleration of the car?

Test Type	N	$\vec{F} \uparrow\uparrow \vec{a}$	$\vec{F}(\uparrow\uparrow, \uparrow\downarrow, \uparrow 0)\vec{a}$	$\vec{F}(\uparrow\uparrow, \uparrow\downarrow)\vec{a}$	$\vec{F}(\uparrow\uparrow, \uparrow 0)\vec{a}$	Other
Test 1	78	89%*	9%			2%
Test 2	40	35%*	13%	15%	20%	17%
Test 3	119	56%*	13%	4%	20%	7%

$\vec{F} \rightarrow \vec{v}$ A soccer player pushes on a soccer ball with her foot. What can you say about the motion of the ball?

Test Type	N	$\vec{F}(\uparrow\uparrow, \uparrow\downarrow, \uparrow 0)\vec{v}$	$\vec{F} \uparrow\uparrow \vec{v}$	$\vec{F}(\uparrow\uparrow, \uparrow\downarrow)\vec{v}$	$\vec{F}(\uparrow\uparrow, \uparrow 0)\vec{v}$	Other
Test 1	191	38%*	58%			4%
Test 2	32	22%*	25%	31%	6%	16%
Test 3	119	11%*	61%	12%	14%	2%

$\vec{a} \rightarrow \vec{v}$ A car is on a hill, and the direction of the acceleration is uphill. What can you say about the motion of the car?

Test Type	N	$\vec{a}(\uparrow\uparrow, \uparrow\downarrow, \uparrow 0)\vec{v}$	$\vec{a} \uparrow\uparrow \vec{v}$	$\vec{a}(\uparrow\uparrow, \uparrow\downarrow)\vec{v}$	$\vec{a}(\uparrow\uparrow, \uparrow 0)\vec{v}$	Other
Test 1	82	37%*	59%			4%
Test 2	40	35%*	28%	20%	8%	9%
Test 3	119	23%*	48%	17%	12%	0%

Note on Notation. All questions provide information about one of three quantities (force, velocity, or acceleration) and ask about another. $\vec{A} \rightarrow \vec{B}$ means that information is given about $\vec{A}$ and the student is asked about $\vec{B}$. $\vec{A} \uparrow\uparrow \vec{B}$, $\vec{A} \uparrow\downarrow \vec{B}$, $\vec{A} \uparrow 0\ \vec{B}$ mean respectively that given information about $\vec{A}$ the student responds that $\vec{B}$ can be in the same direction as $\vec{A}$, opposite $\vec{A}$, or zero given $\vec{A}$ is nonzero. See the introduction for a more complete description.

taking Test 3 tended to be the lowest of the three tests. Second, for Tests 2 and 3, anywhere from 20% to 35% of the students answered either $(\uparrow\uparrow, \uparrow\downarrow)$, i.e. the two vectors can be parallel or antiparallel but not zero, or $(\uparrow\uparrow, \uparrow 0)$, i.e. the vectors can be parallel or zero but not antiparallel, choices which were not available on Test 1.

The most dramatic and perhaps the most interesting difference between the tests appears in the responses to the $\vec{F} \rightarrow \vec{a}$ question. Table 2 shows that almost 90% of the students chose the correct response in Test 1 and less than 10% chose "not enough information." Thus it would seem that students fully understand that net force and acceleration are necessarily parallel, in accordance with Newton's Second Law. However, when given other options, such as in Test 2 or Test 3, 30% fewer students chose the correct answer. Many students chose answers indicating that the net force on an object and its acceleration are not necessarily parallel, even after instruction on Newton's Second Law.

The $\vec{a} \rightarrow \vec{v}$ question shows a similar, though less dramatic, difference. On Test 1 almost 60% of the students incorrectly responded that acceleration and velocity were necessarily parallel, but on Tests 2 and 3 only between 30% and 50%, responded that they must be parallel.

A possible explanation for the difference between Test 1 and Tests 2 and 3 is that the presence of extra answer choices helps remind students of other options for the acceleration/velocity relationship while it incorrectly makes them unsure of the force/acceleration relationship. We have few interviews with students taking Test 1, so we can only speculate about the confidence they have in their answers. In interviews with students taking Test 2, students occasionally chose "sometimes" even though they did not have a good reason in mind. This may account for the greater shift of the response from $\uparrow\uparrow$ into the other models for this test. In any case, it is clear that many students do not have a solid understanding of the qualitative relationship between force and acceleration and between acceleration and velocity.

In addition, we see a parallel effect in the $\vec{F} \rightarrow \vec{v}$ question in Table 2. There is a large shift in the percentage of students answering $\vec{F} \uparrow\uparrow \vec{v}$ on Test 1 into $(\uparrow\uparrow, \uparrow\downarrow)$ and $(\uparrow\uparrow, \uparrow 0)$ on Test 2. However, there is almost no change in $\vec{F} \uparrow\uparrow \vec{v}$ answering in Test 3. We believe this difference is an artifact of the new wording used in place of "net force." Test 1 and Test 2 both used the old net force wording for the question, but Test 3 had the new wording. The new wording changes the pattern of responses from ones consistent with net force and velocity not necessarily being parallel to ones where net force and velocity are necessarily parallel.

RESULTS OF TEST 3

Test 3 is a 15 question multiple choice test. Analysis of the scores does not indicate any significant change in how the students answered throughout the quarter, but the reported results are from quizzes given post instruction only to 119 students.

The results of the student responses is shown in Table 3.

The scores on these questions support previous findings that even after instruction many students do not answer qualitative questions about force and motion correctly, and many choose the common "misconceptions" such as $\vec{v} \uparrow\uparrow \vec{F}$, $\vec{F} \uparrow\uparrow \vec{v}$, and $\vec{a} \uparrow\uparrow \vec{v}$ [5, 2, 4]. In addition these results indicate that some students may understand some of the relationships between force, velocity, and acceleration better than other relationships, and it appears as though there is a directionality to their answering patterns (and thus perhaps to their reasoning). For example, more students answered that $\vec{v} \uparrow\uparrow \vec{F}$ than $\vec{F} \uparrow\uparrow \vec{v}$ and similarly, more answered that $\vec{a} \uparrow\uparrow \vec{F}$ than $\vec{F} \uparrow\uparrow \vec{a}$. Although we did not ask a $\vec{v} \rightarrow \vec{a}$ question on Test 3, students were scoring at 90% correct on Test 1 questions of this type in comparison to 50% on the $\vec{a} \rightarrow \vec{v}$ questions. Furthermore, many more students believe that $\vec{F} \uparrow\uparrow \vec{v}$ than $\vec{a} \uparrow\uparrow \vec{v}$. It is interesting to note that more students respond that if an object has a velocity, then it experiences a net force in the direction of the velocity ($\vec{v} \uparrow\uparrow \vec{F}$) than vice versa, and the logical converse also holds, namely that more students believe that if an object has balanced forces (no net force) acting on it, its velocity is zero. In sum, the results of Table 3 indicate that there are patterns of student answering which may suggest a directionality and hierarchy of student understanding of force and motion. Certainly more varied and deeper assessments of student understanding are needed to investigate this finding further.

TABLE 3. Test 3 Response percentages (N=119). For some question types, two questions with slightly different story contexts were posed; these responses are reported separately. Standard error is approximately 4% for all percentages.

Question Type	$\vec{X}(\uparrow\uparrow, \uparrow\downarrow, \uparrow 0)\vec{Y}$	$\vec{X}(\uparrow\uparrow)\vec{Y}$
$\vec{v} \rightarrow \vec{F}$	11%*	73%
	13%*	64%
$\vec{F} \rightarrow \vec{v}$	11%*	61%
	23%*	61%
$\vec{a} \rightarrow \vec{v}$	36%*	32%
	23%*	48%
$\vec{F} \rightarrow \vec{a}$	13%	56%*
$\vec{a} \rightarrow \vec{F}$	5%	84%*
$\vec{F} = 0 \rightarrow \vec{v}$	14%*	84%
$\vec{v} = 0 \rightarrow \vec{F}$	29%*	60%
Speeding up $\vec{v} \rightarrow \vec{F}$	1%	84%*

*Denotes the correct answer choice.

CONCLUSION

Although a significant number of previous studies have investigated student understanding of force, velocity, and acceleration, there has been no comprehensive study of student understanding of all relationships between these quantities. In order to begin to address this issue, we have developed a multiple choice instrument with very simple conceptual questions that provide information about either force, velocity, or acceleration and asks what can be determined about one of the other two quantities. The test development process revealed a number of issues with student answering. First, increasing the number of possible distracters may significantly decrease the score by reducing the number of false positives. Students also may pick a series of answers that do not logically make sense within a question such as, "it is always moving left, sometimes moving right, and never at rest," and students have difficulties understanding what is intended by the term "net force." Therefore, our research suggests a test, such as Test 3, which takes these factors into account is more valid then tests like 1 and 2 which do not.

Results of Test 3 reveal several interesting findings. First, in addition to the well known student difficulty in understanding the non-causal relation between velocity and force, students have difficulty understanding that force and acceleration do have a causal and directional relationship. Second, consideration of how students answer on all possible relationships between force, velocity, and acceleration reveals that students may have a directional and hierarchical understanding of the relations among these quantities. For example, students may believe that velocity implies force more than force implies velocity. Furthermore students may believe that force implies velocity more than acceleration implies velocity. This kind of hierarchy of understanding may have important implications for instructional design. While the test used for these findings is a quite simple and somewhat superficial assessment of student understanding of the relationship among force, velocity, and acceleration, the results are interesting enough to warrant more investigation using a variety of assessments.

ACKNOWLEDGMENTS

We would like to thank all the students at The Ohio State University who participated in this study. This research is supported by a grant from the Institute of Educational Sciences of the U.S. Department of Education (R305H050125).

REFERENCES

1. I. A. Halloun, and D. Hestenes, *American Journal of Physics* **53**, 1043–1055 (1985).
2. R. K. Thornton, and D. R. Sokoloff, *American Journal of Physics* **66**, 338–352 (1998).
3. J. Clement, *American Journal of Physics* **50**, 66–71 (1982).
4. L. Viennot, *European Journal of Science Education* **1**, 205–221 (1979).
5. D. Hestenes, M. Wells, and G. Swackhamer, *The Physics Teacher* **30**, 141–166 (1992).
6. D. E. Trowbridge, and L. C. McDermott, *American Journal of Physics* **49**, 242–253 (1981).
7. F. Reif, and S. Allen, *Cognition and Instruction* **9**, 1–44 (1992).