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Student understanding of the direction of the magnetic force on a charged particle

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We study student understanding of the direction of the magnetic force experienced by a charged particle moving through a homogeneous magnetic field in both the magnetic pole and field line representations of the magnetic field. In five studies, we administer a series of simple questions in either written or interview format. Our results indicate that although students begin at the same low level of performance in both representations, they answer correctly more often in the field line representation than in the pole representation after instruction. This difference is due in part to more students believing that charges are attracted to magnetic poles than believing that charges are pushed along magnetic field lines. Although traditional instruction is fairly effective in teaching students to answer correctly up to a few weeks following instruction, especially for the field line representation, some students revert to their initial misconceptions several months after instruction. The responses reveal persistent and largely random sign errors in the direction of the force. The sign errors are largely nonsystematic and due to confusion about the direction of the magnetic field and the execution and choice of the right-hand rule and lack of recognition of the noncommutativity of the cross product.

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I. INTRODUCTION

The direction of the magnetic force on a charged particle is a unique and curious phenomenon in the natural world and consequently is a difficult idea for students to understand. In fact it took a number of years for physicists to understand the nature of the magnetic force and arrive at the straightforward expression $\vec{F}=q\vec{v}\times\vec{B}$, starting from early observations of Oersted (see, for example, Ref. 1) in the 1820s to Maxwell's first theoretical description[2,3] to Heaviside's[4] and Lorentz's[5] formulations and the appearance of the right-hand rule in the late 1800s,[6] now commonly used to determine the direction of the force.

In this study we examine two major factors that influence student difficulties with understanding the direction of the magnetic force on a charged particle, a difficult idea for students to understand. The first factor involves the way in which magnetism is represented, usually by magnetic poles or magnetic fields lines. Because these representations are similar to those used for electric fields, this factor highlights students' general confusion of magnetism with electricity. For example, Maloney showed that when asked to describe the force experienced by a charged particle near a magnetic pole, students answered that the particle will experience a force directed toward or away from the pole, even after instruction in magnetism.[7] There are several reasons for these answers, including the beliefs that magnets attract nearby objects,[8] magnets carry a net electrostatic charge in their poles,[8,9] and charged particles, whether stationary or moving, are the source of magnetic fields.[9]

When magnetism is discussed in terms of field lines and magnetic poles are no longer explicitly discussed, a connection to electricity might still be present because students often confuse electric and magnetic fields.[9,10] It has been shown that when a particle is placed in either an electric or a magnetic field, students believe that its trajectory must follow the field lines.[11–14] Students have a tendency to assign a force parallel to the magnetic field lines, similar to the case with electric field lines.[15] Some introductory physics students think of field lines as physical entities.[10] For some students the belief can be so strongly held that they think of field lines as objects that flow. If a charge is drawn on a field line, it will experience a force; if it is drawn off the field line, it will not.[16]

Although the effects of the pole and field line representation on magnetic force problems have been studied separately, here we study them simultaneously. This study will help to determine student sensitivity to the representation of questions and the extent to which students think about poles or fields when presented with either representation. For example, students may picture field lines when presented with magnetic poles or conversely poles when field lines are presented, resulting in similar performance in both representations. It is also possible that the two representations cue different solution paths and the performance might be quite different. In this study we posed both types of questions to help disambiguate the nature of student understanding of fields, poles, charges, and magnets.

The second factor influencing student difficulties with determining the direction of the magnetic force is the necessity of using the unfamiliar and abstract operation of the vector cross product. The resulting magnetic force vector is perpendicular to the field and velocity of the particle, a direction both unexpected and unintuitive for the novice. Knight[17] documented student difficulties with the vector cross product in a mathematical context, and several methods have been suggested to aid students to correctly determine its direction.[18–20] However, the nature of student difficulties with the vector cross product is still unclear. In this paper we study student difficulties with the direction of the cross product, including the noncommutativity of the operation, and examine how these difficulties might lead to difficulties in determining the direction of magnetic force.

In the initial stages of our study we observed that a significant number of students made a “sign error” when determining the direction of the magnetic force—they recognized that the magnetic force was perpendicular to the velocity and magnetic field, but they chose the incorrect sign of the direction. The existence of the sign error is a significant and interesting phenomenon, which provides a more detailed and holistic understanding of student difficulties with fields, poles, and cross products.

II. STUDY ON STUDENT PERFORMANCE IN POLE AND FIELD REPRESENTATIONS

In the first study, student understanding of the direction of the magnetic force for both the magnetic pole and field line representations is studied. The first goal is to replicate and further validate that students believe charges are attracted to magnetic poles and pushed in the direction of magnetic field lines.[21]

All 110 participants were enrolled in the second of the three-quarter, calculus-based, introductory physics sequence at The Ohio State University (OSU). The syllabus for this course consisted of standard topics in electricity and magnetism.

Participants were asked to answer questions about the direction of force experienced by a charged particle moving through a magnetic field. Two representations of magnetic field were tested: Magnetic poles and magnetic field lines (see Fig. 1). The questionnaire was given as a nongraded laboratory quiz approximately 1 week after the instructional unit about magnetic forces. Participants were randomly assigned by laboratory section to one of the two representations: Field line ($N=66$) or magnetic pole ($N=44$). At least two laboratory sections were given each representation.

An answer was classified as “field direction” if it was parallel or antiparallel to the magnetic field. The proportions of students’ top three answer choices are shown in Fig. 2 as a function of the representation. All student answer choices are reported in detail in Table I. The primary result is that, postinstruction, more students believe charges are attracted to magnetic poles (34%) than experience a force in the direction of the magnetic field lines (17%, $\chi^2=3.53$, degrees of freedom (df)=1, and $p=0.06$). This difference might be the main reason why students answer the question more correctly in the field line representation than the pole representation (47% compared to 32%, effect size[22] $d=0.22$, $\chi^2=1.92$, df=1, and $p=0.17$).

For the following questions, use the following arrows to indicate direction: left ←, up ↑, right →, down ↓, into page ×, out of page •. If there is not a solution to a problem, write NO SOLUTION.

A charged particle is placed between two poles of a magnet [*or* in a magnetic field, ”B”], as shown. The particle initially has velocity ”v.” Indicate the direction of the **force** of the particle by drawing an arrow and labeling it ”F.”

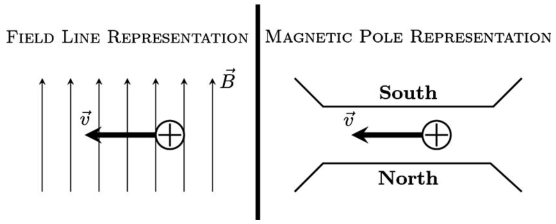


Fig. 1. Text and illustrations for the pole and field representations questions used in the study described in Sec. II.

III. STUDY ON THE EVOLUTION OF DIFFERENCES IN REPRESENTATION

This study was designed to further explore the differences in performance that were discussed in Sec. II. Because it is not clear whether these differences were due to instruction or to prior knowledge, additional tests were given prior to instruction, during the week of instruction, and a much longer time after instruction. The tests also were administered to a larger number of students.

The 366 participants in these tests were enrolled in the calculus-based, introductory physics sequence at OSU. The first three tests were given to students from the second course of the three-quarter sequence (electricity and magnetism), and the last test was given to students from the third course (modern physics topics).

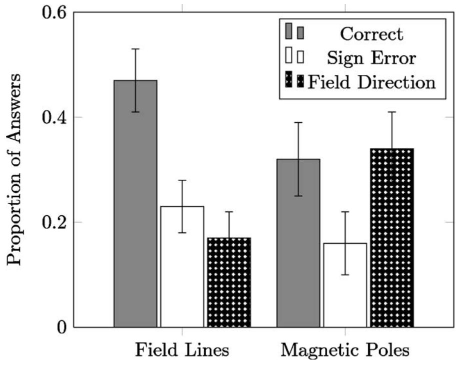


Fig. 2. Proportions of correct, sign error, and field direction answers in the study described in Sec. II for the pole and field representations 1 week after magnetic force instruction. The error bars represent standard errors for a multinomial distribution. (Multinomial errors were used to estimate the uncertainty of the results in the study because the assumptions of the multinomial distribution are similar to the assumptions we made concerning the data: Individual measurements, in this case individual students, are independent of one another and that there is some probability distribution that describes how a student will answer each question.)

Table I. Raw counts of student responses to the two tests administered in Sec. II. Both tests were free response.

Test	Up	Down	Left	Right	Out of page	Into page	Zero	Other	Total
Post, field lines	3	8	2	0	15	31	0	7	66
Post, mag. poles	3	12	2	3	7	14	0	3	44

For each of the first three tests, participants were randomly assigned by laboratory section and tested at only one of the three possible testing times. For the fourth test data were collected only from students who were not enrolled in electricity and magnetism during the previous quarter. The exact time relative to instruction for these students cannot be determined, but because electricity and magnetism are a prerequisite for modern physics, we know that the test was given at least 15 weeks after they received instruction in magnetism. The average number of students in each group was 91, with the smallest group size of 64.

Due to limitations imposed by the course instructors, the format of the questions was altered midway through the study. The first two tests used free response questions, similar to the questionnaire discussed in Sec. II, but the last two tests used multiple choice questions, which are shown in Fig. 3. The questionnaire was administered as a nongraded laboratory quiz at four times: After instruction in electric forces but before instruction in magnetic forces; immediately following instruction in magnetic forces; at the end of the course in electricity and magnetism about 2 weeks following instruction in magnetic force; and in the first week of the course on modern physics.

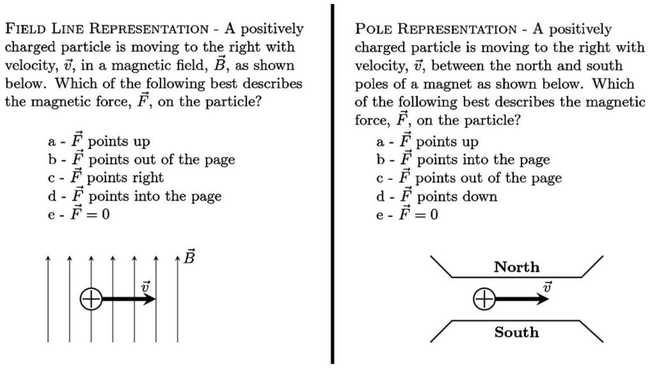
Although the study described in Sec. II used a between-subjects design, the current study used a within-subject design, testing each student with both the magnetic pole and magnetic field line representations. The magnetic pole representation was always presented first.[24] Answers were classified as field direction if they were parallel or antiparallel to the magnetic field.

There are three main results from the student responses represented in Fig. 4.[25] Students did not answer differently between representations on the pretest ($\chi^2=5.22$, df$=3$, and $p>0.15$), but immediately after instruction (and consistent with the results from Sec. II), students performed better in the field line representation (72%) than in the pole representation (48%) ($\chi^2=7.90$, df$=3$, $p=0.05$, and $d=0.34$). After instruction and when the question format was altered, this difference continued: 2 weeks after instruction, $\chi^2=12.50$, df$=3$, $p<0.01$, and $d=0.42$; more than 15 weeks after instruction, $\chi^2=31.58$, df$=3$, $p<0.001$, and $d=0.47$.

The "charged pole" responses appear to rebound after instruction. Because the question formats changed between week 0 and week 2, direct comparisons cannot be made between the initial and final answer proportions. However, we can make reliable comparisons between 2 and more than 15 weeks after instruction (see Fig. 4). There is a significant difference in answering patterns between these two times: Pole representation, $\chi^2=12.78$, df$=3$, and $p<0.01$; field line representation $\chi^2=10.00$, df$=3$, and $p=0.02$. This difference is mostly due to a significant increase in the force-in-direction-of-pole answers from 20% to 38% ($\chi^2=6.12$, df$=1$, $p=0.01$, and $d=0.27$) and an increase in force-in-direction-of-field answers from 6% to 19% ($\chi^2=4.98$, df$=1$, $p=0.03$, and $d=0.22$). These increases are accompanied by a corresponding decrease in the proportion of correct responses with time: Pole representation, $\chi^2=2.47$, df$=1$, $p=0.12$, and $d=-0.18$; field line representation, $\chi^2=0.96$, df$=1$, $p>0.3$, and $d=-0.13$.

Students made sign errors more often in the pole representation than in the field line representation 2 weeks after instruction and later. In particular, a greater proportion of students answered with a sign error in the magnetic pole representation (27%) than the field line representation (15%) ($\chi^2=10.97$, df$=2$, and $p<0.01$) 2 weeks after instruction. This proportion remained unchanged more than 15 weeks after instruction. By combining these results with those in Sec. II, we found that sign errors are not present on the pretest or immediately (up to several days) after instruction but are present in both free response and multiple choice question formats 1 week after instruction and later. This result indicates that the sign errors might be confusion (or forgetting) about the instruction that sets in well after instruction.

In summary, more students learn that the force experienced by a charged particle is not in the direction of magnetic field but perpendicular to both the velocity of the particle and the magnetic field, and they perform better in the


Field Line Representation - A positively charged particle is moving to the right with velocity, $\vec{v}$, in a magnetic field, $\vec{B}$, as shown below. Which of the following best describes the magnetic force, $\vec{F}$, on the particle?

- a - $\vec{F}$ points up
- b - $\vec{F}$ points out of the page
- c - $\vec{F}$ points right
- d - $\vec{F}$ points into the page
- e - $\vec{F}=0$

Pole Representation - A positively charged particle is moving to the right with velocity, $\vec{v}$, between the north and south poles of a magnet as shown below. Which of the following best describes the magnetic force, $\vec{F}$, on the particle?

- a - $\vec{F}$ points up
- b - $\vec{F}$ points into the page
- c - $\vec{F}$ points out of the page
- d - $\vec{F}$ points down
- e - $\vec{F}=0$

Fig. 3. Text and illustrations for both the pole representations and field representation test questions used in the study described in Sec. III. All students answered both versions of the question.

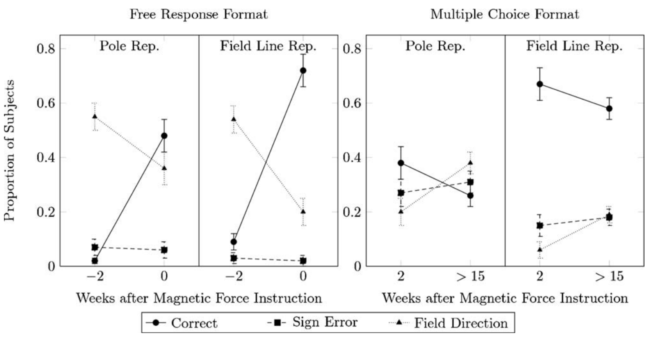
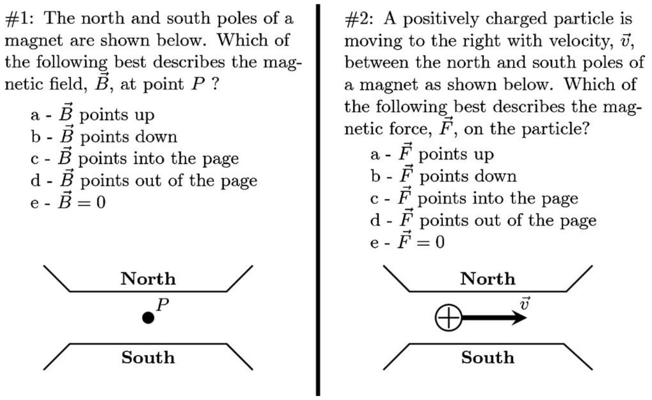


Fig. 4. Student answers from the study described in Sec. III. The error bars represent standard errors for a multinomial distribution.


#1: The north and south poles of a magnet are shown below. Which of the following best describes the magnetic field, $\vec{B}$, at point P ?

a - $\vec{B}$ points up
b - $\vec{B}$ points down
c - $\vec{B}$ points into the page
d - $\vec{B}$ points out of the page
e - $\vec{B}=0$

#2: A positively charged particle is moving to the right with velocity, $\vec{v}$, between the north and south poles of a magnet as shown below. Which of the following best describes the magnetic force, $\vec{F}$, on the particle?

a - $\vec{F}$ points up
b - $\vec{F}$ points down
c - $\vec{F}$ points into the page
d - $\vec{F}$ points out of the page
e - $\vec{F}=0$

Fig. 5. Text and illustrations used in the study described in Sec. IV. Students (N=126) answered both questions in the order depicted.

magnetic field line representation. However, some time after instruction, students begin to answer with sign errors, and it appears that the charged pole and force-in-direction-of-field responses partially rebound to preinstructional levels.

IV. STUDY ON STUDENT UNDERSTANDING OF THE DIRECTION OF A MAGNETIC FIELD

The results described in Sec. III and to a lesser extent in Sec. II indicate that students made sign errors more often in the pole representation than the field representation sometime after instruction. A possible explanation for this difference is that the magnetic pole representation did not explicitly illustrate the presence and direction of the magnetic field, and some students might have been mistaken about the direction of the magnetic field between magnetic poles. In this study we examine student understanding of the direction of the magnetic field between magnetic poles and the resulting direction of force on a charged particle.

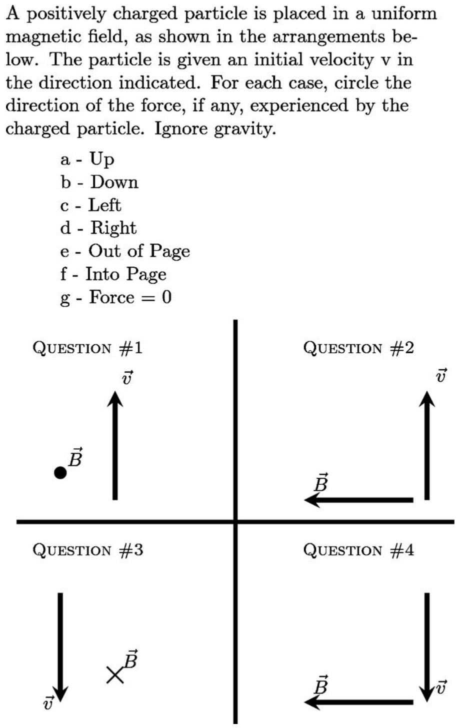
All 126 students who participated in this study were enrolled in an introductory, calculus-based, electricity and magnetism course at OSU. The course from which students were drawn for this study was not used in any other study reported in this article. Students were asked two questions, which are shown in Fig. 5, as part of a graded final exam.

A cross tabulation of student responses to the two questions illustrated in Fig. 5 is shown in Table II.[25] 15% of the students (19/126) incorrectly answered that the magnetic field points from the south to the north pole. By examining the cross tabulation of the results, we find that about half of those who obtained the wrong direction for the magnetic field (10/19) subsequently made a sign error in obtaining the direction of the magnetic force. Thus many of these students might have answered consistently: Their sign errors originated when deciding the direction of magnetic field and then propagated to their force answers. The excess sign errors in the pole representation compared to the field representation in Secs. II and III might be due to errors in deducing the direction of the magnetic field from the magnetic poles.

Table II. Cross tabulation of student responses to the questions depicted in Fig. 5 (N=126).

Proportion of students who claim field points from north pole to south pole (correct)				
Direction of force	Into[a]	Out of[b]	Other	Total
	0.37	0.12	0.09	0.58
Proportion of students who claim field points from south pole to north pole (reversed)				
Direction of force	Into[c]	Out of[d]	Other	Total
	0.03	0.08	0.04	0.15

[a] Correct field direction and correct force direction (consistent).
[b] Correct field direction but incorrect force direction (sign error).
[c] Incorrect field direction but correct force direction.
[d] Incorrect field direction and incorrect force direction (consistent).


A positively charged particle is placed in a uniform magnetic field, as shown in the arrangements below. The particle is given an initial velocity v in the direction indicated. For each case, circle the direction of the force, if any, experienced by the charged particle. Ignore gravity.

a - Up
b - Down
c - Left
d - Right
e - Out of Page
f - Into Page
g - Force = 0

Fig. 6. Text and illustrations used in the study described in Sec. V.

V. STUDY ON WHETHER SIGN ERRORS ARE SYSTEMATIC

Although the results discussed in Sec. IV indicate that a small portion of the sign errors made in the pole representation was likely due to systematic errors when determining the field direction, the majority of sign errors, including those made in the field line representation in Secs. II and III and those not attributed to confusion about field direction in Sec. IV are left unexplained. The study described in this section was designed to determine the extent to which there is a systematic pattern in these sign errors within a given representation. For example, some students may always use their left hands, thus systematically obtaining a sign error. Both written tests and student interviews were administered after instruction on magnetic forces.

A. Written tests

All 174 participants were enrolled in the calculus-based, introductory physics sequence at OSU in a different quarter from those in Secs. II–IV. Students were asked four questions about the direction of a cross product resulting from the

Table III. Cross tabulation of student responses to questions shown in Fig. 6 ($N=174$). Contents of cells represent numbers of students.

	Question 2		
Question 1	Correct	Sign error	Other
Correct	97	11	5
Sign error	17	13	4
Other	16	3	8

multiplication of two perpendicular vectors. Examples are shown in Fig. 6. In these questions the magnetic field was represented by a single vector, as seen in Fig. 6 rather than a series of field lines as on questionnaires administered in Secs. II and III (for example, Fig. 1). The test was administered after instruction in magnetic force and was a required but nongraded quiz in a laboratory session. All students who attended the laboratory participated in this study.

To control for the fact that sign errors might be due to particular orientations of the vectors, the questions included a variety of combinations in which the force or magnetic field are perpendicular to the plane of the page at least once. Orientation might contribute to sign errors because, for example, positioning the right hand to correctly calculate the direction of $\vec{v}\times\vec{B}$ might be more difficult or physically uncomfortable than $\vec{B}\times\vec{v}$. Orientation may also be a factor due to the lack of familiarity with the "into the page" and "out of the page" vector notations.

Student response patterns were similar on all questions ($\chi^2=13.66$, df=9, and $p>0.1$). If there were any differences in student answers due to different orientations of $\vec{v}$, $\vec{B}$, and $\vec{F}$, the differences were less than 10%.

We analyzed the data in two ways to determine whether students made sign errors systematically. One way examined the within-student response patterns for questions 1 and 2 in Fig. 6, the results of which were typical for any pairing of the four questions. A cross tabulation of student answers to the two questions is shown in Table III.[25]

If students made sign errors systematically, then the number of students answering with two sign errors would have been larger than the number of students answering with one sign error. We found that the number of students answering with one sign error (35/174 or 16%) was larger than the number of students with two sign errors (13/174 or 7%) ($\chi^2=10.08$, df=1, and $p<0.01$). Although some students might have systematically answered with a sign error, these results suggest that most students who answered with a sign error did not do so systematically.

The other method used for determining whether there were systematic sign errors was to analyze the response pattern of all four questions. If sign error answers were systematic, the distribution of the number of sign errors should be bimodal with modes at zero sign error answers and four sign error answers. This result does not agree with the observed data, shown in Fig. 7. Conversely, if all students made sign errors in a random, nonsystematic pattern, then the number of sign errors should follow a binomial distribution. For comparison, we included a model based on a binomial distribution in Fig. 7 with the probability of making a sign error at $p=0.2$, which is equal to the total proportion of sign errors made across the four questions. This distribution does not match the mode at zero sign errors, indicating that some students must be systematically answering all four questions correctly. If we assume that one-third of the students are always correct, with the remaining two-thirds answering with sign errors randomly, the model fits the observed distribution fairly well. This observation lends further support to the conclusion that most students do not make systematic sign error mistakes. Note that although the observed data are qualitatively similar to the binomial model, there is a small tail of 5%–10% of students answering systematically with a sign error, which is greater than would be expected by making random mistakes.

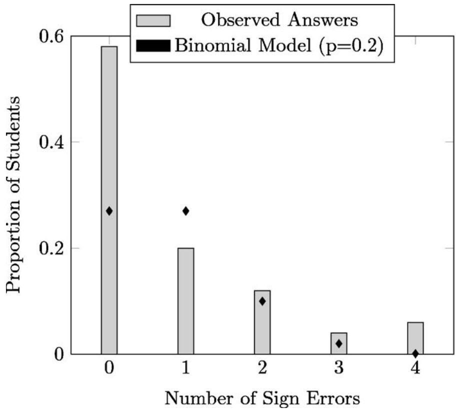


Fig. 7. Histogram showing the number of sign errors made in the four questions in Sec. V. The additional points represent a binomial distribution with $p=0.2$. The binomial model was adjusted to allow 1/3 of all students to answer all four questions correctly without random chance of sign error.

B. Student interviews

Eight students were interviewed. Volunteers were solicited from the third of the three-quarter, calculus-based, introductory physics sequence at OSU. About 60% of the students in this class were enrolled in the E&M course immediately prior to this course. The syllabus for this course includes topics typically covered in geometric optics, sound and waves, and special relativity.

The interview session included questions about the direction of force experienced by a charged particle moving through a magnetic field and questions more generally about cross products, such as those in Figs. 1, 3, 6, and 8. In addition to dimensionality and orientation of vectors being controlled, the representation of magnetic field was controlled so that each student was questioned using the magnetic pole representation, field line representation, and a single vector representation of field. Students were asked to think out loud while solving the questions, with minimal intervention from the interviewer. Clarification questions were asked when appropriate.

There were no indications from any student that the orientation of the vectors helped or hindered their ability to solve the problem. Six of the eight students interviewed answered at least one question with a sign error. The most common sources of sign errors were confusion about which version of the right-hand rule to use and how to correctly use the right-hand rule. For example, one student, who has been enrolled in E&M four quarters or approximately 12 months before being interviewed, gave the following description of how to find the direction of a cross product:

> "…so this is v [points at fingers], and curl toward [B]." [Student then answers F in direction of thumb.]

In this first instance, she correctly applied an appropriate version of the right-hand rule to determine the direction of force. However, when she later answered with a sign error, she explained her method as follows:

> "…There's like two different ways of doing the right-hand rule…sometimes I do it like the B goes into your hand and this [fingers] in the direction of v and your [thumb is F]."

In this second explanation, she used a different form of the right-hand rule and answered incorrectly by reversing the vectors.

The student was capable of correctly applying a right-hand rule to a magnetic force question but did not do so consistently. She used different versions of the right-hand rule for different questions, reversing the vectors when performing one version. Similar confusion was also observed among the other students. At no point was there a reason given for choosing one version of the right-hand rule over another. The student described in the excerpts simply appeared to be guessing. If students who answer with a sign error are guessing, then if all other factors are equal, students would answer with a sign error as often as they answer correctly. Eight students likely did not produce an exhaustive list of reasons for the sign error, but they did present evidence that sign errors are made at random. These reasons also suggest that the order of vectors might not be perceived as an important attribute of the cross product.

VI. STUDY ON STUDENT UNDERSTANDING OF THE NONCOMMUTATIVITY OF CROSS PRODUCTS

Both the written test and student interviews described in Sec. V provided strong evidence that students make sign errors randomly (for example, Fig. 7, with one possible cause being confusion about which right-hand rule to use). Additionally, students might fail to recognize the noncommutativity of vector cross products, which are used to determine the direction of the magnetic field. Although students have experience with noncommutative operators, such as subtraction and division, they have far less experience with noncommutative forms of multiplication, such as cross products and matrix multiplication. This study was designed to determine the extent to which students understand that the cross product is a noncommutative operation.

Table IV. Responses to questions depicted in Fig. 8. If students answered Question 2 as though cross products are commutative, they were more likely to answer Question 1 with a sign error.

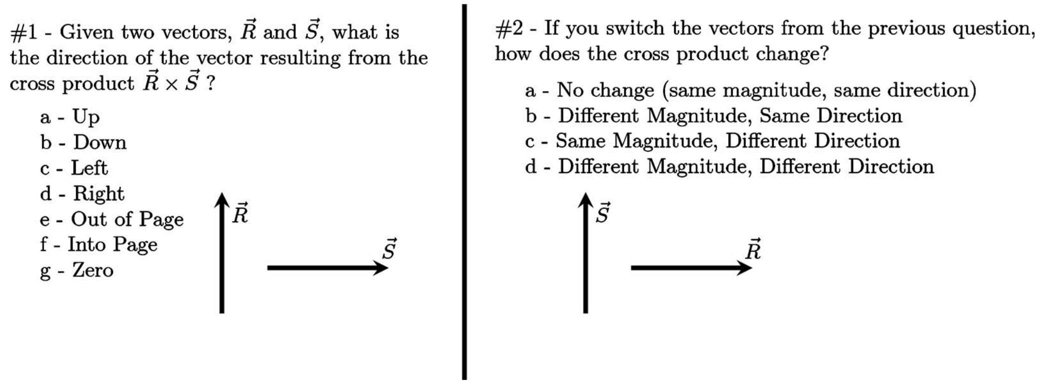
	Question 2		
Question 1	Different direction (correct)	No change (commutative)	Other
Correct	139	22	5
Sign Error	75	47	11
Other	29	27	9

These questions were administered in conjunction with those described in Sec. III and used the same participants. Students first answered the questions reported in Sec. III and then the questions shown in Fig. 8. These questions were administered as a part of a laboratory quiz graded only for participation. As with the study described in Sec. III, students answered these questions at one of four times.

Perhaps surprisingly, student answers did not reliably change over the course of the quarter (Question 1: $\chi^2 = 10.15$, df=6, and $p > 0.1$; Question 2: $\chi^2 = 10.96$, df=6, and $p = 0.09$), so the results were pooled into a single sample. Of the 365 students questioned, 26% answered as though the order of vectors does not matter when computing a cross product.

A cross tabulation of student responses to the two questions in Fig. 8 is shown in Table IV.[25] Students who answered Question 2 as though the cross product were commutative (answer "a") were more likely to have answered with a sign error on Question 1 than students who answered Question 2 correctly ($\chi^2 = 21.94$, df=1, and $p < 0.001$). This lack of attention to the order of the cross product multiplication by some students might not only explain why a significant fraction of students makes sign errors but also explain why the sign errors are not systematic.

There are two important points to note from Table IV. Only about 40% of the students making sign errors also answer that the cross product is commutative. Thus although the lack of understanding of noncommutativity of the cross



#1 - Given two vectors, $\vec{R}$ and $\vec{S}$, what is the direction of the vector resulting from the cross product $\vec{R} \times \vec{S}$?

a - Up
b - Down
c - Left
d - Right
e - Out of Page
f - Into Page
g - Zero

#2 - If you switch the vectors from the previous question, how does the cross product change?

a - No change (same magnitude, same direction)
b - Different Magnitude, Same Direction
c - Same Magnitude, Different Direction
d - Different Magnitude, Different Direction

Fig. 8. Questions used to test students' understanding of the noncommutativity of the cross product. Although these questions were given at four times, student answers did not change between tests.

Table V. Cross tabulation of student responses to the field line representation in Sec. III and the generic math representation in Sec. VI, corresponding to the third test from Sec. III. More students answer the generic math question with a sign error than the field line question.

	Generic math rep.		
Field line rep.	Correct	Sign error	Other
Correct	22	17	5
Sign error	3	5	2
Other	7	2	15

product might explain why some of the students make sign errors, it does not explain why the majority of them do. Additionally, it is important to note from Table IV that students who do not understand the noncommutativity of the cross product are more likely to make a sign error than answer correctly, whereas we would expect sign errors to be equally probable as correct answers for these students. Why these students tend to make more sign errors is not clear. These results suggest that a small fraction of the population (around 5%–10%) does not recognize the noncommutativity of the cross product and systematically make sign errors.

Because these questions were administered in conjunction with those described in Sec. III, a cross tabulation of student responses to both the generic cross product question, Fig. 8, and the field line representation of the magnetic force question, Fig. 3, can be calculated and is shown in Table V (using data only from the third test time in Sec. III). The main difference in performance between the two questions is that 56% of the students answered the magnetic field question correctly versus 41% for the generic mathematical question. This difference is largely due to more students making a sign error in the generic mathematical question (31%) than the magnetic field question (13%; McNemar,[27] $\chi^2=8.45$, df=1, and $p<0.01$). Among those students who answered the magnetic field question correctly, a similar number answered the generic question correctly as answered with a sign error ($\chi^2=0.02$, df=1, and $p>0.5$). Although most students know that a cross product results in a vector that is perpendicular to the two vector factors (requiring use of the right-hand rule), they are unsure of how to apply the right-hand rule in the generic case. The magnetic force version of the right-hand rule has likely been repeated frequently throughout instruction (for example, "stick your fingers in the direction of v and curl in the direction of B…"), and this familiarity and repetition may help to prevent errors. Additional practice with cross products or development of methods less prone to error than the right-hand rule might be helpful in addressing all of the issues discussed. For example, Nguyen and Meltzer suggested a tool constructed from an index card.[20]

VII. CONCLUSION

This study has revealed a rich and robust array of student difficulties with understanding the direction of the magnetic force on a charged particle in a magnetic field. Trends in student answers were consistent between the various studies and independent of course offering and instructor. The traditional university level introductory E&M physics courses we studied were somewhat successful in teaching students the correct direction of the magnetic force on a charged particle moving through a magnetic field but were significantly less successful for isomorphic questions presenting charges near magnetic poles. In addition, there is some indication that the charged pole responses that are prevalent prior to instruction reappear several months after instruction.

By analyzing student response patterns in more detail, which was not possible in earlier studies such as in Ref. 7 due to limited answer choices, we also found an interesting and persistent signal of the sign error in 15%–30% of responses postinstruction. This number is somewhat misleading: When given a series of four similar questions, over 40% of the students made a sign error at least once, illuminating a significant student difficulty that warrants attention during instruction.

Our results suggest at least three causes for the sign error. About 15% of students thought that the magnetic field points from the south pole to the north. A substantial fraction of these students correctly applied the right-hand rule to the incorrect field direction, resulting in a systematic sign error in their magnetic force responses.

The overwhelming majority of students did not make sign errors systematically. This lack of systematic rendering of sign errors led to uncovering two other causes for sign errors. One is a confusion in choice and execution of the several right-hand rules available. Although most students recognized that the magnetic force is in the direction perpendicular to the velocity and field after instruction, inconsistent execution of the proper right-hand rule led to an unsystematic sign error.

The last identified cause of the sign error arises from the fact that even after instruction, about one-quarter of the students did not recognize that reversing the order of the vectors in the cross product reverses the direction of the resultant vector. This reversal can lead to a nonsystematic sign error. These students are more likely to make a sign error than those who understood the noncommutative nature of the cross product.

Besides the issue of sign errors, we can speculate on possible reasons for the differences in student responses between the pole and field line representations. Solving the magnetic pole representation requires the additional steps of identifying both the presence and direction of the magnetic field. Increasing the number of steps increases the overall probability of making an error. For example, failure to recognize the presence of a field may have resulted in lack of cuing of the Lorentz equation $\vec{F}=q\vec{v}\times\vec{B}$, or an error in deducing the direction of the field from the poles can result in an incorrect answer. Also, students might have been more familiar with the magnetic field line representation because questions with magnetic field are likely more practiced and emphasized than questions with magnetic poles during instruction. In the course textbook[29] field lines are used to represent the presence of a magnetic field much more frequently than magnetic poles. Reports from the lecturers also support this claim. In addition, students have had concrete experience with the attraction and repulsion of magnets, which have similarities with the attraction and repulsion of static charges. Thus the magnetic pole representation might have caused more confusion of magnetism with electricity than abstract representations of fields, with which student have had less experience.

Although the design of this study did not explicitly examine how specific forms of instruction might affect students' understanding of magnetic force, we point out two implications for instruction. First, instructors cannot assume that magnetic field and pole representations are equivalent and interchangeable from the student's perspective. To help students understand the nature of these two representations and the physics of the relation between them, instructors might, for example, ask students to explicitly compare given situations in which only field lines are present with isomorphic situations in which only magnetic field sources are present (for example, permanent magnets and current-carrying wires). Because a slight decrease back to the original misconception was observed after instruction, more so in the magnetic pole representation, instructors should look for opportunities to include magnetic poles and magnetic forces in subsequent instruction by either using magnetism as a context for a contemporary physics topic, such as particle detection, or by periodically reviewing concepts relating to magnetism.

The second implication for instruction is related to the emergence and persistence of sign errors after instruction in magnetism. Because cross products are a novel mathematical operation for many students, more emphasis on the nature and execution of the cross product may be needed, including explicit emphasis on the noncommutative nature of the cross product and identifying and practicing proper right-hand rules. The latter point is particularly important in courses where the instructor teaches multiple right-hand rules. Some sign errors were due to a misunderstanding about the direction of the field between magnetic poles. For these students, requiring the comparison between isomorphic arrangements of field lines and field sources or some other instruction designed to illustrate the relation between field lines and field sources could be beneficial.

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[22] The effect size was calculated using Cohen's d with a pooled standard deviation (see, for example, Ref. 23).

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[24] In a previous study, we demonstrated that there is no significant difference in performance when the order of the two representations is varied (Ref. 26).

[25] See supplementary material at http://dx.doi.org/10.1119/1.3386587 for raw counts in this study.

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