International Journal of Science Education
Vol. 32, No. 14, 15 September 2010, pp. 1829–1851

RESEARCH REPORT

Some Consequences of Prompting Novice Physics Students to Construct Force Diagrams

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We conducted a series of experiments to investigate the extent to which prompting the construction of a force diagram affects student solutions to simple mechanics problems. A total of 891 university introductory physics students were given typical force and motion problems under one of the two conditions: when a force diagram was or was not prompted as part of the solution. Results indicated that students who were prompted to draw the force diagram were less likely to obtain a correct solution than those who were not prompted to solve the problem in any particular way. Analysis of the solution methods revealed that those students prompted to use a diagram tended to use the formally taught problem-solving method, and those students not prompted to draw a force diagram tended to use more intuitive methods. Students who were prompted to draw diagrams were also more likely to depict incorrect forces. These results may be explained by two factors. First, novice students may simply be more effective using intuitive, situational reasoning than using new formal methods. Second, prompting the construction of a force diagram may be misinterpreted by the student as a separate task, unrelated to solving the problem. For instruction, the results of this study imply that ignoring students' prior abilities to solve problems and their necessary developmental stages in learning formal problem-solving techniques may lead to serious mismatches in what is taught and what is intended to be learned.

Keywords: *Problem solving; Diagrams; Free-body diagrams; Mechanics*

Introduction

Force diagrams are often used as a tool to solve problems involving Newton's laws of motion. Also called free-body diagrams, force diagrams are a generic pictorial representation using arrows to indicate the vector nature of the forces acting on the "free

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ISSN 0950-0693 (print)/ISSN 1464-5289 (online)/10/141829–23

DOI: 10.1080/09500690903199556

body" of interest (Court, 1993). They are standard in high school and university-level physics textbooks, and they are commonly taught as part of a formal step-by-step procedure to solve problems involving force and motion (e.g., Cutnell & Johnson, 2004; Halliday, Resnick, & Walker, 2008). Previous studies have investigated the spontaneous use of force diagrams by students in courses emphasizing their use and found a positive correlation between diagrams and solutions: Students who drew diagrams correctly were very likely to correctly solve the problem, and students who drew no diagram or an incorrect one were somewhat likely to have incorrect solutions (Kohl, Rosengrant, & Finkelstein, 2007; Rosengrant, Van Heuvelen, & Etkina, 2004; Van Heuvelen, 1991).

Instead of studying students' spontaneous use of force diagrams, in this study we investigated how *prompting* students to draw a force diagram affected their solutions. In particular, we presented students with simple mechanics problems and compared the performance of those who were asked to draw a force diagram representing the problem compared to those who received no such prompting. This design was selected in order to help us to answer three questions. First, does prompting a diagram increase students' success at obtaining the correct solution? Second, does prompting a diagram change the nature of the students' solution method? Third, what does student answering in these conditions tell us about student understanding of mechanics? Careful examination of student performance in detail is essential for determining whether instructional goals are achieved, and can be helpful in designing materials and methods to improve instruction. Thus, the goal of this study is not aimed at determining the effectiveness of a particular instructional intervention; rather it is aimed at studying in detail a particular aspect of student problem-solving performance in a typical classroom.

There are a number of reasons to believe why prompting students to draw force diagrams will help them to solve mechanics problems. First, as stated before, successful force diagram drawing is correlated with a correct solution. While this does not prove that force diagrams facilitate correct solutions, it is certainly consistent with such a hypothesis. Second, a significant number of studies that instructionally emphasize (among other things) either the use of diagrams in general or force diagrams, in particular, in the solution process have demonstrated significant success in improving student problem-solving in mechanics (Clement, 1993; Heller & Reif, 1984; Huffman, 1997; Larkin, 1983; Savinainen, Scott, & Viiri, 2005; Van Heuvelen, 1991; Ward & Sweller, 1990). Finally, from a more global perspective, asking students to draw a diagram is one small step that fits into a larger scheme of general problem-solving methods. An ample array of educational researchers have advocated and even demonstrated that teaching problem-solving by providing structured scaffolding of problem-solving methods can be very effective in helping students learn how to solve math and science problems (Larkin, Mcdermott, Simon, & Simon, 1980; Polya, 1945; Reif & Heller, 1982; Shoenfeld, 1978; Sweller & Cooper, 1985).

On the other hand, there are also reasons to believe that prompting a diagram may not facilitate a correct solution. First, students may use their intuition and everyday conceptual understanding to successfully solve problems in particular situations, especially ones with which they have some familiarity. For example, when students are presented with a novel but somewhat physically intuitive problem, Iszak (2004) found that the students are able to generate their own representations and criteria to enable the construction and judgment of validity of equations that model the problem. Hall, Kibler, Wenger, and Truxaw (1989) found that when solving algebraic word problems, students make significant use of models that are not related to algebraic formalism, but rather the situation. Thus, instead of using formal algorithms, students first used situational reasoning to build a model, then constructed equations for "routine formal manipulation." Finally, Koedinger and Nathan (2004) presented students with problems in the form of a story problem or an equivalent set of algebraic equations. Surprisingly, they found that the students solving the story problem performed better. The explanation for this difference stems from the students' use of informal, situational reasoning when solving the story problems; for example, order of operations errors are much less likely in the story problem condition.

Second, prompting students to draw a force diagram may cue them to use the formally taught solution method (which usually begins with the construction of a force diagram), and since they may simply not be as practiced or familiar with this formal solution method, they may consequently have a relatively high rate of failure in executing the method. This is similar to studies finding that students are more successful at reasoning tasks when they are couched in familiar contexts (e.g., Wason & Shapiro, 1971).

Finally, another potential issue with prompting a force diagram is that students may have difficulty in perceiving the goal or purpose of the force diagram as an expert would see it. Consequently, students may have less success in obtaining the correct solution because, ironically, students may regard the requirement of drawing a correct diagram as an "extra" academic exercise to be done to get a good grade, thus distracting them from the notion that the diagram is meant to help with self-consistency and increased success in solving the problem. Several have observed these kinds of attitudes among students in physics classes (e.g., Adams et al., 2006; Halloun & Hestenes, 1998; Hammer, 1994; Redish, Saul, & Steinberg, 1998). However, as pointed out by Hammer and Elby (2003), while students recognize that logical consistency and generalizability can be a valid and useful attribute of a problem-solving method, the instructional context, including the teaching method and the grading, does not necessarily motivate the student to think of the presented formal problem-solving methods in this way. This variability in student responses has been explained in terms of *resources* activated by the context and in turn accessed by the student (Hammer, 2000; Hammer, Elby, Scherr, & Redish, 2005; Sabella & Redish, 2007). In particular, prompting the construction of a force diagram may activate an epistemological resource such as "follow a relatively meaningless algorithm" when the instructor would prefer that the student access the resource of "ensure global consistency."

In sum, the construction of force diagrams is part of a well-established and successful method for solving problems in mechanics, and teaching students how to construct and use force diagrams as part of a formal problem-solving procedure is standard in introductory physics courses. Nonetheless, there are reasons to believe that prompting introductory physics students to draw force diagrams could both help and hinder them in obtaining the correct solution.

Experiment 1

The first experiment compares problem-solving performance between students who were asked to draw a formal diagram on a particular problem and students who were presented with the same problem but received no such prompting to draw a diagram.

Participants

Two populations of undergraduate students at the Ohio State University, a large public research university, participated in Experiment 1. The first population consisted of 435 students in a traditional introductory calculus-based physics course populated mostly by science and engineering majors. The instructors of the course are well-regarded, typical traditional lecturers (physics faculty) who frequently use force diagrams in solving example problems in lectures, and force diagrams are also common in example solutions in the textbook (Halliday et al., 2008). The course consisted of three traditional lectures and one recitation per week in which quizzes were administered and example problems were practiced. Grades in this course were based on homework, weekly quizzes, a midterm, and a final exam. These assessments were a mixture of multiple choice and show-work problems, heavily emphasizing "end of the chapter" problems similar to those found in the textbook. The questions were almost entirely quantitative. There was very little emphasis on conceptual questions.

The second population consisted of 315 college freshmen enrolled in an honors-level, calculus-based introductory physics course. The course emphasized explicit, research-based problem-solving techniques more than the traditional course (e.g., using the workbook by Van Heuvelen, 1996) including significant modeling and practice drawing force diagrams on every relevant problem. In addition to three interactive lectures per week, the honors class also included significant cooperative group work in practicing problem-solving during their recitation class twice per week, using specially selected education research-based context-rich problems (Heller & Hollabaugh, 1992). The grading structure was similar to the traditional course; however, there was significantly more emphasis on conceptual questions in the assessments (e.g., drawing diagrams, making qualitative comparisons, etc.). The students in the honors course have undergone a very competitive process to enter into the program and have significantly higher scores on measures such as ACT scores and high school grades compared to the traditional students.

Design and Procedure

We presented one of two mechanics problems, numbers 1 or 2 in Table 1, to students randomly placed in one of two conditions. In the first condition (denoted ND, for No Diagram prompted), the students were simply asked to solve the problem and show their work. In the second condition (denoted D, for Diagram prompted), the students were asked to first draw a force diagram relevant to the problem then solve the problem and show all of their work. The conditions were assigned randomly within a classroom, such that students in both conditions were in the same room. Each problem, including both conditions, was given to both the

Table 1. Problems used in this study

Problem 1

Mary Kate is pushing on a box with a force of 480 N in one direction and Ashley is pushing the box with a force of 340 N in the opposite direction. The box is not moving or beginning to move. There is friction between the box and the floor, and the coefficient of static friction is μ_s = 0.4 and the coefficient of kinetic friction is μ_k = 0.25.

(1) [a]Draw a diagram indicating all of the forces on the box. Be sure to label each force clearly.

(2) What is the minimum mass that the box can be in order for it to remain motionless? Show your work.

Problem 2

Three siblings, Margaret, Dan, and Liz, are playing in the basement. With some rope, they attached three boxes together in a line like a train. Liz sits in the first box, Dan in the second and they put the dog Rex in the third box. Margaret grabs on to the first box and pulls the "train" around the basement. When the kids (and the dog) are sitting in their box, each box has a total mass of 30 kg, and the coefficient of friction for the boxes on the basement floor is μ = 0.2. At one point, Margaret is pulling horizontally and the "train" is moving with constant velocity v = 2.0 m/s on the level basement floor.

(1) [a]Draw a force diagram clearly indicating the forces on Dan's box.

(2) How much force is the rope from Liz's box pulling on Dan's box?

Problem 3

A 0.2 kg hockey puck is sliding (there is friction) on the ice at 8 m/s in the positive "x" direction, directly toward the opponents goal.

(1) [a]Draw a diagram indicating all of the forces on the hockey puck. Be sure to label each force clearly.

(2) Assuming that the friction coefficient between the puck and the ice is 0.05, what is the magnitude and direction of the block's acceleration? Show your work.

Problem 4

Diagram required condition

A basketball is rolling on the gym floor toward the stands. The friction between the basketball and the ground is so small it may be ignored. Draw a force diagram indicating the forces on the basketball.

No Diagram required condition

A basketball is rolling on the gym floor toward the stands. The friction between the basketball and the ground is so small it may be ignored. What are the forces on the basketball?

[a]This part was omitted in the no-diagram-prompted (ND) condition.

traditional and honors student populations. For the traditional population, each question was administered to a separate lecture section in a different quarter. For the honors class, the problems were given to the same group of students in different weeks of the same term (each time the condition was random) after the units on Newton's laws were covered. Each problem was given as a paper and pencil quiz at the beginning of a lab class, and they were given 20 minutes to solve the problem. The questions were graded for participation only. It is our experience that students work earnestly on these problems and performance is similar to that on a for-credit test. A cursory inspection of the test responses in all groups confirmed this.

Coding of Student Responses

For each problem, student diagrams and written solutions were categorized by two physics instructors who have taught the subject matter for a number of years. Each reviewer examined the responses of a subset of students and determined categories for diagrams and solutions. The solutions were coded by analyzing equations and words written by the student. The diagrams were coded separately. The categories of the two coders were very similar in each case, and it was quickly realized that the solutions could be simply coded into a small number of categories (three or four), which are discussed in detail for each problem in the Results section. The responses were then coded independently. The inter-rater reliability was above 90% for the coding of both problems. The diagrams and solutions for which there was no initial independent agreement were proportionally distributed among the categories. These were discussed, and agreement was easily reached on the category of each response.

In addition to categorization of the solution methods, the final numerical answers were categorized as *correct* if they agreed with the correct final numerical answer or if it was clear that there was only a trivial, non-content related error, such as a simple arithmetic error, in the final solution (this happened less than 3% and the results in the following sections changed negligibly if even these errors were included). Otherwise they were marked as incorrect.

Interview Data

In order to cross-check that the student written protocols were interpreted in a valid manner, a sample of 16 students were given one of the studied test questions and they were asked to solve the problem and think aloud while they were solving it. The think-aloud sessions were video-taped and the written solutions were collected. Each of the written protocols were categorized as discussed in the section below, and this categorization was compared to the think-aloud narration by the students. The categorization given to the written protocols matched the categorization given to the think-aloud data for every case analyzed, thus further supporting the validity of the original categorization of the written protocols. Examples of student comments are provided in the sections below.

Results

Comparing scores. Perhaps the most surprising result from student responses is that in all but one of the groups, the diagram-prompted condition scored significantly lower than the condition in which the diagram was not prompted, as represented in Figure 1. In particular, for Problem 1 (see Table 1) students in the traditional class had a higher average score in ND than for D (74% and 64%, respectively, $t(326) = 1.97$, $p < 0.05$, Cohen's $d = 0.22$). For students in the honors class, there was no appreciable difference in average score (89% for ND, 90% for D, p = ns), but these scores were also close to ceiling, so one would not expect large differences. For Problem 2 (see Table 1), a similar pattern appeared. In the traditional class for Problem 2, the average score in ND was higher than for D (60% and 40%, respectively, $t(105) = 2.03$, $p < 0.05$, $d = 0.41$). For the students in the honors class for Problem 2 the score in ND was higher than for D (74% and 59%, respectively, $t(310) = 2.83$, $p < 0.01$, $d = 0.32$).

Comparing solution paths. As mentioned earlier, student solutions were categorized into three or four distinguishable and fairly coherent solution paths. The first

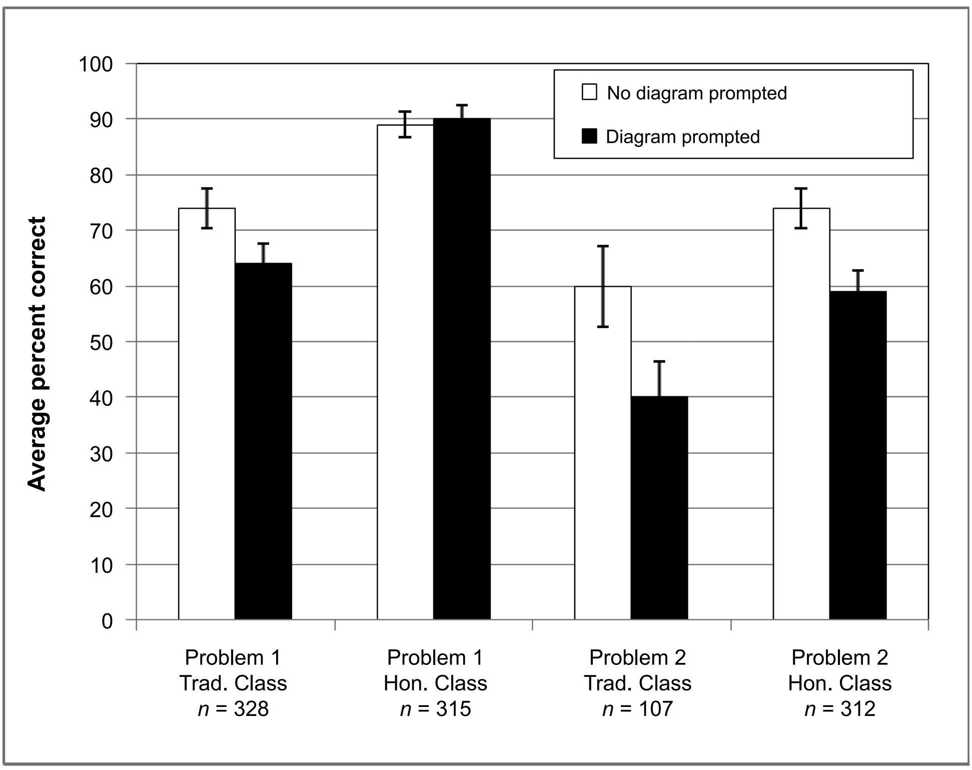

Figure 1. Percentage of students in each condition scoring correctly on Problems 1 and 2 for both traditional and honors class. Error bars represent standard error of mean

solution method category followed the formally taught solution method explicitly using $\Sigma F = ma$, as outlined in Table 2 and commonly found in textbooks (e.g., Cutnell & Johnson, 2004; Halliday et al., 2008). While there may be small variations on this method, this commonly taught formal procedure is to draw a force diagram, explicitly write Newton's second law $\Sigma F = ma$, then use the diagram to find the correct terms in the equation and solve the equation. The second major solution path category is characterized by both the absence of the explicit expression $\Sigma F = ma$, and the use of correct, physically intuitive concepts specific to the problem. The specific intuitive method is outlined in Table 2 for each problem and explained in more detail below. It should be noted that most or all students in this category use the same intuitive idea to solve the problem, thus indicating a common pattern. The third solution category, typically around 10–20%, followed a somewhat incoherent, miscellaneous method and about 90% of students following this solution path obtained an incorrect answer. Finally, for Problem 2, an additional solution category followed a coherent but incorrect solution path, and this was labeled separately.

Tables 3 and 4 present the percentage of students answering in each solution category for Problem 1 and Problem 2, respectively. In addition, Figure 2 displays the percentage of students in each condition and in each class that follow the formal $\Sigma F = ma$ solution path (as opposed to taking any other path). Figure 2 highlights the second most notable empirical result of this study: students in D tend to follow a more formal $\Sigma F = ma$ solution path than students in ND. Conversely students in ND are more likely to follow an intuitive solution method.

It is beneficial to consider the results from each problem separately. In Problem 1, the main difference between the conditions is in the number of students choosing either the method that explicitly employs the concept $\Sigma F = ma = 0$ (e.g., see Figure 3a), or the method that uses a two-step process: first finding the net force of the two sisters, then setting this net force equal to friction, with no explicit expression of $\Sigma F = ma$ present (see Figure 3b). For the vast majority of students using $\Sigma F = ma$, the expression "$\Sigma F = ma = 0$" was explicitly written at the top left

Table 2. Outline of solution paths used in Problems 1 or 2

Formal solution path for force and motion problems	Intuitive "two-step" solution for Problem 1	Intuitive "combining" solution for Problem 2
1. Draw a picture representing the problem 2. Draw a force diagram indicating all forces on object(s) of interest 3. Write $\Sigma F = ma$ for each relevant dimension and each relevant object 4. Use force diagram to identify terms in $\Sigma F = ma$ 5. Solve equation(s)	1. Draw the box and the forces of the two sisters 2. Find the difference between the forces of the two sisters 3. Equate that difference with the friction force 4. Solve equation(s)	1. Draw the train of three identical connected boxes, and draw forces on one box 2. Find the friction force on one box 3. Combine the last two boxes together to find force needed to overcome friction of both 4. Solve equation(s)

Table 3. Percentage of students using various solution methods for Problem 1

Condition	Solution method: Formal	Intuitive	Miscellaneous
Traditional class (n = 328)			
No diagram prompted	32	55	13
Diagram prompted	44	42	14
Honors class (n = 315)			
No diagram prompted	64	34	2
Diagram prompted	71	27	2

hand side of the working space just under the diagram (for both conditions); thus, it is likely that it was the first thing written after the diagram and the expressions that followed almost always followed from this equation. The think-aloud interviews confirm the ordering and the formal $\Sigma F = ma$, strategy. For example, directly after completing the force diagram, one student says "so what do we know … force equals mass times acceleration." Another student said after completing the diagram, "then I'll set up the net forces in each direction is equal to zero."

In contrast, the two-step process, while certainly valid, is more concrete and specific to this problem. One could judge that the students using this path had some admirable physical insight that the forces must balance in this situation. Further, students using this solution path broke up the problem into smaller, easier to intuitively understand chunks, starting with the difference between the forces of the two sisters. This method was thus labeled as *intuitive*.

This intuitive problem-solving process is similar in nature to the findings of Sherin (2001) who examined student use of equations while solving simple mechanics problems. Sherin proposes that rather than learning a "rigorous and routinized" method of applying equations and formally manipulating them, students have an intuitive understanding of the meaning of the equations which guides their reasoning. Thus, students express their understanding in basic "symbolic forms" that are simple equations which represent a simple physical understanding of the situation

Table 4. Percentage of students using various solution methods for Problem 2

Condition	Solution method: Formal	Intuitive	Common Incorrect	Miscellaneous
Traditional class (n = 108)				
No diagram prompted	15	45	17	23
Diagram prompted	35	25	18	22
Honors class (n = 312)				
No diagram prompted	43	45	3	9
Diagram prompted	61	24	3	12

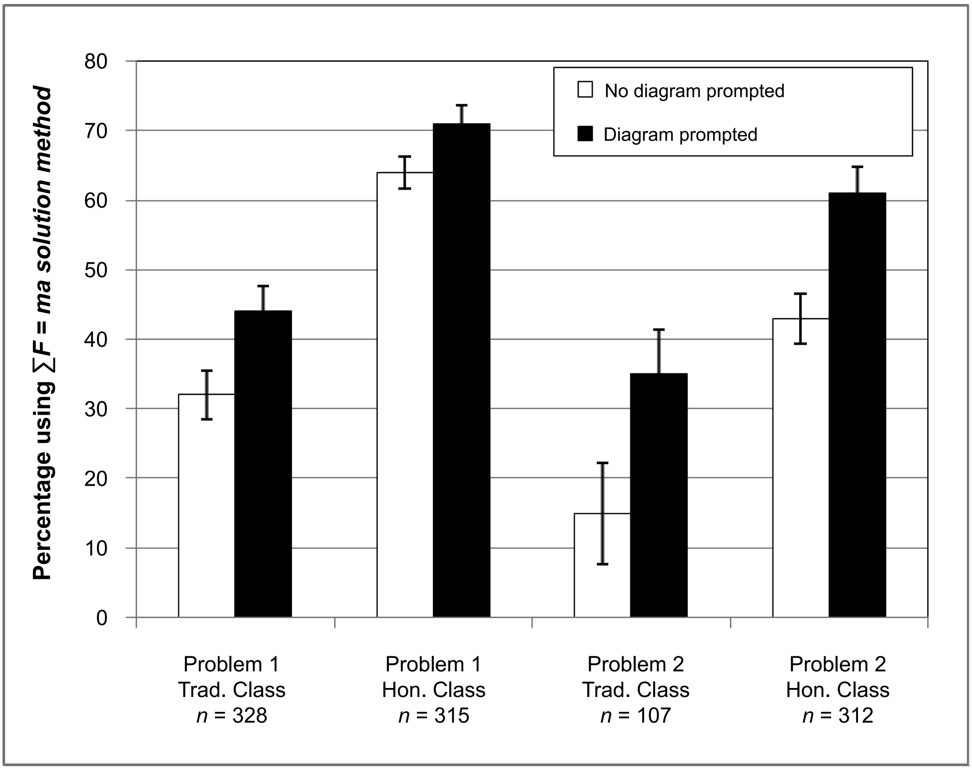

Figure 2. Percentage of students in each condition that follow the formal solution path (as opposed to taking an intuitive or miscellaneous path) on Problems 1 and 2 for both traditional and honors class. Error bars represent standard error of mean

(such as upward force equals downward force), as opposed to writing down equations which represent a formal physical principal. In addition, Sherin observes that the construction of equations and problem-solving are not completely linear processes and that there is some dynamic interaction between the two. To further support this view, we found that a significant fraction of students in ND (17%) used two diagrams (Figure 3c), which diagrammatically represent the two-step equation process and a physically intuitive understanding.

Student interviews also supported this interpretation of intuitive thinking. One student stated after reading the problem "there are two opposing forces, so obviously you just subtract them." Another said (referring to the sister's forces): "the difference is 140 Newtons, and the friction force has to be opposite of that because it's not moving." Neither student even mentioned $\Sigma F = ma$. For the traditional class, the percentage of students choosing the formal solution path was significantly greater in D (44%) compared to ND (32%), ($t(326) = 2.16$, $p < 0.05$, $d = 0.25$). For the honors class, the trend is the same, though not significant, perhaps because the students in both conditions were close to ceiling as seen by the scores (above). In D,

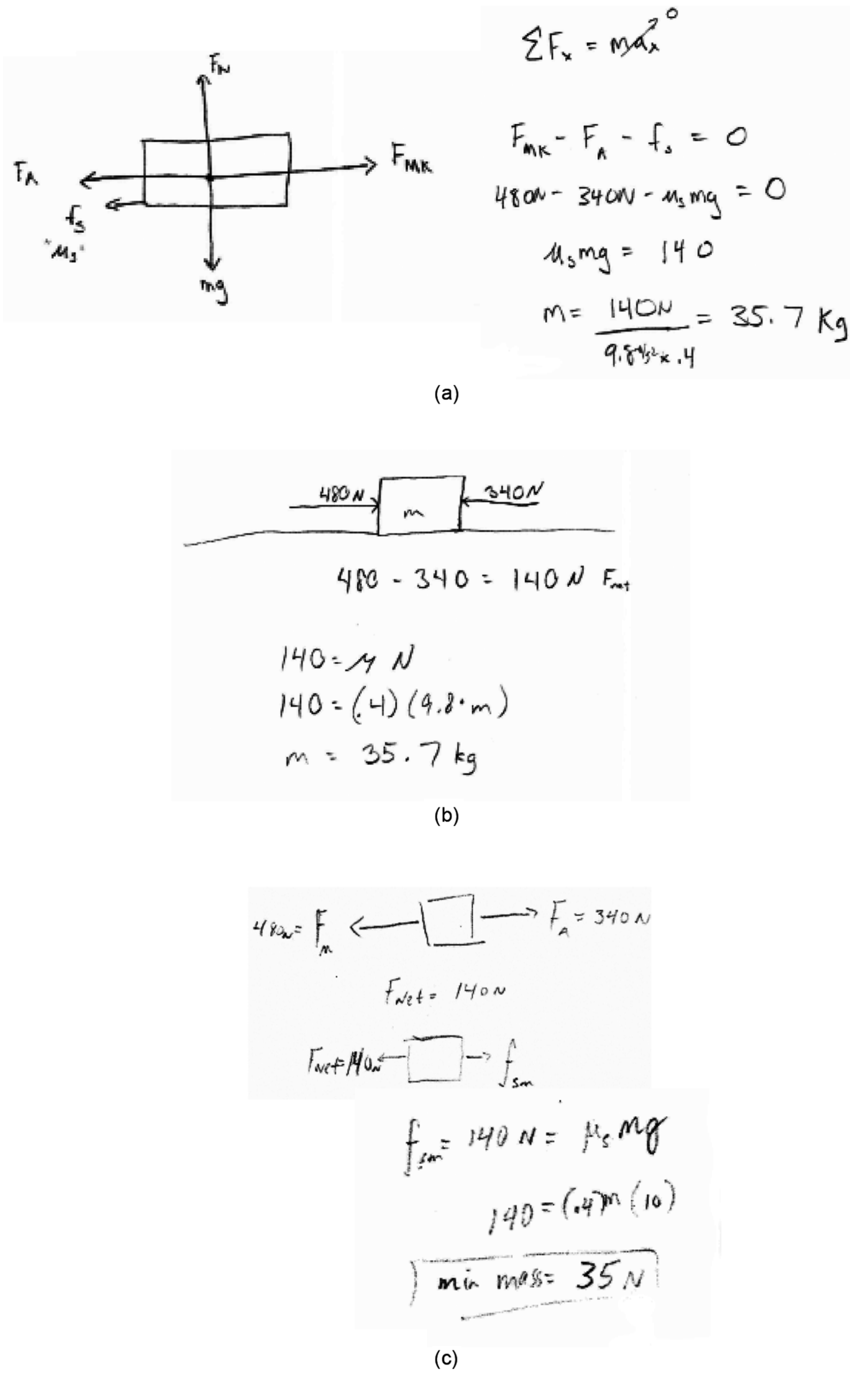

Figure 3. Examples of formal and intuitive student written solutions for Problem 1: (a) formal solution, explicitly using $\Sigma F = ma = 0$ (Diagram-prompted condition); (b) intuitive solution (No-Diagram-prompted condition); and (c) intuitive, two-step diagram/solution (No-Diagram-prompted condition)

71% chose the formal solution path compared to 64% in ND, ($t(313) = 1.47$, $p = 0.14$, $d = 0.15$) (see Figure 2).

In Problem 2 the same pattern emerged, only stronger. Students in D chose a formal solution path significantly more than students in ND. For the traditional class, 35% of students in D used the formal solution path compared to 15% in ND, ($t(105) = 2.39$, $p < 0.05$, $d = 0.48$). For the honors class, 61% in D used the formal solution path compared to 43% in ND, ($t(310) = 3.20$, $p < 0.01$, $d = 0.37$). In Problem 2, the formal solution method was very similar to Problem 1 in that Newton's second law was explicitly written $\Sigma F = ma = 0$ near or at the top of the working space below the diagram, and the expressions below this usually logically followed (see Figure 4a). The main intuitive path involved the realization that in order to find the tension on the box in the middle, one could consider the system as the middle box and the end box, and the solution is immediately solvable as simply twice the friction on one box (see Figure 4b). This could reasonably be seen as a valid and desirable physical insight into the problem, albeit specific to this kind of problem. Finally, some students answered incorrectly by considering only the friction on one box and ignoring the rest of the boxes. Clearly these students did not have a good grasp of the problem, and these solutions were labeled *common incorrect*, since this method was the most common method that was incorrect.

In sum, the students in D used a more formal $\Sigma F = ma$ solution method—the method taught in class—significantly more frequently than students in ND. The fact that students in D tend to use the more formal path is consistent with our assumption of the purpose of prompting the diagram: that it cues the formal solution process. Students in ND often chose an informal, physically valid and intuitively appealing solution path. The fact that students in D score lower will be addressed in sections below.

Comparing diagrams drawn. All students in D drew a force diagram for Problems 1 and 2. For ND, 95% students in both classes drew some kind of diagram depicting forces in Problem 1. For Problem 2, 75% of students in ND drew some kind of force diagram in the traditional class. Of the 25% of the students who do not draw a diagram, 90% of them drew some kind of picture representing the problem situation. In the honors class, regardless of condition more than 90% drew a force diagram and all of them drew at least some kind of figure representing the problem. This is also consistent with other findings that it is not uncommon for students to spontaneously draw force diagrams when solving a problem (Kohl et al., 2007; Rosengrant et al., 2004). Interestingly enough, the average score for the few students (about 10 in each class) that did not draw a diagram in ND was equal to or better than those that did draw diagrams. Thus, for these students, it does not appear that failure to draw a diagram was hindering their success on the problem.

To gain more insight into the relation between students' diagrams and solutions, we coded in detail the diagrams from the traditional students for Problem 1. Figure 5 presents the solution paths of students who drew particular diagrams, revealing two notable features of student diagrams and solutions. The first is that in

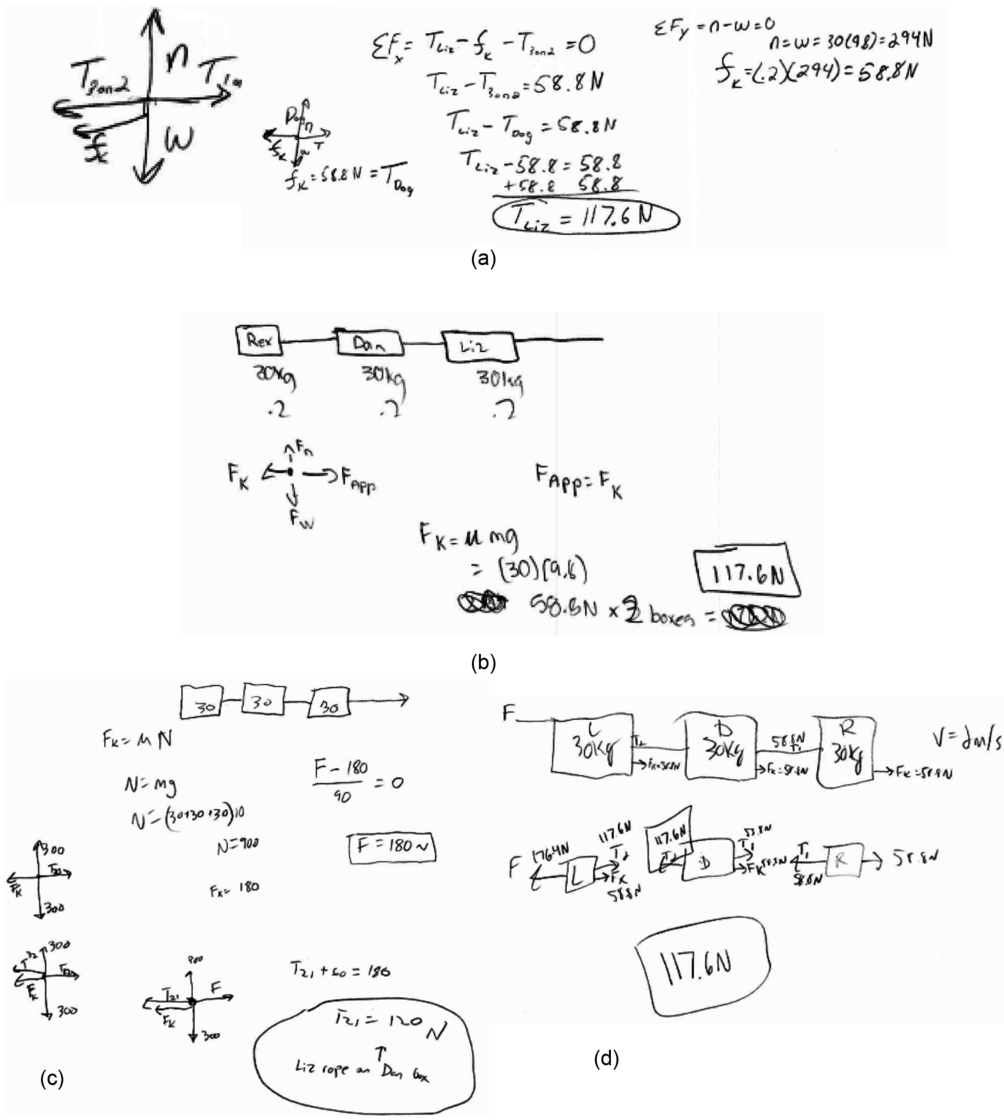

Figure 4. Examples of formal and intuitive student written solutions for Problem 2: (a) formal solution explicitly using $\Sigma F = ma = 0$ (Diagram-prompted condition); (b) intuitive solution combining two boxes (No-Diagram-prompted condition); (c) intuitive solution combining three boxes (No-Diagram-prompted condition); and (d) intuitive solution using diagrams (No-Diagram-prompted condition)

both conditions, students who draw a formally correct diagram tend to use the formal solution path compared to the intuitive path (56% vs. 24% in D and 69% vs. 17% in ND), and conversely, students who draw an informal diagram depicting only the forces of the two sisters tend to use the intuitive two-step solution, which first finds the difference between the two sisters' forces (100% in D, though this

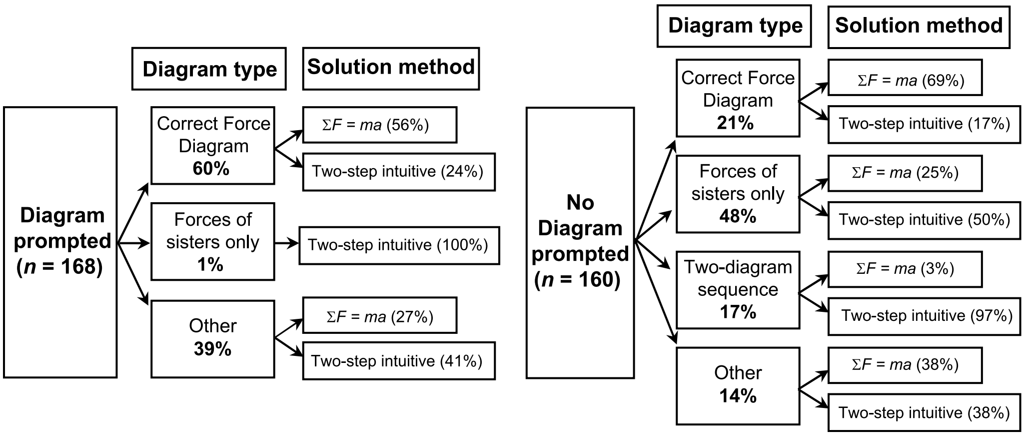

Figure 5. Constructed diagrams and corresponding solution methods used by students in both conditions in Problem 1 in the traditional class

represents only one student, and 50% vs. 25% in ND). In fact, it is interesting to note 17% of students in ND actually drew two diagrams, thus diagrammatically representing the two-step equation solution path (e.g., see Figure 3c). In fact, almost all of these students (97%) used the two-step solution path, thus showing for these students a very high consistency between the diagrams they drew and the equations used in their solution path.

The second notable feature of the student diagrams and solutions depicted in Figure 5 is that 48% of the students in ND drew the informal diagram depicting only the forces of the two sisters and another 17% drew the informal two-diagram combination, whereas only one person (< 1%) in D did either of these.

These two features taken together would suggest that prompting a diagram tends to prompt the construction of a formal diagram and thus a formal solution, while no prompting at all tends to elicit an informal diagram and a more intuitive solution.

Problem 2 also displays a similar kind of pattern in which students tend to draw more informal diagrams depicting what they presumably interpret as salient features of the problem. For example, over 90% of the students in ND in both classes drew a picture of the problem separately from a force diagram, whereas only about 60% in both classes draw one in D. Since the students in ND tend to solve the problem more intuitively, this picture-drawing may be either indicative or causal of the beginning of an intuition-based strategy. In fact, it is interesting to point out that a very small number of students in ND in the honors class (8 out of 134) solved the problem without writing down any equations; rather, they step through a process of reasoning from the diagram they drew and writing correct values, presumably calculated in their minds or on a calculator, next to the relevant forces (see Figure 4d). In D, only 2 out of 160 did this. Albeit small numbers, it still supports the above data that students in ND tend to reason intuitively rather than use the formally taught process.

Evidence for cuing incoherence. Since drawing a formal force diagram is part of the common formally taught problem-solving method, it is not surprising that prompting students to draw a force diagram may prompt them to follow the next steps in the formal solution process. However, how this results in poorer performance warrants closer investigation. We want to focus on evidence for one of the mechanisms that may be causing poorer performance, namely that the prompting of the force diagram is misinterpreted by the student as a task that is separate from solving the problem. The splitting up of the problem into two parts—drawing the diagram and finding the numerical solution—may implicitly or explicitly distract the student from making coherent sense of the whole problem and cause them to use a formal algorithm which they do not see as coherent, but rather compulsory (e.g., Redish et al., 1998). Thus, when the diagram is prompted, the process of "solving" the problem would consist of drawing a diagram to get points, then once that is completed, follow the $\Sigma F = ma$ procedure to get more points. Or as one interview student (S1) in D put it after completing the diagram and moving to the next part:

> S1: OK, go on to the next part? (Interviewer nods, student reads part 1b out loud.) So the sum of all the forces … umm (writes down $\Sigma F = ma$) … I just like to write that. Makes me feel good, cause I know that I got at least one thing correct.

This is to be contrasted to ND, in which the problem is directly posed and the students are more likely to immediately determine a path toward a solution. For example, after reading the problem, an interview student (S2) in ND immediately said:

> S2: Well, draw a picture. Just makes it easier (draws a box). It's an unknown mass and static friction because it's not moving. Someone's pushing on the box 480 N. Someone else is pushing on the box 340 N (draws two arrows on opposite sides, pushing in). Um, the difference between those is what's acting on it.

Here, S2 is simply making physical sense out of the situation and focusing on the salient fact that the sisters are pushing on either side with unequal forces, so another force (friction) must make up for the difference. Thus, the direct question may be more likely to result in a more a coherent, intuitive, sense-making method that includes the construction of both a diagram and equations together aimed at solving the problem.

Determining coherence: comparing diagrams and solution paths. Another way to determine the extent to which prompting a diagram affects the coherence of the student solution is to examine the consistency between a students' diagram and equations. If prompting a force diagram and asking for the numerical solution cues two separate goals, then students in D may tend to construct diagrams that are not consistent with (or relevant to) their constructed equations. On the other hand, for students in ND there may be a comparatively higher level of consistency between the diagram and the equations constructed because there tends to be a more globally coherent reason to draw a diagram: to solve the problem.

While, as mentioned earlier, there are a significant number of students in D who construct diagrams and equations that are mutually consistent, there are slight hints that there tends to be more agreement between diagrams and solutions in ND than in D in Experiment 1. The first hint is, as mentioned earlier, 17% of students in ND in Problem 1 drew a type of informal diagram (actually two diagrams) that was 97% predictive of the solution path. Even for those who drew a formally correct diagram, a higher percentage of students in ND (69%) used the corresponding formal $\Sigma F = ma$ solution path, compared to only 56% of those in D. This suggests that students in ND tend to have higher consistency between their diagrams and solutions.

Conversely, the second hint involves blatant inconsistency between diagrams and equations for students in D. In Problem 1, a number of students in D drew incorrect features on their diagrams that were not included in their equations, and this did not happen as frequently in ND. The text of the Problem 1 mentions both static and kinetic friction, though only static friction is relevant, since all objects remain motionless in the problem. Nonetheless, 11% of the students in D explicitly depicted both static and kinetic friction (e.g., see Figure 6) compared to 2% in ND, a significant difference ($t(326) = 3.3$, $p < 0.01$). Thus, when a force diagram was required, some students seemed compelled to depict both forces in their diagram perhaps because both forces were mentioned in the text and these students assumed that both forces must be important and included. However, many of these students may not "really believe" that both forces are completely relevant to solving the problem since less than half (8/18) actually used both forces in any equations constructed for the solution. In ND only three students depicted both forces in the diagram, but all of them used both forces in their solution.

Both of these examples suggest that students in ND tend to construct diagrams in order to integrate them into their equations more so than students in D, while at least some students in D tend to draw explicitly incorrect features in their diagrams that they then ignore when constructing their equations. This supports the explanation that at least for some students prompting a force diagram results in students interpreting it as a separate task from solving the problem. In Experiment 2, we tested this further to see if this phenomenon could be replicated.

Experiment 2

Experiment 2 was designed to replicate the modest evidence from Experiment 1 that at least for some students, prompting the construction of a force diagram for solving a simple mechanics problem can cue students to treat the construction of the diagram as a separate goal from solving the problem. For example, it may cue them into the goal of drawing a diagram to maximize credit as an end unto itself, without considering the context of solving the problem.

In Problem 1 of Experiment 1, students more often depicted an incorrect extra force in their diagram when a diagram was required compared to students spontaneously constructing a force diagram. Furthermore, only half of these students in D used the extra force in their equations to solve the problem. In order to create

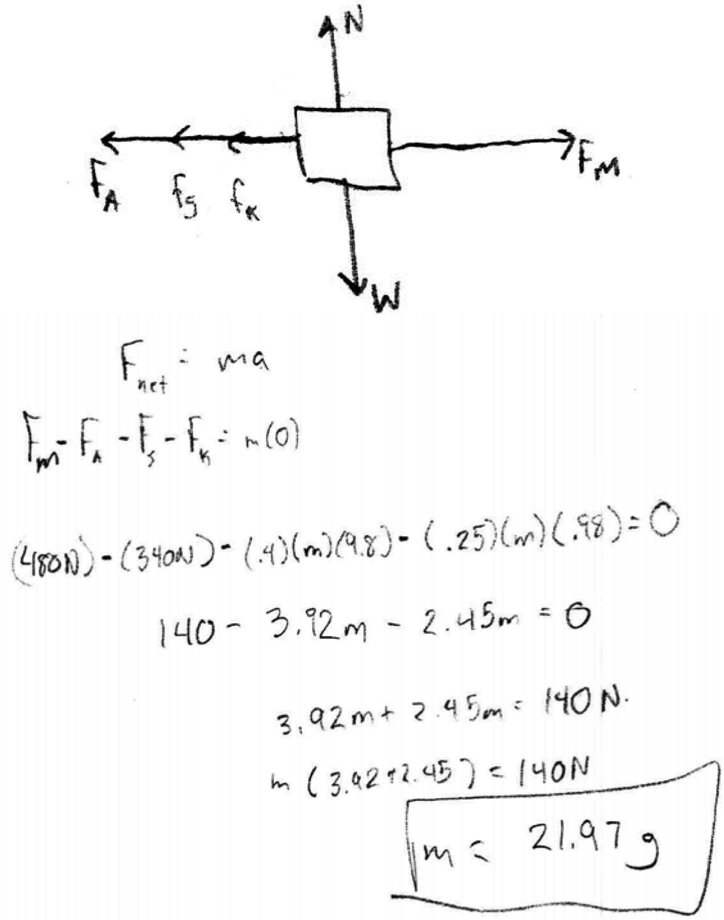

Figure 6. Example of a student solution for Problem 1 in the diagram-prompted condition that explicitly depicted both static and kinetic friction forces mentioned in the stated problem. The kinetic friction is spurious and incorrect since the box is not moving

another similar situation, Problems 3 and 4 in Experiment 2 (see Table 1) use a well-known context in which students may be likely to include an extra incorrect force, namely for an object in motion. For a moving object, introductory-level students often include an extra (and incorrect) "force of the object" if there is no other explicitly mentioned force in the direction of motion, such as gravity, or a labeled push or pull by an agent (Clement, 1982; Halloun & Hestenes, 1985; Viennot, 1979). For example, Figure 7 shows a students' diagram for Problem 3 that incorrectly displays a "force of the puck" in the direction of motion. The students commonly (and incorrectly) explain that this extra force in the direction of motion must be included because an object cannot be moving in a given direction unless there is a force acting on it in that direction.

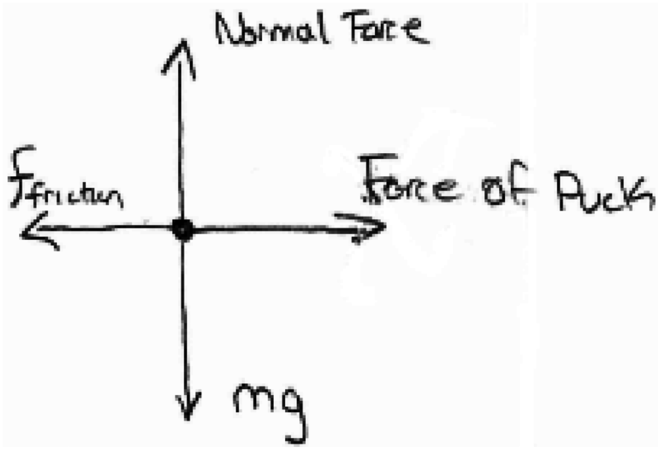

Figure 7. Example of a student diagram in Problem 3 in the diagram-prompted condition that explicitly depicted an extra and incorrect "force of puck" in the direction of motion

Participants and Design

The participants in Problem 3 are the same sample of (traditional) students as for Problem 1. The students given Problem 4 were a separate group, from a different term in the school year. The design was identical to Experiment 1, with two conditions D and ND. The only difference was that Problem 4 was a conceptual question that did not require a calculation. In this case, students in D were simply asked to draw a diagram, whereas students in ND were simply asked to indicate the forces on an object (free response).

Results

Figure 8 shows that significantly more students in D depicted an incorrect force of motion compared to ND in both problems, (for Problem 3: 20% in D and 8% in ND, $t(326) = 3.27$, $p = 0.001$, $d = 0.35$; for Problem 4: 28% in D and 5% in ND, $t(123) = 3.70$, $p < 0.001$, $d = 0.65$). In Problem 3, only half (16/32) of the students who depicted force of motion in D actually used this extra force in their equations. Most of the rest (10/32) used the formal solution path, $\Sigma F = ma$, completely ignoring the force of motion depicted in the diagram. In ND, 10 of the 13 students who depicted force of motion used this extra force in their solution, and only 1/13 of the remaining used the formal $\Sigma F = ma$ solution path, and ignored the depicted force of motion. Therefore, as in Experiment 1, students in D tended to depict an incorrect force more often, yet tended to use what was depicted in their diagrams relatively less often than those students in ND.

Problem 4 simply asked students to identify all of the forces on the moving object. Nonetheless, the same pattern was found: students in D depicted an incorrect force

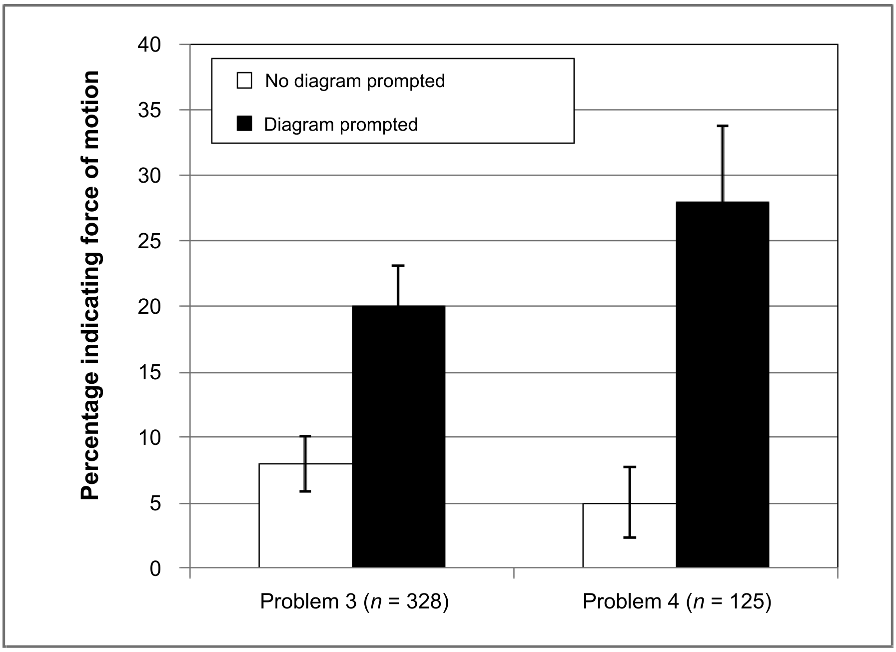

Figure 8. Percentage of students in each condition indicating an extra and incorrect force in the direction of motion for Problems 3 and 4. Error bars represent standard error of mean

of motion more often than in ND. Even in this conceptual problem, 53% of the students in ND drew a force diagram, yet only 9% (3/34) of these students depicted force of motion compared to 28% (17/61) of the students in D. This highlights the possibility that neither asking for a list of forces nor drawing a force diagram itself elicits the force-of-motion misconception, but rather it is specifically *prompting* a force diagram that elicits the incorrect force. This supports the idea that there is something particular about specifically prompting the construction of a force diagram that tends to cause students to treat the diagram as a goal unto itself, separate from other tasks.

General Discussion

Experiment 1 provides evidence that when presenting introductory physics students with a simple mechanics problem, scaffolding a solution by first prompting the students to draw a force diagram may result in lower success in obtaining the correct answer compared to no prompting at all. This is somewhat surprising given that force diagrams are part of the standard, formally taught solution process aimed at facilitating success, and this kind of scaffolding in tests is not uncommon.

Why does prompting a diagram tend to decrease success in finding the correct answer? Upon closer examination, differences appear between the solution methods of students who have been prompted to draw a force diagram compared to those who have not, and two possible reasons for the decrease in success begin to emerge. First, it is observed that many students in both conditions use less formal and more intuitive or everyday reasoning to find a solution. The students achieve this by using their familiarity with the specific context of the problem to build their informal, situational reasoning. For example, instead of starting from the general principle of $\Sigma F = ma$, students use features specific to the problem, such as "the object is stationary," to employ the common concept of "forces are balanced." Interestingly, this study finds that students with no force diagram prompting tend to use intuitive, situationally specific reasoning more often that students who were required to draw a force diagram. Could it be that students with no diagram prompting are more successful because they tend to use more intuitive reasoning at which they are more successful? As mentioned earlier, there are certainly studies which indicate that there are situations where using informal yet familiar reasoning can lead to greater success (Hall et al., 1989; Koedinger & Nathan, 2004; Wason & Shapiro, 1971). This is also reminiscent of the studies of Luchins (1942) who found that mechanistic learning of an algorithm impeded a students' ability to solve problems that could be solved more easily without the algorithm.

This is not meant to suggest that teaching a formal solution method is counterproductive or even that students' intuitive, everyday problem-solving abilities are better than formal ones. Rather, this is meant to suggest that when teaching formal problem-solving methods, it may be important to consider that students may already have their own methods, and that an approach acknowledging developmental stages in formal problem-solving may be more effective. Certainly the implicitly or explicitly understood methods students use may be fundamentally flawed, or applied in the wrong context. Nonetheless, whether their methods are physically valid or not, it is not unreasonable to expect that this informal problem-solving knowledge will interact (e.g., interfere) with what is formally taught. This is similar to what has been observed in student learning of scientific concepts: if the instructor ignores students' prior knowledge, there will likely be serious mismatches between what is taught and what is learned (e.g., Bransford, Brown, & Cocking, 1999; McDermott, 1991). This may also call into question what test questions such as those used in this study are rewarding. To the extent that students perform worse when they follow the formal solution method, it would seem reasonable for students to favor an intuitive and successful method and be reluctant to use a formal and less successful method.

Experiments 1 and 2 provide evidence for another reason for the difference in success between the conditions: Prompting the construction of a force diagram may tend to cue undesired epistemological resources in novice students. For the questions studied here, requiring the students to construct a force diagram appeared to not only cue students to depict incorrect forces that would not otherwise be depicted, but it may also have cued at least some students into the mindset that constructing the diagram and solving the problem are two separate tasks. This is consistent with a number of studies documenting that novices often display the epistemological belief that problem-solving consists of finding the right equation and that global coherence is not needed (e.g., Redish et al., 1998). These results also corroborate a more general view that each problem context may cue a specific set of resources (epistemological or otherwise) that may inhibit the utilization of other possibly useful resources available to the student. For example, Sabella and Redish (2007) compared problem-solving performance between students who all have been given a typical mechanics problem to solve, but one group had an additional set of qualitative questions added to the problem. They found evidence that the additional questions may cue students into the wrong "knowledge structure" and prevent them from solving the problem. Put in term of understanding students' developmental stages of problem-solving, it may be that instructors often assume that students already understand the value and purpose of formal problem-solving methods as an expert does, when in fact they may not.

This study also indicated that the difference in performance between conditions existed for both the traditionally taught students as well as the honors students who were taught with more education research-based methods. The honors students performed better overall, but the pattern of performing worse when the force diagram was prompted remained. This would suggest that the improved problem-solving methods still do not address the issues raised here. For example, the results suggest that the improved instructional methods may not significantly improve students understanding of the nature and purpose of force diagrams as part of a necessarily coherent method to solve problems.

Finally, it could be argued that the results of this study risk passing judgment too soon on the usefulness of teaching a formal solution method. Quite possibly, the prompting of force diagrams may serve longer term goals, and since students will in time learn to master the formal solution method this intermediate state may not be as relevant. There are two responses to such a comment. First, these questions were posed toward the end of a course which is terminal for many students. While it is true that students going on to study more mechanics may fare well in the long run, the remainder of the students appear to possess only partial and inefficient knowledge and skills and risk being left with the impression that formalism is largely academic and not practically useful. Second, a primary reason for studying students in detail is to examine the process of learning in its intermediate stages, not simply at novice and expert states. These detailed observations can help us to understand the learning process so that we may optimize learning and customize instruction to the specific topic. In this case, understanding what students already know about solving mechanics problems and how they progress through the process of learning a formal solution method can help us to better teach students holistic skills—including the use of intuition—that are necessary for solving problems.

Acknowledgments

This research is partially supported by a grant from the Institute of Education Sciences, US Department of Education (No. R305H050125). The author would like to thank Steve Stonebraker for helpful conversations.

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