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# Patterns of Response Times and Response Choices to Science Questions: The Influence of Relative Processing Time

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**Abstract**

We report on five experiments investigating response choices and response times to simple science questions that evoke student "misconceptions," and we construct a simple model to explain the patterns of response choices. Physics students were asked to compare a physical quantity represented by the slope, such as speed, on simple physics graphs. We found that response times of incorrect answers, resulting from comparing heights, were faster than response times of correct answers comparing slopes. This result alone might be explained by the fact that height was typically processed faster than slope for this kind of task, which we confirmed in a separate experiment. However, we hypothesize that the difference in response time is an indicator of the *cause* (rather than the result) of the response choice. To support this, we found that imposing a 3-s delay in responding increased the number of students comparing slopes (answering correctly) on the task. Additionally a significant proportion of students recognized the correct written rule (compare slope), but on the graph task they incorrectly compared heights. Finally, training either with repetitive examples or providing a general rule both improved scores, but only repetitive examples had a large effect on response times, thus providing evidence of dual paths or processes to a solution. Considering models of heuristics, information accumulation models, and models relevant to the Stroop effect, we construct a simple relative processing time model that could be viewed as a kind of fluency heuristic. The results suggest that misconception-like patterns of answers to some science questions commonly found on tests may be explained in part by automatic processes that involve the relative processing time of considered dimensions and a priority to answer quickly.

*Keywords:* Response time; Fluency; Heuristics; Dual processes; Stroop effect; Science misconceptions; Graph comprehension; Science concepts

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## 1. Introduction

One of the most important empirical findings of science education research in the last half century is the fact that, when asked simple scientific questions, people often get them wrong in regular, patterned ways. More specifically, thousands of empirical studies have established that when conceptual questions about simple natural phenomena are posed to students, their answers are often contrary to scientists' answers, remarkably similar to those of other students, and resistant to traditional instruction (see compilations in Kind, 2004; McDermott & Redish, 1999; Pfundt & Duit, 2000). For example, students often believe, even after traditional instruction, that an upward traveling ball must have a net upward force acting on it (Clement, 1982). These patterns of "incorrect" answers to science questions permeate virtually every aspect of science education.

While patterns in response *choices* to science concept questions have been well-studied, the extent to which there are also corresponding patterns in other response metrics, such as response *times*, is relatively unknown. This additional information about response times is potentially important because it may help to gain insight into mechanisms contributing to the pervasive phenomenon of incorrect answering patterns to science questions.

Therefore, this study is aimed at investigating two questions. First, are there interesting patterns in response times that correspond to specific response choices to simple conceptual science questions known to commonly evoke incorrect responses? Experiments 1 and 2 provide strong evidence that such patterns exist. This finding leads to the second question: What can response time data reveal about the mechanisms involved in generating patterns of incorrect responses? Through a variety of contextual and instructional manipulations in Experiments 3–5, we investigate this question and argue that implicit, automatic processes involving relative processing time of different solution paths (as measured by response time) may play a significant role in generating response choice patterns to at least some science questions typically used in testing and instruction.

We will propose a simple model of incorrect answer patterns based on relative processing time of different solution paths. This model will emerge from the introductory discussions on dual process theories, heuristics, and response time models for simple tasks, and in the course of the experiments. The model is explicitly introduced in Experiment 3. Finally, we summarize the empirical results, discuss how they compare to the simple model we introduce in this study, discuss other prevalent related models and phenomena, and discuss the multiple points of significance of our findings.

### *1.1. Response patterns and dual process models*

Cognitive psychologists have long noted that for a variety of tasks there appears to be two distinct ways to arrive at a response, and in many cases these two paths lead to different responses (cf. the *Criterion S* of Sloman, 1996) that may be associated with different response times. One kind of response tends to be fast, implicit, intuitive, automatic, relatively effortless and is ascribed to being a result of System 1 processes. The other response tends to be slower, explicit, and controlled and is thought to come from a System 2 process (e.g., Evans, 2008; Kahneman & Frederick, 2002; Stanovich & West, 2000). Interestingly, studies tended to concentrate on cases in which the fast implicit response was the incorrect response. Classic examples used to study the possible existence of such dual processes include optical illusions, the Stroop task (e.g., MacLeod, 1991), the Wason selection task (e.g., Evans, Newstead, & Byrne, 1993), and answers to simple questions involving statistical inference (Tversky & Kahneman, 1974). Note that the Stroop task is of particular relevance to this study and will be discussed in more detail in the final discussion section at the end of the paper.

Can dual process theories help to explain response patterns to science concept questions? The general idea that implicit and explicit processes are at work in the answering of science questions related to physical phenomena has been proposed and investigated in the past, though such studies (which are relatively scarce) have focused more on the format of the question or the type of response rather than the speed of the response to determine the implicit or explicit nature of the processes involved in responding (cf. Kozhevnikov & Hegarty, 2001). For example, differences in answering on verbal tasks compared to equivalent physical tasks have been attributed to the utilization of a higher-level conceptual system compared to a lower level perceptual system based on every day perceptual experience (Oberle, McBeath, Madigan, & Sugar, 2006; Piaget, 1976). Similarly, differences in predictions of motion using static versus dynamic diagrams has been attributed to explicit reasoning versus perceptual knowledge that is based on common experience (Kaiser, Proffitt, & Anderson, 1985; Kaiser, Proffitt, Whelan, & Hecht, 1992; Rohrer, 2003; however, see also Thaden-Koch, Dufresne, & Mestre, 2006). Another example is *representational momentum*, in which the image of an apparently moving object is recalled in a position shifted forward (Freyd, 1987; Freyd & Finke, 1984a; Hubbard, 1998). The observers are not aware of the distortion; thus, their response is considered as originating in implicit knowledge. Kozhevnikov and Hegarty (2001) found that even physics experts' implicit knowledge as measured by representational momentum is non-Newtonian. They proposed that implicit knowledge may affect explicit answering under certain constraints such as time limitations.

In this study, we will measure the relative response times for correct and incorrect responses which can provide further evidence for faster more automatic and implicit processes at work (cf. De Neys, 2006). Perhaps more important, such measurements can also help to identify specific mechanisms underlying such automatic implicit processes. This will be discussed in the following sections on heuristics and response time models.

### *1.2. Response patterns and heuristics*

Based on work by Simon (1955), judgment and choice have been explained in terms of *bounded rationality*, namely, that people make rational decisions that automatically include real-world constraints such as limited time and limited access to information. This idea has led to the hypothesis that people use fast and efficient *heuristics* to make choices. While heuristics are often quite successful in achieving tasks, in other cases heuristics can lead to biases that cause systematic errors. For reviews of the topic of heuristics and biases, see Gigerenzer (2008), Kahneman (2003), and Gilovich and Griffin (2002).

The heuristics explanation is related to the dual system perspective in that heuristics tend to be regarded as an automatic, bottom-up process rather than an analytic explicit reasoning process (e.g., Evans, 2008; Kahneman, 2003; Stanovich & West, 2000). Heuristics are an attempt to more carefully model automatic, fast and efficient processes. However, as pointed out by Gigerenzer (2008), early attempts at characterizing specific heuristics lacked sufficient specificity necessary for rigorous theoretical consistency and empirical testing. For example, the *availability heuristic* was first described by Tversky and Kahneman (1973) as the fast heuristic by which people judge the probability of events by their "availability." Kahneman (2003) later discussed the related notion of *accessibility* or "ease with which particular mental contents come to mind" as being a critical factor in determining which dimensions are used in making judgments, though he also acknowledged that this was not a sufficiently detailed theoretical account.

There has been some progress in establishing testable predictions from models of heuristics that bolster the scientific usefulness of the heuristics hypothesis (Gigerenzer & Brighton, 2009). In fact, response time data can provide crucial evidence for distinguishing between the use of automatic processes such as heuristics as opposed to more deliberate reasoning processes (e.g., De Neys, 2006; Evans, 1996). Bergert and Nosofsky (2007) recently modeled response choices and response times for a generalized version of the well-studied Take-the-Best heuristic (Gigerenzer & Goldstein, 1996) and a prevailing generalized model of (explicit) rational decision making for a multi-attribute decision task and compared the results to participant data. They found that both models yielded similar response choices, but the response times were more closely matched to the Take-the-Best heuristic.

### *1.3. A heuristic using processing time as a factor*

Much of the work on explicit and implicit processes discusses the response time as being a result or output of the process employed. However, the notion of heuristics versus deliberate reasoning is partially motivated by the hypothesis that heuristics are used *because* they are faster. Therefore, we would like to consider the possibility that processing time is part of the *input* in the decision mechanism itself because value is implicitly placed on answering quickly.

Such a mechanism might be viewed as a way to define the notion of *accessibility*: The smaller the processing time, the more accessible the dimension. A similar approach focusing on processing time has been taken by Schooler and Hertwig (2005) in their work on the *fluency heuristic* (see also Hertwig, Herzog, Schooler, & Reimer, 2008). For a two-choice task, the fluency heuristic can be stated as follows: If one of two objects is more fluently processed, then infer that this object has the higher choice value. Schooler and

Hertwig, whose work focused on memory retrieval, interpreted fluency via processing time; therefore, this heuristic effectively chooses the option that is processed the fastest.

Here, we are interested in considering a similar version of the fluency heuristic by applying the idea of *relative processing time* to explain response patterns to simple science concept questions. The framework of the model (described in more detail in Experiment 3) is the following: Accumulate information from the two dimensions available in the question (even though one happens to be relevant, and the other irrelevant), and base the response on the dimension processed first. This relative processing time model is rooted in the priority to answer quickly. Note that if both dimensions are processed before the choice is made and the dimensions lead to different responses, then some other decision process must be used to choose between the responses. In Experiments 3–5, we will consider this relative processing time model as a kind of fluency heuristic to explain response patterns.

### *1.4. Response time models*

Response time models may provide an avenue to delve yet deeper into basic mechanisms behind automatic processes (such as manifest by the fluency heuristic) contributing to answering patterns. One can consider two classes of response time models, roughly distinguished by either involving more rule-based, multiple processes with large response times on the order of seconds, or more automatic, single processes with small response times < 2–3 s. At least some of the longer timescale multistep processes are likely comprised of a collection of shorter time scale processes, but how they might be unified into one model is still an open question. Multi-step models are exemplified by the highly successful model Active Control of Thought-Rational (ACT-R; Anderson, 1993; Anderson & Lebiere, 1998), which is a well-developed cognitive architecture that includes both procedural (rule-based) and declarative tasks. It has been used to model response choices and response times for a wide range of tasks (e.g. Anderson, Fincham, & Douglass, 1999), including for graph reading tasks (Peebles & Cheng, 2003).

Complex model and cognitive architectures such as ACT-R may ultimately be best suited to coherently model the overall results of this study and address the fact that the task of answering science concept questions may involve several steps and take at least several seconds to answer. However, it is the second, finer-grain class of models, those relevant to single processes and short reaction times that are currently better suited to more carefully *characterize* and explain the response time data in this study. For example, with only a small number of parameters these short timescale models can characterize and predict the shape of the response time distributions for simple tasks, which may carry important information not described by the mean (Balota & Yap, 2011; Heathcote, Popiel, & Mewhort, 1991; Ratcliff & Murdock, 1976).

Over the last 50 years, there have been a large number of such quantitative response time models for simple two-choice or multichoice tasks, largely motivated to explain the well-known speed-accuracy tradeoff phenomenon (Brown & Heathcote, 2008; Luce, 1986; Ratcliff & McKoon, 2008; Townsend & Ashby, 1983; Usher & McClelland, 2001). We may rule out one category of these models: the simple fast-guess model. This model assumes that responding is governed by two processes: guessing, which is fast and random, or stimulus-controlled, which is slower (Ollman, 1966). For the science concept question studies here, student responses are not dominated by guessing. We will show that students responding correctly or incorrectly tend to respond consistently and not randomly.

Another category of simple-task models, namely information accumulation models or "sequential sampling models" (e.g., Brown & Heathcote, 2008; Ratcliff & McKoon, 2008; Usher & McClelland, 2001) may be relevant to simple multiple-choice science questions like the one used in this study, especially after students answer many similar questions in a row, possibly allowing the answering process to become more simplified and automated. In these models, once the question is presented, a stochastic (e.g., Ratcliff & McKoon, 2008; Usher & McClelland, 2001) or steady (e.g., Brown & Heathcote, 2008) accumulation of information occurs until the accumulation has reached a predetermined critical level for a response. Most of these have been able to accurately model with a relatively small number of parameters a wide range of response time phenomena including the speed accuracy tradeoff, the shape of the response time distribution, and response times for correct versus incorrect responses. Furthermore, these models may be more than just phenomenological. They are consistent with some neural models and data (Ratcliff & McKoon, 2008).

The most relevant single-task response time models for the task in this study are those in which there are *n* information accumulators (one for each response choice) and each accumulator has its own average information accumulation rate, decision boundary value, and average starting point. The accumulator that reaches its boundary first (i.e., "wins") is the one chosen for the response, analogous to a race. An example of simple model is that of Brown & Heathcote, 2008. Note that different response choices are likely to have different response time distributions, reflecting different average accumulation rates, initial conditions, and final boundaries. Also note that this model is consistent with the fluency heuristic mentioned in the previous section. This includes the fact that accumulator models must resort to some other decision mechanism to mediate a "tie." We will consider general features of the information accumulation model throughout the paper, including in the construction of a simple model outlined in Experiment 3.

### *1.5. Response patterns and science education*

While studies in cognitive psychology have devoted significant attention to automatic and implicit processes when considering patterns of incorrect responses, this is not the case in the field of science education (see also Heckler, 2011). Perhaps the most common explanation for incorrect answering patterns in science education stems from the inference that the patterns are caused by relatively "higher level" mental structures such as concepts, schemas, mental models, or loose collections of pieces of knowledge (e.g., Carey, 2009; Driver & Erickson, 1983; McCloskey, 1983; Novak, 2002; Smith, diSessa, and Roschelle (1993); Stavy & Tirosh, 2000; Vosniadou, 1994; Wellman & Gelman, 1992). One prevalent explanation is that the answering patterns arise from somewhat coherent and generally applied "misconceptions" or "naïve theories" cued by the question and constructed by students from their everyday experience. A student, for example, may answer that an upward moving ball must have a net upward force because he/she has developed a coherent "impetus" theory (i.e., misconception) that all moving objects must have a force in the direction of their motion (e.g., Halloun & Hestenes, 1985).

The key issue here is that many of these approaches tend to *infer* that incorrect answering patterns are primarily a result of some relatively high order mental structure such as a naïve theory or concept. Clearly it is reasonable to conclude that *if* students held coherent, incorrect theories (i.e., misconceptions), then they would likely answer relevant questions in patterned incorrect ways, assuming the misconception is applied consistently (cf. Keil, 2010). However, this is not necessarily the case: Patterns of incorrect answers may also be due to other causes. Thus, we will often refer to patterns of incorrect answering as *misconception-like* answers, because the patterns may not be due to misconceptions per se.

Here we investigate the possibility that misconception-like answers could also stem from or at least be significantly influenced by implicit, automatic processes involving relative processing time that direct the student toward "undesired" answers in regular ways, and sometimes may have little to do with consistently applied explicit concepts (see also Heckler, 2011). These automatic processes may help people to function in the everyday world (e.g., answer quickly), but they may present barriers for answering scientific questions and acquiring scientific knowledge.

It is important to note that our focus on implicit, automatic processes as a significant influence on answering patterns to science questions is to be seen as complementary to (rather than in conflict with) most existing explanations in science education research which primarily focus on top-down, higher level mental structures or processes. Furthermore, knowledge of the role of bottom-up mechanisms may also prove useful for interpreting student responses to science questions typically used in testing and instruction and for designing instruction to improve student performance on difficult science questions.

### *1.6. The science concept questions used in this study*

The set science concept questions used in this study is based on a well-known student difficulty with interpreting graphs commonly used in math and physics courses at the high school and introductory university level (Beichner, 1994; Kozhevnikov, Motes, & Hegarty, 2007; Mcdermott, Rosenquist, & van Zee, 1987; Mokros & Tinker, 1987). Students often interpret graphs as a physical picture and commonly confuse the physical meaning of *height* and *slope* at a point on a line. When a variable of interest corresponds to the slope of a line at a given point, students instead often attend to the value (i.e., the height) of the point on the line rather than the slope at that point. For example, when students are presented with a position versus time graph for an object and asked “At which point does the object have a higher speed?”, many incorrectly and consistently answer according to the higher point rather than the greater slope (Mcdermott et al., 1987).

A useful way to consider the general structure of the questions posed in this study is presented in Fig. 1. There are chiefly two available dimensions, *height* and *slope*, that the responders may implicitly or explicitly consider when determining their response. Importantly, novice students may utilize dimensions not scientifically valid according to experts because the students may perceive these dimensions as relevant. In the case of the graphs questions, *slope* is the “correct,” scientifically relevant dimension to utilize, while *height* is the irrelevant dimension that will lead to the incorrect answer for the target questions in this study.

Therefore, we propose that the two available dimensions lead to two different solution paths and potentially two different response choices and response times. We will demonstrate that the graph questions used in this study follow this structure (Fig. 1), namely that (a) the questions have two prominent available dimensions for consideration; (b) there are two populations of students who consistently choose one or the other dimension to determine their answer; and (c) the response choices are associated with different characteristic distributions of response times. We will study the nature of the differences in these response times in more detail in order to construct a simple model of misconception-like responses to the simple graph task.

## 2. Experiment 1: The relative processing speed of slope and height

Experiment 1 has two goals. The first goal is to establish that the population studied is able to recognize and extract *slope* and *height* information from a point on a line on a simple, generic graph. This was done by asking students to compare the relative heights or relative slopes of two points on a curved line on a graph. The second goal is two determine whether there are any within-student differences in processing time distributions for answering *slope* or *height* questions.

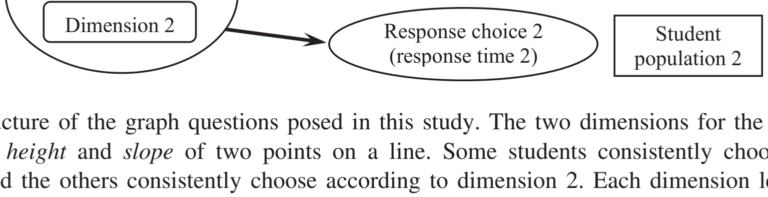


Fig. 1. The structure of the graph questions posed in this study. The two dimensions for the graph questions are the relative *height* and *slope* of two points on a line. Some students consistently choose according to dimension 1, and the others consistently choose according to dimension 2. Each dimension leads to a different response choice and response time.

## 2.1. Method

### 2.1.1. Participants

Eighteen undergraduates enrolled in an undergraduate calculus-based introductory physics class participated, receiving partial credit for participation.

### 2.1.2. Materials, design, and procedure

Testing was presented to individual participants on a computer screen in a quiet room. Participants were presented with examples depicting various position time curves for a car (e.g., Appendix A). For each graph, two points on the curve were marked, indicating the position and time of the car at two different times. Participants were asked to determine as quickly as they could without making a mistake either which point was higher, or at which point the slope was greater. The test was administered in blocks of nine questions of the same type (compare height or compare slope). Question type blocks were presented in an alternating sequence, with two blocks for each question type, for a total of four blocks (36 questions), each with a set but random order of questions.

## 2.2. Results and discussion

Students successfully answered both *slope* and *height* questions. The mean score for the compare-height and the compare-slope questions was >97% for both types. Because the response times in the first two blocks were initially relatively high and decayed to an asymptote within 3–4 questions and the times were near a steady asymptote in the final two blocks, we only compared the response times in the third and forth blocks. The mean response time was significantly lower for the compare-height questions ($M$ = 788 ms) versus the compare-slope questions ($M$ = 1,216 ms), paired-sample $t(17) = 7.04$, $p < .001$, $d = 1.28$.

In order to more carefully parameterize the shape of the response time distributions for each condition by more than just the mean, we fit the curves to an ex-Gaussian function, which has been demonstrated to be a useful fit to response time curves for a variety of tasks (Balota & Yap, 2011; Heathcote et al., 1991; Ratcliff & Murdock, 1976). This method will allow us to disambiguate factors that contribute to the shape of the curve as well as quantitatively parameterize and compare curves. The ex-Gaussian function is a combination of a Gaussian and exponential decay function:

$$f(t, \mu, \tau, \sigma) = \frac{e^{\left(\frac{\sigma^2}{2\tau^2} - \frac{t-\mu}{\tau}\right)}}{\tau\sqrt{2\pi}} \int_{-\infty}^{\frac{t-\mu}{\sigma} - \frac{\sigma}{\tau}} e^{-\frac{y^2}{2}} \mathrm{dy}$$

The parameter $\mu$ can be thought of as the peak of the distribution, $\tau$ as the decay constant for the tail of the distribution, and $\sigma$ as contributing to the width. For data that perfectly fits the ex-Gaussian distribution, the mean, $M$, is given by $M = \mu + \tau$, and the variance by $s^2 = \sigma^2 + \tau^2$.

We applied maximum likelihood methods to the observed response times to find estimates of the three parameters, presented in Table 1. Standard errors of the parameter estimates were computed using the Fisher information matrix (Lehman & Casella, 1998).

The results in Fig. 2 and Table 1 provide a clear indication that the peak, variance, decay and mean times are larger for the slope comparison task compared to the height comparison task. In sum, the results establish that the participants are able to compare slopes and heights of points on a simple graph, but in this context it takes significantly longer to compare the slopes at two points on a line than to compare the heights.

### 2.3. Caveat for data analysis

There is one important caveat to the ex-Gaussian fitting employed for all of the response time data in this paper. Typically, parameters are fit for a single participant individually, and this is the usual assumption of response time models, such as information accumulation models. Here, we will determine parameters for populations of students,

Table 1
Experiments 1, 2, 4, and 5 response time results, in milliseconds (SE in parentheses)

| | | | MLE of ex-Gauss. Parameters | | |
|---|---|---|---|---|---|
| Group | Response | Mean | $\mu$ | $\tau$ | $\sigma$ |
| Experiment 1 | | | | | |
| Compare height | Correct | 727 (21) | 553 (10) | 175 (18) | 37 (8) |
| Compare slope | Correct | 1,140 (30) | 877 (43) | 262 (46) | 205 (30) |
| Experiment 2 | | | | | |
| Math | Incorrect | – | – | – | – |
| Math | Correct | 2,949 (80) | 1,360 (47) | 1,566 (82) | 316 (38) |
| Kinematics | Incorrect | 3,457 (203) | 946 (82) | 2,418 (192) | 259 (65) |
| Kinematics | Correct | 3,826 (101) | 1,711 (71) | 2,081 (115) | 528 (57) |
| Elec. potential | Incorrect | 2,553 (180) | 631 (26) | 1,890 (143) | 35 (23) |
| Elec. potential | Correct | 3,662 (208) | 1,168 (149) | 2,425 (230) | 464 (135) |
| Experiment 4 | | | | | |
| Control | Incorrect | 3,053 (171) | 453 (63) | 2,508 (162) | 222 (52) |
| Control | Correct | 3,215 (138) | 1,139 (98) | 2,041 (150) | 593 (75) |
| Example tr. | Incorrect | 1,295 (106) | 510 (54) | 764 (102) | 49 (55) |
| Example tr. | Correct | 2,077 (89) | 963 (37) | 1,099 (82) | 145 (30) |
| Experiment 5 | | | | | |
| Control | Incorrect | 2,922 (161) | 609 (37) | 2,248 (131) | 157 (27) |
| Control | Correct | 4,482 (146) | 1,574 (69) | 2,825 (158) | 332 (58) |
| Example tr. | Incorrect | 1,620 (147) | 712 (26) | 799 (109) | 35 (55) |
| Example tr. | Correct | 2,202 (96) | 1,018 (48) | 1,159 (97) | 159 (42) |
| Rule tr. | Incorrect | 2,829 (368) | 863 (118) | 1,831 (333) | 104 (104) |
| Rule tr. | Correct | 3,213 (166) | 1,414 (83) | 1,751 (154) | 281 (70) |

*Note*. The small number of incorrect responses in the Experiment 2 Math condition resulted in inconclusive values for the parameters shown.

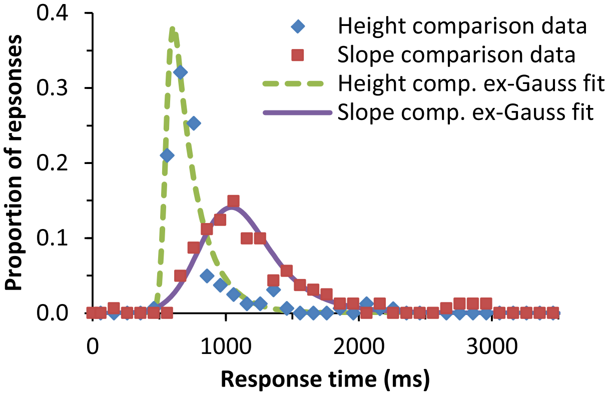


Fig. 2. Experiment 1 response time distributions for height and slope comparison tasks. Maximum likelihood fits to an ex-Gaussian function for each task is included.

assuming that this measures average values for response time parameters. Ratcliff (1979) describes a way to “Vincent average” response time curves for groups of students which has the advantage of tending to preserve the functional shape of the curve and having group parameter values which are the averages of the parameter values of individual participants. However, the number of trials per student (8) in this study makes this method marginal at best. Therefore, we simply pooled the data in order to have a large enough set to reliably perform maximum likelihood estimations. Naturally, simple pooling of response time data will have some confounding effect on the shape and the parameter values determined. Nonetheless, most of the differences in response time curves in the experiments here are large enough to maintain confidence in the ultimately qualitative conclusions made from this study. Cleary, improvements to this study would include increasing the number of within-student trials so that techniques such as Vincent averaging would decrease the uncertainty in parameter values.

## 3. Experiment 2: Response choices and response times to physics graph questions

Experiment 2 consists of a preliminary Experiment 2a and the main Experiment 2b, which were designed to achieve three goals. The first goal is to demonstrate misconception-like response patterns relating to the well-known slope-height confusion for a set of physics graph questions. That is, the goal is to demonstrate that a significant subpopulation of students *consistently* answer a given set of similar graph questions according to *height* when the correct response requires answering according to *slope*.

The second and most important goal of this experiment is to determine whether there are patterns of response times associated with patterns in response choices. Specifically, were response times corresponding to incorrect responses faster or slower than correct response times?

The third goal is to examine the extent to which student *familiarity* with the question influences the response choices and the response times. Experiment 1 demonstrated that when students were asked to compare either "slopes" or "heights" on a line on a simple graph, they had no difficulty. And yet it is well-documented that students have a slope-height confusion with physics graph questions; therefore, lack of familiarity may play a role in the confusion. For an unfamiliar question context, students may fail to recognize that the quantity of interest (such as speed) is associated with *slope* on the graph, and instead base the solution path on the more "available" (i.e. more rapidly processed) quantity *height*.

Therefore, we constructed three analogous question sets which were identical in structure yet were imbedded in different physics contexts, varying in familiarity. Experiment 2a establishes the relative familiarity of the three physics contexts, and Experiment 2b measures the response patterns and response times for the three contexts. Both experiments employed a between-subjects design with three conditions (contexts): math graphs, kinematic graphs, and electric potential graphs. Each condition presented a series of graphs and participants were asked to compare two points on a curved (or straight) line on the graph. Fig. 3 presents examples of the graphs in the three conditions, including the question posed for each graph.

The series of graphs presented in the three conditions were identical, except for the labels on the axes. The questions contexts for each condition were also conceptually analogous. The math graph condition asked for a comparison of the *slopes* at two points (magnitude of slope = $|dx/dt|$); the kinematic condition asked to compare *speed,* which is the slope for the position-time graph (speed = $|dx/dt|$); and the electric potential condition asked to compare the magnitude of *electric field*, which is the slope of the electric potential ($V$)-position ($x$) graph (magnitude of electric field = $|dV/dx|$).

### 3.1. Experiment 2a: Relative familiarity ratings

In this preliminary experiment, we established the relative familiarity of the three graph contexts. We expect that the order of familiarity from highest to lowest would be math,

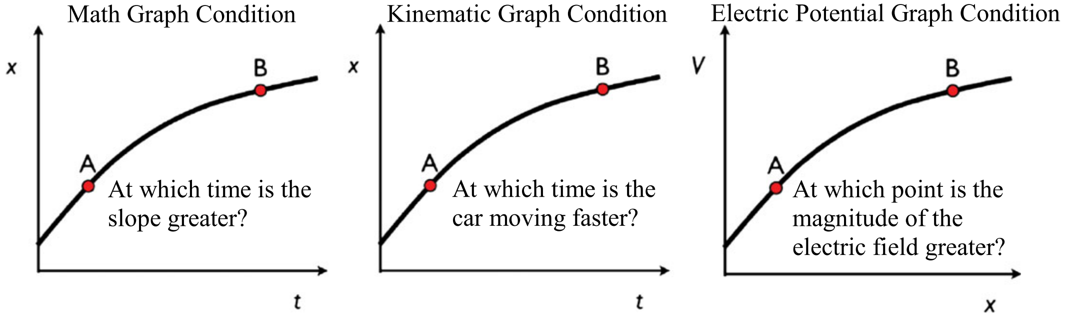

Math Graph Condition
x
t
A
B
At which time is the slope greater?
Kinematic Graph Condition
x
t
A
B
At which time is the car moving faster?
Electric Potential Graph Condition
V
x
A
B
At which point is the magnitude of the electric field greater?

Fig. 3. Examples of the graphs and questions used in the three conditions in Experiment 2. The answer choices for all three were "A," "B," or "the same at A and B." This is an example of a target (incongruent) question in which the higher point has the smaller slope.

kinematic, and electric potential graphs. Math graphs are likely to be most familiar because such generic graphs and the concept of slope are commonly introduced in standard curricula before and throughout high school and in college math. Next, kinematic graphs are typically introduced in high school physical science courses and used frequently in the prerequisite university-level physics course (mechanics). Finally, electric potential graphs would likely be the least familiar, since most participants likely saw them for the first time in the physics course in which they were enrolled at the time of the study.

Sixty-three students enrolled in the second course of introductory calculus-based physics (electromagnetism) were randomly assigned to one of three graph contexts, receiving partial credit for participation. In a pencil and paper task, students were asked to answer one of three graphs questions in Fig. 3. Next, they were asked, “How would you rank your familiarity with the question above?” and given a scale of 1 (Not at all familiar) to 5 (Very familiar).

The familiarity ratings are presented in Table 2, and the trends follow the expected pattern, with math graph rated as the most familiar and electric potential graphs rated as least familiar. The three familiarity ratings are significantly different, $F(2, 62) = 27.8$, $p < .001$, and a post hoc Bonferroni adjusted comparison reveals significant differences between the mean familiarity rating of the electric potential graphs and the math graphs ($p < .001$, $d = 2.3$) and between electric potential graphs and the kinematic graphs ($p < .001$, $d = 1.4$); however, there was no reliable difference between mean rating of math graphs and the kinematic graphs ($p = .40$, $d = .5$), though the means followed the expected trend.

In addition, in the electric potential graph condition a comparison of the mean familiarity ratings according to score on the question (about 70% answered correctly) revealed a significant difference in the familiarity ratings of students answering correctly ($M = 3.3$) versus students answering incorrectly ($M = 2.3$), $t(20) = .005$, $d = 1.4$. In the other two conditions, <10% of students answered incorrectly, preventing any meaningful comparison; however, these few that answered incorrectly also chose the lowest familiarity rating recorded in their condition. These results indicate that students answering incorrectly (consistent with answering according to *height* rather than *slope*) tend to be those who are least familiar with the question.

## 3.2. Experiment 2b: Response choices and response times

### 3.2.1. Participants

The participants were primarily engineering majors enrolled in one of two undergraduate calculus-based introductory physics courses that comprise a sequence covering

Table 2
Mean familiarity ratings of the three studied question contexts

| Condition | Mean (SE) Familiarity Rating | *n* |
|---|---|---|
| Math graph | 4.71 (0.12) | 21 |
| Kinematic graph | 4.35 (0.18) | 20 |
| Electric potential graph | 3.05 (0.19) | 22 |

classical mechanics and electromagnetism. Participants received partial course credit for participation which was >95% of all students enrolled in course (about 350 each course). Only some of these students, chosen randomly, participated in this particular study.

Participants were assigned to one of three conditions. For the math graphs condition, 28 participants were chosen from the mechanics course and 49 from the electromagnetism course, for a total of 77 participants. For the kinematics graphs condition, 94 participants were chosen from the mechanics course. For the electric potential graphs condition, 38 students were chosen from the electromagnetism course.

#### *3.2.2. Procedure, materials, and design*

All testing was presented to individual participants on a computer screen in a quiet room. They proceeded through testing at their own pace, and their response choices and response times were electronically recorded.

In each condition participants were presented with a series of graphs and asked to compare the slopes, speed, or electric field at two points on each graph, such as in Fig. 3. Participants were given no feedback as to the correctness of their answers. Once they answered a question, the next question was immediately displayed on the screen.

The question set contained 16 graphs with various line shapes on which two points were placed for the comparison task (see Appendix B). Each graph could be placed in one of four categories. First, there were 10 graphs of various shapes in which the higher point had a lower slope. These are the “incongruent” target questions, and are of the most interest because they represent the archetypical difficult question, where the relative slopes and heights conflict. Second, there were two graphs in which the higher point had a higher slope (“congruent” questions). Students might be expected to answer these correctly since relative slope and height are in accord. Third, there were two graphs in which both points had the same slope but different heights, and finally there were two graphs in which the two points had the same height but different slopes. These last two graph types could be considered special cases of “incongruent” questions. The response choices and response times for these last two question types were similar to the target questions, but for purposes of focusing on the target question type, were not included in the analysis below. All four kinds of graphs were included to provide some variety in the graphs and the responses, and to mimic more closely a natural test environment.

The 16 graphs were placed in a fixed random order for all participants in all conditions. Experiment 4 uses a design to counterbalance for order, with similar results to Experiment 2. Therefore, we are confident that the results here are not an artifact of question order. The graphs were constructed such that the correct response was sometimes “A” or “B” or “Equal,” and the correct answer was not always the lower or higher point or the point on the right or left.

#### *3.2.3. Results: Analysis of response choices*

We will focus on the performance on the “incongruent” target questions, namely those graphs in which the higher point has a lower slope. These type of questions are important

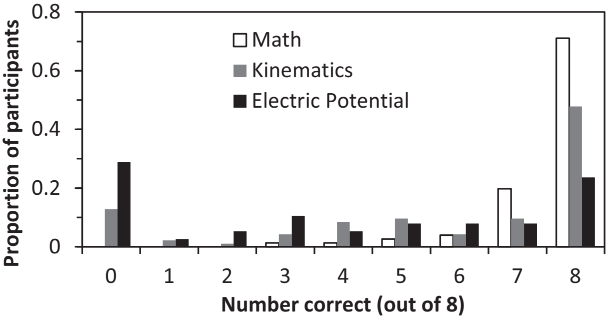


Fig. 4. Score distributions for Experiment 2b.

for investigating graph difficulties, since the correct answer choice is opposite of the common "misconception" that, for example, "the higher point has greater speed."

Fig. 4 presents the score distributions for each condition. There is significant difference between the score distributions for the three graph types, $\chi^2(16) = 54.6$, $p < .0001$. The answering patterns were largely bimodal rather than normally distributed, with most students either choosing the correct answer or the incorrect answer almost all of the time. For example, for the electric potential graphs, 31% of students answered at least 7 of 8 target questions correctly, 32% answered only zero or one question correctly, and about 30% of the students answered in a binomial distribution consistent with guessing one choice or another. Over 95% of the incorrect answers were the main misconception-like distracter, namely the point with the higher value; few responses indicated that the points had the same electric field. Therefore, these questions, especially unfamiliar electric potential questions, evoke consistent misconception-like responses in a significant fraction of students.

While the distributions are bimodal, it is still informative to report the mean scores, which depended strongly on the condition, as shown in Table 3. This confirms one of the results of Experiment 2a, namely that the less familiar the graph context, the higher the proportion of misconception-like responses.

Another kind of evidence to support the fact that students answering incorrectly consistently chose the misconception-like distracter comes from a comparison of scores on the congruent versus incongruent questions. Table 3 shows the mean scores for both question types. In all three conditions, participants scored higher on the "congruent" questions in

Table 3
Mean proportional correct for target (incongruent) questions and congruent questions

| Condition | Target (SE) | Congruent (SE) |
|---|---|---|
| Math graph | .94 (.01) | 1.00 (0) |
| Kinematic graph | .72 (.07) | .85 (.03) |
| Electric potential graph | .47 (.06) | .78 (.05) |

which the relative slopes and heights led to the same (correct answer), compared to the incongruent questions in which the relative slopes led to the correct answer and the heights led to the incorrect answer (paired *t*-tests, *p*s < .003). These results and the question types are similar in structure to the Stroop task and will be discussed in the General Discussion.

### *3.2.4. Results: Analysis of response times*

We present the response time results in two ways. First, to provide a picture of the scale and dynamics of the response times, we present trial-by-trial median response times for each condition in Fig. 5. As expected, the figure shows a typical and expected rapid decrease in response times for the first few questions, followed by a slow decrease (on average) for the remaining questions. Roughly speaking, the median response times range between 7 and 15 s for the few two responses, and range between 2 and 4 s for the last 13 questions. To characterize a central value of the distributions, we use the median time to reduce effects of outliers (Ratcliff, 1993). Fig. 5 demonstrates that the evolution of the median response times for the three conditions were roughly similar in shape and magnitude. However, there was a difference in averages of the median response times for the last 13 questions between the three conditions, $F(2) = 10.4$, $p < .001$. Specifically, a post hoc analysis reveals that the median response times are smaller for the electric potential graph condition ($M = 2{,}305$ ms) compared to the kinematic graph condition ($M = 3{,}311$ ms), Bonferroni adjusted $p < .001$, $d = 1.8$ and compared to the math graphs condition ($M = 3{,}091$ ms), Bonferroni adjusted $p = .006$, $d = 1.6$. Therefore, averaged over all responses, the least familiar question type was answered the most rapidly.

Since both the scores and the familiarity ratings of electric potential graph questions were significantly lower compared to the other two graph question types, this suggests that the lower average response times for this condition may be due to the possibility that incorrect responses had faster response times. Therefore, we analyzed the response time data a second way: By comparing the response times of correct versus incorrect answers for the last 8 of 10 target questions (when the response times are "settled" into roughly

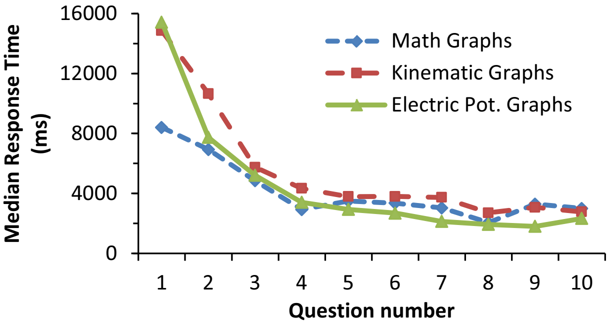


Fig. 5. Experiment 2b median response times for the first 10 questions. The response times leveled off quickly after 2 questions.

asymptotic means). Fig. 6 presents the distribution of these response times for each condition, separated out by response times for correct and incorrect responses. In addition, Table 1 presents the three fitted response curve parameters (μ, τ, σ) and the means for correct and incorrect responses for each graph type.

The most important result from the data presented in Fig. 6 and Table 1 is that response times for incorrect answers are faster than response times for correct answers for the kinematic and electric potential graphs conditions (Mann–Whitney *U* test used because of long tails in distribution, *p*s < .0001). This difference is highlighted via the peaks of the distributions, as measured by μ in Table 1. The peaks of the distributions for correct answers are around 1,100–1,700 ms. For incorrect answers, the peaks are around 600–900 ms; hence, the incorrect answers are about 500–700 ms earlier than for the correct answers. There are so few incorrect responses for the math graphs that no reliable comparisons can be made for that case. Interestingly, unlike the score results, there does not appear to be any large or systematic trends in correct and incorrect response times that correlate the familiarity ratings.

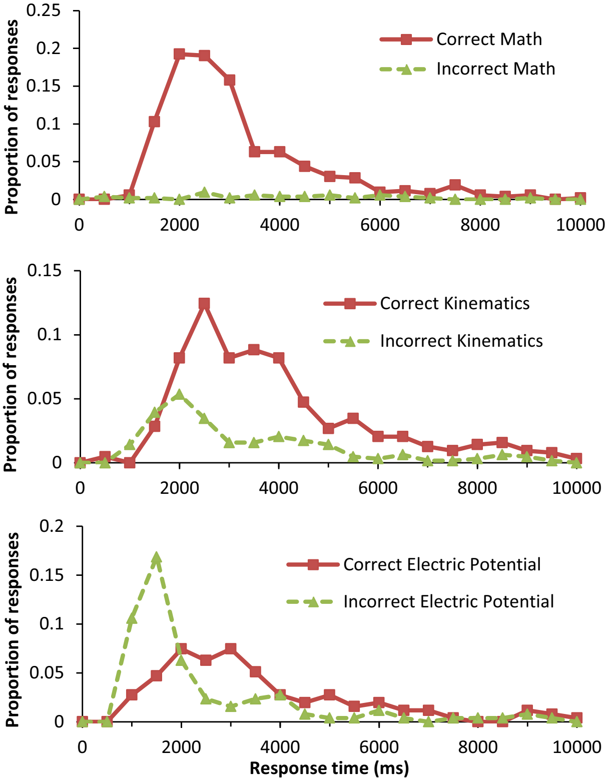


Fig. 6. Experiment 2b response time distributions for correct and incorrect answers to target questions for all three question contexts. The total number of correct or incorrect answers for each condition is proportional to the area under each curve.

Note the caveat mentioned earlier that the response times for both between student and within student data were pooled together. Since the majority of students answered virtually all questions either correctly or incorrectly, one could interpret the difference between the correct and incorrect response times in two (nonmutually exclusive) ways. First, the difference may be due to some fundamental difference between the two populations in processing speeds for the two solution paths. Second, the populations may have similar solution path processing speeds, but they are simply utilizing different solution paths. While we will not rule out the first possibility, we provide evidence supporting the second and assume this interpretation is valid. For example, Experiment 1 demonstrated that all students can compare slopes and heights, and that, within-student, processing height is faster than slope (for this task). In addition, Fig. 2 does not show any evidence of two populations, such as bimodal peaks. Other evidence will come from experiments 3–5 which will demonstrate that changing the question context or kind of training changes the response choices and response times in ways that are consistent with the assumption that there is not a fundamental difference in slope and height processing speeds between student populations.

## 4. Experiment 3: Imposing a delay in responding

The results of Experiments 1 and 2 demonstrate that response times of misconception-like responses were shorter than those of correct responses, and the underlying task necessary for determining the misconception-like response (comparing heights) took less time than the task necessary for determining the correct response (comparing slopes). Furthermore, as familiarity with graph context decreased, the misconception-like response patterns increased. A simple explanation for these results is that students who are unfamiliar with a particular graph context use height as the default dimension to base a response, and because height happens to be processed faster, the response times are relatively short.

However, we propose a more causal rather than coincidental relationship between the response choice and the relative processing time of the available dimensions. That is, let us consider the relative processing time model discussed in the introduction as the *cause* of the incorrect response choices. The outline of this model is shown in Table 4. In this model, we assume an implicit goal to answer quickly. For the task in this study, we propose that students unfamiliar with the question context will implicitly consider both height and slope as sufficiently plausible dimensions for determining the answer. The model assumes separate information accumulators for each dimension. Experiment 1 suggests that the height accumulator reaches its critical boundary faster than the slope accumulator (for the graph questions studied). Since the height is processed faster, the responder (who is biased to answer rapidly) will tend to base the response on height, which is incorrect. As for the slope dimension, this information requires more time to process, and the response may often be already completed before the slope accumulator reaches its critical level and is thus excluded from the response. If for some reason both processes are completed before the response is set, then some other decision process,

Table 4
A simple relative processing time model that can result in misconception-like response patterns to a science concept question

|  |
|---|
| 1. Two dimensions are considered relevant by the responder (only one is relevant). |
| 2. Both dimensions accumulate information in parallel; time to process may be different. |
| 3. As time progresses, if only one dimension is processed, utilize it. |
| 4. If both dimensions are processed completely, employ a different method for choosing which dimension to utilize. |

*Note*. This model assumes that the question effectively presents one relevant and one irrelevant dimension, such as slope and height for the graph question in this study. Misconception-like answering patterns occur when the irrelevant dimension is processed faster than the relevant dimension, and utilization of the irrelevant dimension results in an incorrect response. This model could be considered as a kind of fluency heuristic, and it is consistent with information accumulation models of response times.

perhaps one that involves more explicit reasoning, must be used to choose between the responses determined by the two dimensions.

The purpose of Experiment 3 is to further test this model by imposing a time delay before responding. In particular, Experiment 3 has a Delay condition in which participants are first presented with the question and are permitted to answer only after a short delay. Imposing a delay can provide enough time to allow for the processing of both the faster, incorrect solution (comparing heights) and the slower, correct solution (comparing slopes). Then, information from both the correct and incorrect dimensions should be available for determining a response, rather than information from only the incorrect dimension. This manipulation could result in answering correctly, more frequently compared to a control (no delay) condition. In a sense, the imposed delay will effectively neutralize the constraint of answering quickly and allow the more slowly processed dimension to compete more effectively in the decision making process.

The delay was set to 3 s since after the first few trials, the majority of participants who answered correctly in Experiment 2 did so within 3 s. Note that during the first two trials, average times were of order 10 s. Since the response times for the first two trials is several times larger than the 3 s delay, we expect that the delay would have very little effect on scores compared to the control condition for these first two trials.

Finally note that for Experiment 3 only electric potential graphs were used, since these graphs were rated as the least familiar and exhibited misconception-like responses most frequently. Therefore, any effects from a delay should be most observable for these graphs.

### *4.1. Method*

#### *4.1.1. Participants*

A total of 57 undergraduates similar to those in Experiment 2 enrolled in the second-semester course (electromagnetism). Participants were randomly assigned to one of two conditions: 28 in the Delay condition and 29 in the Control condition. There was found no significant difference between the average course grades of participants in the Delay (*M* = 2.13) versus Control (*M* = 2.21) conditions, *t*(53) = .45, *p* = .66, *d* = .08.

### *4.1.2. Procedure, materials, and design*

The procedure was similar to Experiment 2b. Participants in both conditions were presented with the same graphs as in the electric potential graph condition in Experiment 2b. However, in the Delay condition participants were presented a screen with the following message: "On each slide, you will see a question with a message at the bottom of the screen." At first the message will read: "Take a moment to determine your answer." While this message is displayed, you will not be able to answer the question. After a couple of seconds, the message will change and prompt you for an answer. Please press the key that corresponds to your answer at that time." Participants were then given a simple math-fractions problem as an example of the delay, and they proceeded to the graph questions. Participants in the Control condition also received the initial practice simple math fractions problem, but with no delay. In sum, the Control and Delay conditions were presented with the same questions as in Experiment 2b, but the students in the delay condition were required to wait 3 s before responding.

## *4.2. Results and discussion*

Similar to Experiment 2, we report here the results of the target incongruent questions. Since there is a significant change in response times for the first couple of questions (e.g. see Fig. 5), we analyzed the scores for the first two questions, which had an average response time of approximately 16 s for both the control and delay condition (including the 3 s delay), and the remaining 8 target questions which had an average response time of approximately 4 s for both conditions, including the 3-s delay.

For the first two questions, there was a small but unreliable difference between the scores in the Delay condition (*M* = 48%) and the Control (*M* = 40%), Mann–Whitney *U*(55) = 366, *Z* = .76, *p* = .46, *d* = .19, as expected.

However, for the remaining questions participants in the Delay condition (*M* = 70%) scored significantly higher than those in the Control condition (*M* = 47%), Mann–Whitney test *U*(55) = 247, *Z* = 2.24, *p* = .025, *d* = .64. The score distribution for the two conditions is shown in Fig. 7. Therefore, the delay had little if any effect for the first two questions when the response times were much larger than the delay time, but the delay did affect student responses once the response times were comparable to the delay time.

The difference in score between the Delay and Control conditions implies that a significant fraction of students were able to answer correctly if they were constrained to wait before responding, but without this constraint, these students would choose the common, fast misconception-like response. The improved score of the Delay condition provides support for the hypothesis that misconception-like answers are influenced by a bias to answer quickly, utilizing the dimension that is processed the fastest, to the exclusion of more slowly processed dimensions.

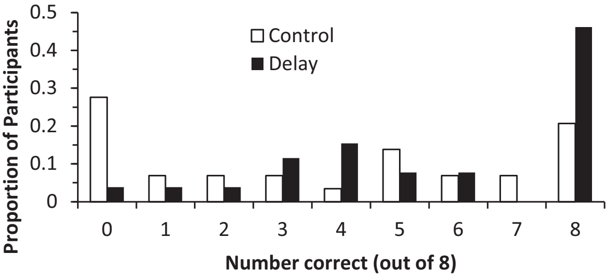


Fig. 7. Experiment 3 score distribution for the last eight target questions, when average response times were of the same order as the 3s delay.

One might also interpret the difference in answering between the conditions as due some other cause. For example, in the Delay condition, the delay and the cue “take a moment to determine your answer” might motivate the students to exert more effort in finding the correct solution. However, if this explanation were valid, one would then expect a significant increase in performance in the Delay condition for the first two questions as well, but this was not the case.

## 5. Experiment 4: Including repetitive training

In Experiment 3, the score improved when a short delay in responding was imposed. This was interpreted as providing time for both fast-incorrect and slow-correct dimensions to be processed, allowing the two dimensions to compete for the response. In Experiment 4, we improve scores a different way: by providing *training* prior to the test. The question then becomes, if training improves scores, does it also change the relative response times between correct and incorrect answering? According to the relative processing time model, one way to improve scores would be to decrease the processing time of the correct response relative to the processing time of the incorrect response. According to the model, one might reasonably expect that training has the effect of speeding up the processing of the dimension associated with the correct response so that it may better compete with the dimension associated with the incorrect response. Experiment 4 investigates this possibility.

Therefore Experiment 4 has two main goals. First, it is to replicate the results of Experiment 3, namely that an imposed delay improves scores. Second, it is to determine whether training that improves scores also decreases the relative response times (from which we infer processing times) of correct versus incorrect answers compared to no training.

### 5.1. Method

#### 5.1.1. Participants

A total of 138 undergraduates similar to the population in Experiment 3. Participants were randomly assigned to one of three conditions: 35 in the Example training condition, 37 in the Delay condition, and 35 in the Control condition. There was no significant difference between the average course grades of participants in the Example training ($M$ = 2.66), Delay ($M$ = 2.82), and Control ($M$ = 2.64) conditions, $F$(2, 135) = .37, $p$ = .69.

### 5.1.2. Procedure, materials, and design

The procedure was similar to Experiment 3, with two exceptions. First, we added a condition that included training prior to the test. Second, in order to address a concern in the design of Experiments 2 and 3, we counterbalanced the test for question order by having two (random) test orders. The two test orders were the same for each condition.

A critical goal of the Example training condition was to improve the scores on the target questions by providing multiple practice trials with feedback on simple examples similar to the target questions. This training condition was based on some of our related pilot studies in which this training sequence was found to be effective in improving scores on the target questions. In the Example training condition participants were presented a series of 32 graph questions with immediate feedback (providing the correct answer) after each response (see Appendix C). The training questions were similar to the target (incongruent) test questions in that the text of the question is the same as the test question (At which point is the electric field greater?), but the graphs were slightly different. The training questions consisted of graphs with two points (on a curved line) at the same height but different slopes and graphs with two points on a curve line, with one point higher and having a higher slope. The average score for the entire training condition was 86%, indicating they learned the task, and the average time to complete training was approximately 2 min. Between training and the test was an unrelated task lasting between 5 and 10 min.

## 5.2. Results and discussion

### 5.2.1. Analysis of scores

As done in Experiment 3, we will separately consider the scores for the first two questions and for the last eight. Similar to Experiment 3, as expected, the average score for the Delay condition ($M$ = 60%) was not reliably higher than for the Control ($M$ = 51%), Mann–Whitney test $U$(103) = 1,351, $Z$ = .99, Bonferroni adjusted $p$ = .68, $d$ = .21. However, there was a larger difference between the Example training ($M$ = 71%) and Control ($M$ = 51%) scores for the first two questions, Mann–Whitney test $U$(101) = 862, $Z$ = 2.25, Bonferroni adjusted $p$ = .05, $d$ = .5. This is also expected, since the training should improve scores on all of the test questions.

The score distributions for the remaining target questions reveal that the scores of the three conditions were different, Kruskal–Wallis $K$(2) = 10.2, $p$ = .006. In a replication of Experiment 3, the Delay condition ($M$ = 70%) scored higher than those in the Control condition ($M$ = 51%), Mann–Whitney test $U$(103) = 1,527, $Z$ = 2.18, Bonferroni adjusted $p$ = .06, $d$ = .48. Furthermore, the Train condition ($M$ = 75%) scored higher than the Control ($M$ = 51%), Mann–Whitney test $U$(101) = 771, $Z$ = 2.87, Bonferroni adjusted

$p = .008$, $d = .6$. In sum, the increase in scores due to an imposed delay in responding was replicated, and there was also an increase in scores due to repetitive example practice, as expected.

### 5.2.2. Analysis of response times

The response time distributions and the ex-Gaussian fitted parameters for the two conditions are presented in Fig. 8 and Table 1. The Fisher matrix estimates of the standard errors as well as visual inspection of the graphical distributions allow us to make reasonably reliable inferences about trends in the data. For example, the data reveal that training had a clear overall effect of reducing the mean response times by over 1 s compared to the Control condition. An examination of the fitted parameters reveals that this reduction was predominantly in the reduction of the "decay" times $\tau$ for both correct and incorrect responses, and there was also a small reduction in the "width" $\sigma$.

However, the result of most interest to this study arises from the peaks of the distributions, as parameterized by $\mu$. The data indicate that training did not significantly change the peak response time for the incorrect responses, but training did significantly decrease the peak response time for correct responses. The difference between the peak times for incorrect responses for the Example training ($\mu$ = 510 ms) and Control ($\mu$ = 453 ms) conditions is 57 ms, which is within the pooled standard error of the peak times for incorrect responses (59 ms). However, the peak-time difference for correct responses for

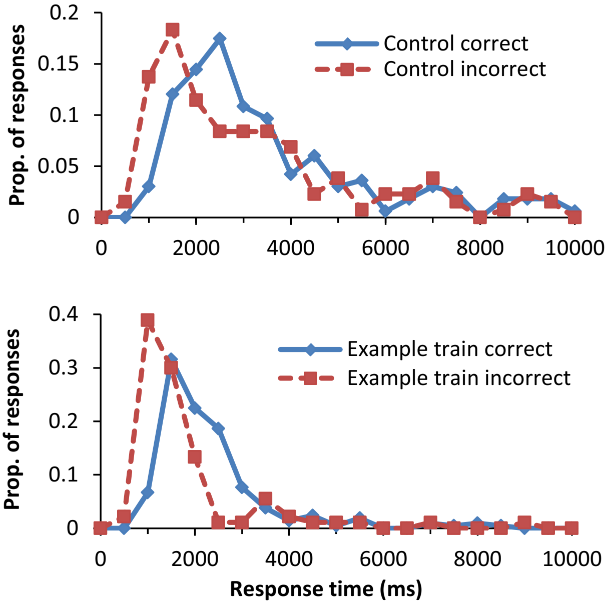


Fig. 8. Experiment 4 response time for the Control and Examples training conditions. Training reduced the difference in incorrect versus correct response times compared to control.

the Example training (μ = 963 ms) and Control (μ = 1,139 ms) is 176 ms, which is more than twice as large as the pooled standard error for correct responses (74 ms).

In other words, training decreased the *difference* in the peak response times between correct and incorrect responses (453 ms) compared to the difference for control (686 ms). This reduction in the difference between the peak-times for the correct versus incorrect responses is also evident from the response time distributions in Fig. 8.

In sum, the data provide support for the relative processing time model in that training not only improved scores, but it also reduced the relative peak response time between the correct and incorrect responses. In particular, the incorrect peak times did not change, but the correct peak times were reduced by training. These results strengthen the idea that this particular kind of training resulted in more correct responses by reducing the processing time of the dimension associated with the correct response, thereby allowing it to better compete with the dimension associated with the incorrect answer choice.

## 6. Experiment 5: Repetitive training versus rule training

The relative processing time model (Table 4) proposes that incorrect responders consider both the relevant and irrelevant dimensions and utilize the irrelevant dimension because it is processed faster. How does the model account for correct responders? Experiment 4 demonstrated that one way to increase the score is to decrease the processing time of the relevant dimension via repetitive training such that the relevant dimension competes better with the irrelevant dimension. In Experiment 5 we will investigate another way implied by the relative processing model to increase the score, namely to remove the irrelevant dimension from consideration for the response.

The irrelevant dimension will be suppressed from consideration by providing an explicit general rule stating that only the relevant dimension must be used to respond correctly. The result should be an increase in score. However, one might not expect any change in response time compared to control, since it is not clear that an explicit rule can affect the processing time of the automatically processed relevant and irrelevant dimensions. Both may still be processed, but only one is now considered for the response.

In addition, as noted in the discussion in Experiment 3, the improved scores in the delay condition suggest that some student understand the task and may “know” the correct answer, but somehow do not choose it, perhaps because they have a priority to answer quickly. We would like to test this possibility in another way by providing the students with an explicit task to determine whether they can identify the correct (written) rule for finding the answer. Experiments 3 and 4 suggest that there may be students who can identify the correct written rule but do not use this rule in the graph task, perhaps because they choose the faster-processed dimension instead of the correct, slower processed dimension.

Therefore, Experiment 5 has three main goals. First, it is to replicate the results of repetitive examples training in Experiment 4 to improve our confidence in the results. Second, Experiment 5 will compare response times for repetitive example training with explicit rule-based training to determine if the results agree with our relative processing time model. Both kinds of training are expected to improve the average score. However, as seen in Experiment 4 repetitive example training is expected to decrease the response time of correct responses compared to control, yet rule training may not. Third, it is to determine whether students who are answering incorrectly can nonetheless recognize the correct written rule, thus supporting our hypothesis that something other than explicit knowledge of the rule is playing a role in their response.

## *6.1. Method*

### *6.1.1. Participants*

A total of 71 undergraduates similar to the population Experiment 3. Participants were randomly assigned to one of three conditions: 25 in the repetitive Example training condition, 22 in the Rule training condition, and 24 in the Control condition.

### *6.1.2. Procedure, materials, and design*

The procedure was similar to Experiment 4, with the addition of a condition that included Rule training prior to the test. The Rule training condition consisted of a simple rule stating that the electric field can be determined from the slope of the electric potential versus position graph followed two multiple-choice comprehension questions with corrective feedback (see Appendix D for the actual text used). This sequence was then repeated once to better ensure learning of the rule. All students in this condition indicated "yes" when prompted whether they understood the rule, with average score was 91% on the two multiple-choice questions, indicating that the student successfully learned the rule. Testing on the target questions followed directly after training on the rule.

We also added a multiple-choice question at the end of the graphs tasks which asked students to choose which explicit rule would determine the correct response. This question was used to determine whether students had some level of explicit understanding of the graph tasks and could identify the correct rule, namely that a comparison of *slopes* determines the correct response.

## *6.2. Results and discussion*

### *6.2.1. Analysis of scores*

The score distributions on the (last eight) target questions for all three conditions are shown in Fig. 9. As expected, both training conditions improved the average score. The Example training condition ($M$ = 78%) scored higher than those in the Control condition ($M$ = 53%), Mann–Whitney test $U(71) = 178$, $Z = 2.6$, Bonferroni adjusted $p = .018$, $d = .58$. Furthermore, the Rule training condition ($M$ = 83%) scored higher than the Control ($M$ = 53%), Mann–Whitney test $U(71) = 144.5$, $Z = 2.8$, Bonferroni adjusted $p = .012$, $d = .75$.

### 6.2.2. Analysis of rule recognition question

Since we are interested in the extent to which students (whether or not they answer the graphs task correctly) could explicitly identify the correct rule, namely that comparing only slopes determined the correct answer (see Fig. 10), we accepted and answers *c* or *g* in Fig. 10 (virtually no students answered *e*). Because of an error in data collection, data for this question was only recorded for the control condition (24 students). Nine students answered at least 7 of 8 graph task questions incorrectly, and of these nine students, six of them (i.e. two-thirds) answered the rule recognition *correctly*. This suggests that a majority of students who are answering the graph questions incorrectly nonetheless recognize the correct rule; they are simply not applying it. This supports our hypothesis that at least some students prefer to answer quickly (and use the rapidly processed dimension) rather than answering correctly and use the more slowly processed dimension. For the 15 students answering the majority of graph questions correctly, 13 (the overwhelming majority) of them recognized the explicit rule.

### 6.2.3. Analysis of response times

The response time distributions and the ex-Gaussian fitted parameters for the three conditions are presented in Fig. 11 and Table 1. There are several important results from

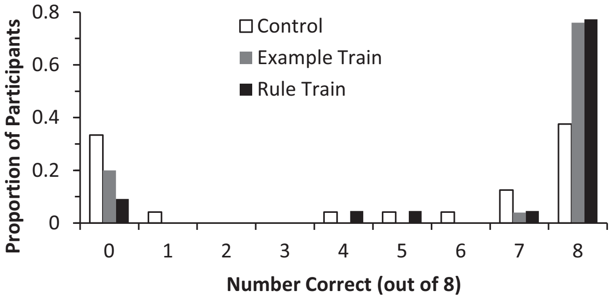
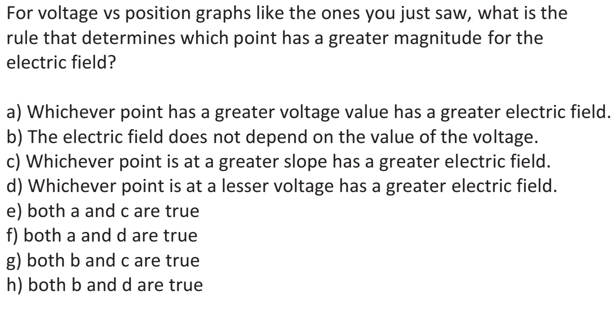


Fig. 9. Experiment 5 score distributions.



For voltage vs position graphs like the ones you just saw, what is the rule that determines which point has a greater magnitude for the electric field?

a) Whichever point has a greater voltage value has a greater electric field.
b) The electric field does not depend on the value of the voltage.
c) Whichever point is at a greater slope has a greater electric field.
d) Whichever point is at a lesser voltage has a greater electric field.
e) both a and c are true
f) both a and d are true
g) both b and c are true
h) both b and d are true

Fig. 10. Experiment 5 rule recognition question. The question was administered after all other tasks. Answers *c* and *g* were counted as correct.

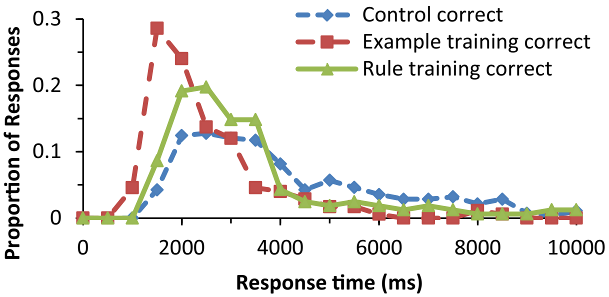


Fig. 11. Experiment 5 response time distributions. Note that example training significantly reduces response times while rule training only marginally reduces response times compared to control.

this data. First, a comparison of the peak times for the Control and Example training conditions reveals results similar to Experiment 4. While there is a small difference (103 ms) between the peak times for incorrect responses for the two conditions, there is a larger and significant decrease of 556 ms in the peak time for the correct responses. This difference is much larger than the pooled standard error of the peak times (59 ms) for correct responses of these two conditions. In other words, similar to Experiment 4, there was reduction in the difference between correct and incorrect peak times for Example training (306 ms) compared to the difference for the Control (965 ms).

The second and more important result is that Rule training affected the peak response times less than the Example training. There are two ways to see this. First, Rule training did not reduce the peak time for correct answers nearly as much as Examples training did. For correct answers the peak for Rule training was only 160 ms less than Control, which is only about twice the pooled standard error (76 ms) and is thus only marginally significant. As stated above, Example training reduced the peak time by 556 ms, which is much larger, about ten times the pooled standard error (59 ms). Second, Rule training did not reduce the difference between correct and incorrect peak times as much as Example training did. For Rule training, the difference between correct and incorrect peak times is 551 ms, which is larger than the 315 ms difference for Example training (though it is still smaller than the 965 ms difference for Control).

In sum, while Rule and Example training both increased scores on target questions to the same extent, they did not affect the response times similarly. In a replication of Experiment 4, we found that Example training significantly reduced the peak response time for correct answers and the difference between peak correct and incorrect times compared to Control. In contrast, Rule training at most only marginally reduced the peak times for correct responses and consequently did not reduce the difference between peak correct and incorrect response time as much as Example training. These results are also supported by inspecting the response time distributions in Fig. 11.

The results of the comparison between Examples and Rule training support our relative processing time model, namely that Rule-based training increases the score by affecting the dimensions considered for the response, but does not appear to significantly affect the relative processing time, and the Example-based training does significantly affect the relative processing time, and this in turn affects the response choice.

## 7. General discussion and conclusions

There were two main objectives of this study. The first was empirical: to determine, in various testing contexts, whether there may be interesting patterns of response times corresponding to response choices for science questions known to exhibit patterns of incorrect responses (i.e., misconception-like responses). The second was more theoretical: to analyze the empirical results in light of existing response time explanations and mechanisms and to test a simple model for misconception-like response choices and response times that emerged in the course of the experimental results. The results are discussed below.

### *7.1. Empirical results*

All experiments in the study were administered after the relevant physics instruction to students enrolled in an introductory physics class. The main empirical results of this study are as follows:

1. When asked to compare "heights" or "slopes" on a simple graph, all students were successful, and response times for comparing heights were faster than times for comparing slopes (within student). (Experiment 1)
2. For a set of physics graph questions which asked for a comparison of a quantity represented by the slope, most students *consistently* either compared heights on the graph (incorrect) or they compared slopes on the graph (correct). (Experiments 2–5)
3. The lower the familiarity of the question context, the more likely a student would consistently compare heights (incorrect) rather than slopes. (Experiments 2a, 2b)
4. The response times for the incorrect responses were faster than the correct responses. (Experiments 2–5)
5. Imposing a delay on the response increased the accuracy. (Experiments 3 and 4).
6. Example training increased accuracy and significantly decreased the difference between correct and incorrect answer response times. (Experiments 4 and 5)
7. Explicit rule training increased accuracy but only marginally decreased time of correct response, still leaving a large difference between correct and incorrect response times. (Experiment 5)
8. The majority of students answering the graphs task incorrectly could nonetheless identify the correct explicit rule for determining the correct answer. (Experiment 5)

In sum, we have found interesting patterns in response times that correspond to response choices to the simple but standard science concept question used in this study. Note that the response times in this study are short, on the order of 1–3 s. These empirical results indicate that the relative processing time of relevant available dimensions may play an important role in response patterns to science concept questions, and some students appear to know the correct answer (or the correct rule) but answer incorrectly because answering quickly is a high priority. Furthermore, these results highlight an interesting difference between the effect of rule versus example training. While both kinds of training increase the score, example training reduces the difference in processing time of incorrect and correct responses more than rule training.

### *7.2. Explanations of empirical results*

We have constructed a simple relative processing time model to help explain the misconception-like patterns of incorrect answers and the corresponding response times to a simple science concept question. The most important components of the model (see Table 4) are (a) many students at least implicitly consider both relevant and irrelevant dimensions available in the question, (b) as time progresses, if one and only one of these dimensions is processed, then students will utilize this dimension, and (c) if more than one dimension is processed, then some other mechanism must be used to choose between them. This relative processing time model may be a result of an implicit priority to respond quickly. As demonstrated in Experiments 3 and 4, some students are able to respond accurately, but instead answer rapidly, even when there is no explicit constraint or reason to do so. This is further supported in Experiment 5, in which it was demonstrated that the majority of students answering incorrectly nonetheless recognized the correct explicit rule, but they simply were not applying it in this context in which a rapidly processed but incorrect dimension is competing with a slower processed correct dimension.

#### *7.2.1. The relative processing time model as a heuristic*

This relative processing time model can be regarded as a kind of fluency heuristic (Hertwig et al., 2008; Schooler & Hertwig, 2005) and is a more specific version of the general idea of an availability heuristic (Kahneman, 2003; Tversky & Kahneman, 1973). The core idea of the fluency heuristic is to place high value on speed, thus favoring the solution that is processed the fastest. However, to fully explain misconception-like answering, it is also necessary to assume that students will consider incorrect solution paths and that these paths are faster than the correct paths. Furthermore, the fastest processed solution is not always the one to be chosen, as evidenced by the improved scores when a delay in responding was imposed. This result can be explained in terms of competition. If at the time a solution is processed there are no other solutions processed, then there is no competition, and this first-processed solution is utilized. However, if more than one solution is processed before an answer is chosen, then these solutions must compete, and the outcome of this competition must be determined via some other mechanism (e.g., another heuristic). In a sense, the imposed delay might be interpreted as a kind of forced *disfluency* (e.g., Alter, 2013), inducing the participant to use some other perhaps more explicit process.

Besides imposing a delay in responding, this model implies two other ways to improve scores, which we investigated in Experiments 4 and 5. The first is to speed up the processing of the correct solution path via repeated practice. The second was employ explicit rule training to remove the incorrect solution path from the student's consideration, thus denying the initial assumption of the model that both incorrect and correct solutions paths are considered. This might therefore be seen as a demonstration of the dual processes or two paths to the same solution (fast and automatic versus slow and controlled).

#### *7.2.2. The relative processing time model as an information accumulation model*

The relative processing time model is also consistent with the general structure and assumptions of more fine-grained information accumulation models often used to explain and describe response time data, as discussed in the introduction. From this perspective, information about the relevant and irrelevant dimension accumulates simultaneously, and the accumulator that reaches its critical boundary first wins. Such models predict within-student ex-Gaussian-like response time shapes, and they can describe the fairly complicated phenomenon of response times for simple tasks with only a few parameters. Clearly these response time models are at a fine grain level and would need to be adapted to the more complex (e.g., multi-step) science question task studied here. This includes the need to explain how decisions are made when both accumulators have reached the boundary.

### *7.3. Do students "Just not understand the task or the topic"?*

The term *understand* is often poorly specified in education and the learning sciences. For the science topics in this study, is *understanding* demonstrated by correctly answering the graphs tasks in this study? Is it identifying the correct rule? Is it providing a detailed verbal explanation? It is being able to answer quickly? All of these meaning are useful in different circumstances. In educational practice, understanding is determined via instructors, who make inferences about "understanding" (e.g., the achievement of an instructional goal) based on student performance on an assigned task. Therefore it is important to carefully characterize how students respond to typical test questions, such as the ones in this study.

One might attempt to "explain" incorrect responses by saying that the incorrect responders do not *understand* the task or the topic. However, this ambiguous statement does not explain why there are *patterns* to their incorrect answers, or why incorrect responders can identify the correct rule for the task, but do not apply it. In contrast, for the relative processing time model explored here, the less familiar a student is with the graph task, the more the relative processing time matters, and this is what produces the pattern of incorrect answering. This model is consistent with other models in cognitive psychology that explain incorrect answering pattern phenomena such as the Stroop effect. Note that in such experiments, one does not speak of participants "not understanding" Stroop-like tasks, rather the patterns are influenced by automatic processing time issues.

Therefore, the implication of the influence of bottom-up processes on answering patterns is to call into question what is practically meant by *understanding*. If the performance on science question tasks is inevitably influenced by unconscious, automatic, bottom-up processes, then our characterization of understanding a science concept must include both explicit reasoning and automatic, bottom-up processes. One might say that both "System 1" and "System 2" are a necessary part of what we operationally mean by understanding a science concept, as they both may influence performance on any task relevant to the science concept. Indeed, a significant portion of expert science knowledge may be implicit (cf. Evans, Clibbins, Cattani, Harris, & Dennis, 2003).

Furthermore, if bottom-up processes do play an important role in what is meant by understanding of a science concept, then this suggests that one should utilize methods of instruction that align these process with goals of explicit reasoning. For example, students may be better able to *understand* the meaning of tangent slopes on a graph if they can process them as quickly as positions on a graph. This idea of aligning instruction with automatic processes has been explored by a number of researchers (e.g., Brace, Morton, & Munakata, 2006; Goldstone, Landy, & Son, 2010; Kellman, Massey, & Son, 2010).

### *7.4. Comparison to the Stroop task*

The Stroop effect has held a long-established importance in building models of responses to tasks. In the prototypical Stroop task, participants are asked to name the color of the letters comprising a color word name. Incongruent trials consist of colored ink that does not match the color word, such as the word "blue" in red ink, and congruent trials consist of ink that does match the word. Response times are typically higher and accuracy is typically lower (though often close to perfect) in incongruent versus congruent trials. Furthermore, response times for color naming on incongruent trials are typically longer than for "control" trials in which the word is not the name of a color (for reviews, see MacLeod, 1991; MacLeod & MacDonald, 2000).

This increased response time and reduced accuracy are typically interpreted as *interference* of the word-reading process on the color naming process (note that color does not interfere with word reading). There have been two prevalent and possibly related explanations for the Stroop effect interference. The first explanation is based on relative processing time. Since it was well established that word reading is faster than color naming, it was hypothesized that word reading "interferes" with color naming in the incongruent trials (e.g., Dunbar & MacLeod, 1984; Dyer, 1973). The idea is that, during decision, when the two dimensions provide conflicting responses, time is needed to resolve the conflict. However, the relative processing time explanation is unable to account for all of the Stroop task data. For example, manipulation of the processing time of the fast or slow dimensions via training or change in format does not always lead to an expected reversal or disappearance of the Stroop effect (Dunbar & MacLeod, 1984; Macleod & Dunbar, 1988).

While the speed of processing model explains many Stroop effect observations, the data as a whole appears to be better explained by the related concept of relative automaticity and relative strength of processing pathways (Cohen, Dunbar, & McClelland, 1990; Macleod & Dunbar, 1988; MacLeod & MacDonald, 2000). In this account, processes are considered to be on a continuum from automatic (and usually faster) to controlled and slower. The more automatic processes tend to interfere with the more controlled processes, but the controlled processes do not tend to influence the automatic processes. Cohen et al. (1990) have built a parallel distributed processing connectionist model to account for much of the Stroop effect data. In this model, the relative strength of processing pathways determines the outcome of competing responses. The relative strength also determines the relative automaticity and roughly the response time of the process.

### *7.4.1. Is the graph task a Stroop task?*

There are a number of points to consider for this question.

1. The structure of the graph task in this study is similar to the Stroop task. There are incongruent questions (i.e., the target questions of this study) in which two dimensions (slope and height) provide conflicting answers, and there are congruent questions in which the slope and height provide the same answer. Furthermore, the irrelevant dimension (height) is processed faster than the relevant dimension.
2. While the structure may be similar to a Stroop task, it is not clear whether all of the response data of this study are similar. On one hand, the scores for incongruent trials are lower than for congruent trials, like the Stroop task. However, post hoc, we analyzed the response times for Experiments 1 and 2b and found no significant difference between congruent and incongruent trials. It should be noted that these experiments were not designed to measure response time differences between congruent and incongruent trials. For example, there were only two congruent trials in the test in Experiment 2b, compared to 10 incongruent trials. Thus, at most we can conclude that the response time differences, if any, were not large. Note also that Stroop effects can be strongly influenced by the proportion of congruent to incongruent trials.
3. The concept that response times are longer for incongruent trials *because* of interference (i.e., competition) is not an explicit part of the relative processing time model in this paper, though some accumulation models such as by Usher and McClelland (2001) do include competition between accumulators. Stroop effect models would explain the observed longer processing time of slope compared to height on incongruent trials in this study as due to interference from the conflicting height and slope dimensions. If the interference explanation is correct, a modification of our model, which assumes that processing slope (in this context) takes inherently longer to process than height, would be required. The interference explanation could be tested by determining whether correct responses for congruent trials are reliably faster than incongruent trials. As mentioned in the previous point, we have found no such evidence in our study, but further experiments are needed.
4. The increase in accuracy on incongruent questions with familiarity in Experiment 2b and with repetitive training in Experiments 4 and 5 is consistent with the relative processing time and the relative automaticity model of the Stoop effect.

5. One way to directly challenge the relative processing time model would be to find an instance in which the incorrect dimension was processed slower than the correct dimension yet there were still significant numbers of incorrect responses. Such cases were found for the Stroop effect (e.g., Dunbar & MacLeod, 1984), and these counterexamples were one reason for the evolution of the Stroop explanation to the idea of relative automaticity and relative strength of processing pathways rather than a simple relative processing time model. If the graph task is a kind of Stroop task, then a more general model should include relative automaticity rather than only relative processing time.
6. If the graph task and other similar science concept questions are to be considered as a Stroop task, then it is one in which the accuracy is zero for a significant number of students and 100% for another significant fraction. Typically in Stroop tasks, the accuracy is very high, often close to 100% for all students. Therefore the graph task probes a potentially interesting region of parameter space.

In sum, the graph task appears to be closely related (if not isomorphic) to the Stroop task, and the relative automaticity and relative processing strength model of Stroop task may be another model to consider as an explanation for the graph task in this study. The prevalent Stroop effect models do not appear to have major fundamental contradictions with information accumulation models or the fluency heuristic model. In fact the relative processing time concept used in this study is still an important part of Stroop effect explanations and matches well with many aspects of the relative automaticity model. Perhaps the Stroop models, fluency heuristic, and information accumulation models may be incorporated or seen as manifestations of a unified model for a wide range of tasks in which there are patterns of incorrect answers or delays in responding for incongruent questions.

### *7.5. Relevance to research on processing of visual-spatial displays*

Since the science concept questions in this study involved graphs, it is important to consider related research and models on graph perception and comprehension. Certainly there is a considerable amount of research on visual-spatial displays, particularly on two dimensional graphs (for a review see Hegarty, 2011). For example, there has been work on understanding the interaction between top-down versus bottom-up in processing of graphs (e.g., Shah & Freedman, 2011), and animations of visual displays (e.g., Kriz & Hegarty, 2007).

We discuss two points regarding the relation of previous work on visual displays to this study. First, we presume that the relative processing model discussed in this study and its consequences is not necessarily unique to tasks involving graphs or other visual-spatial displays. While the graph question used here certainly highlights the importance of the relative processing speed of certain graphical features, other tasks involving, for example, written or spoken words may also be influenced by relative processing speed and the general idea of fluency or availability.

That being said, our second point is that our findings and the relative processing model do not appear to be fundamentally inconsistent with the very general and flexible models of visual display comprehension such as proposed by Kriz and Hegarty (2007). Furthermore, our findings and methods of experimentation may help to expand and refine such models. While there have been studies modeling response time as a parameter itself, (e.g., Gillan & Lewis, 1994; Peebles & Cheng, 2003), here we model response time as a *causal* factor that determines which dimensions are utilized. That is, the faster processed dimension will be utilized, even at the cost of accuracy. One might interpret this approach as a model of saliency, in that relative saliency might be operationally defined in terms of relative processing time, and the more salient features dominate utilization. However, in models of visual display comprehension, saliency is also determined in terms of relative amount of attention captured, which is not the same parameter as relative processing time. Therefore, including the relative processing time of dimensions may be another important measure in models of visual display comprehension.

### *7.6. Significance of the findings*

From a broad perspective, this study investigated the important domain of misconception-like responses to a simple science concept question, one that might be found on a typical classroom or standardized test. We present novel findings of patterns in response times on the few-second time scale that correspond to either consistently correct or incorrect response choices to a science concept question.

This finding on response times has led to the identification of a basic processing time mechanism and a simple model that may at least partially explain the origins of incorrect answer patterns to some science questions. Specifically, the response patterns suggest that automatic processes involving relative processing time of competing relevant and irrelevant dimensions plays a significant role in misconception-like response patterns for the graph question studied here and perhaps for a large class of science concept questions.

The identification of an automatic mechanism involving relative processing times for science concept questions is significant for three main reasons. First, the data and proposed model are consistent with the general framework and models of several different areas in cognitive psychology in which response time plays a critical role, namely heuristics, sequential sampling models of response times, and the Stroop effect. It also indicates that models of visual display comprehension may need to more explicitly include relative processing time as a factor. The results here may further help to develop a unified model which explains response choices and response times for a wide variety of tasks. Second, much of science education has focused on explicit reasoning when investigating and addressing the topic of incorrect response patterns to science questions. The results here indicate that the study of the influence of automatic, implicit processes involving relative processing time on responses to science questions may also yield important results. Third, the elaboration of mechanisms underlying student responding to science questions may be of practical use for instruction and assessment. In other words, understanding basic mechanisms may help in more first-principled design of instructional materials and assessments for understanding science concepts.

For example, in addition to the observation of patterns in response times, this study also found that imposing a small delay in responding can significantly improve student scores; perhaps this is a kind of disfluency effect. This improvement implies a kind of false positive, namely that students may in fact explicitly know the correct answer, but implicitly they place higher value on answering quickly. In addition, we provide evidence that many students who answer incorrectly can identify the correct rule, but they are not applying it. These results call into question how scores on this and other similar questions should be interpreted. If student answering is a sensitive function of response time, how does this affect how assessments should be designed and administered? As mentioned in a previous section, what does this mean about how we define and measure the meaning of *understanding* of a particular topic?

Finally, the results of Experiments 4 and 5 highlight interesting differences between the results of instruction involving repetitive practice examples with simple corrective feedback versus instruction on an explicit rule. Both appear to be effective in improving scores, yet the difference in response times between the two kinds of training indicates that the repetitive training results in a speed-up of the processing of correct response so that it better competes with the incorrect response, and the benefit is purely a change in automatic processing. In contrast, the results suggest that the rule training does not speed up the processing of the correct answer, rather explicitly removes the incorrect answer from the competition. Which result is a more desirable outcome may depend on the instructional goals; what is interesting is that the response times provide evidence that the different kinds of instruction can affect explicit and implicit mechanisms in different ways. Better knowledge of these mechanisms may help us to design instruction to improve student understanding of science concepts.

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## References

Alter, A. L. (2013). The benefits of cognitive disfluency. *Current Directions in Psychological Science*, *22*(6), 437–442.

Anderson, J. R. (1993). *Rules of the mind*. Hillsdale, NJ: Erlbaum.

Anderson, J. R., Fincham, J. M., & Douglass, S. (1999). Practice and retention: A unifying analysis. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *25*, 1120–1136.

Anderson, J. R., & Lebiere, C. (1998). *The atomic components of thought*. Mahwah, NJ: Erlbaum.

Balota, D. A., & Yap, M. J. (2011). Moving beyond the mean in studies of mental chronometry: The power of response time distributional analyses. *Current Directions in Psychological Science*, *20*, 160–166.

Beichner, R. J. (1994). Testing student interpretation of graphs. *American Journal of Physics*, *62*, 750–762.

Bergert, F. B., & Nosofsky, R. M. (2007). A response-time approach to comparing generalized rational and take-the-best models of decision making. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *33*, 107–129.

Brace, J. J., Morton, J. B., & Munakata, Y. (2006). When actions speak louder than words: Improving children's flexibility in a card-sorting task. *Psychological Science*, *17*, 665–669.

Brown, S. D., & Heathcote, A. (2008). The simplest complete model of choice response time: Linear ballistic accumulation. *Cognitive Psychology*, *57*, 153–178.

Carey, S. (2009). *The origin of concepts*. New York: Oxford University Press.

Clement, J. (1982). Students' preconceptions in introductory mechanics. *American Journal of Physics*, *50*, 66–71.

Cohen, J. D., Dunbar, K., & McClelland, J. L. (1990). On the control of automatic processes: A parallel distributed processing account of the Stroop effect. *Psychological Bulletin*, *97*, 332–361.

De Neys, W. (2006). Dual processing in reasoning: Two systems but one reasoned. *Psychological Science*, *17*, 428–433.

Driver, R., & Erickson, G. (1983). Theories-in-action: Some theoretical and empirical issues in the study of students' conceptual frameworks in science. *Studies in Science Education*, *10*, 37–60.

Dunbar, K. N., & MacLeod, C. M. (1984). A horse race of a different color: Stroop interference patterns with transformed words. *Journal of Experimental Psychology: Human Perception and Performance*, *10*, 622–639.

Dyer, F. N. (1973). The Stroop phenomenon and its use in the study of perceptual, cognitive, and response processes. *Memory and Cognition*, *1*, 106–120.

Evans, J. St. B. T. (1996). Deciding before you think: Relevance and reasoning in the selection task. *British Journal of Psychology*, *87*, 223–240.

Evans, J. St. B. T. (2008). Dual-processing accounts of reasoning, judgment, and social cognition. *Annual Review of Psychology*, *59*, 255–278.

Evans, J. St. B. T., Clibbins, J., Cattani, A., Harris, A., & Dennis, I. (2003) Explicit and implicit processes in multicue judgment. *Memory and Cognition*, *31*, 608–618.

Evans, J. St. B. T., Newstead, S. E., & Byrne, R. M. J. (1993). *Human reasoning: The psychology of deduction*. Hove, England: Erlbaum.

Freyd, J. J. (1987). Dynamic mental representations. *Psychological Review*, *94*, 427–438.

Freyd, J. J., & Finke, R. A. (1984). Representational momentum. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, *10*, 126–132.

Gigerenzer, G. (2008). Why heuristics work. *Perspectives on Psychological Science*, *3*, 20–29.

Gigerenzer, G., & Brighton, H. (2009). Homo heuristicus: Why biased minds make better inferences. *Topics in Cognitive Science*, *1*, 107–143.

Gigerenzer, G., & Goldstein, D. G. (1996). Reasoning the fast and frugal way: Models of bounded rationality. *Psychological Review*, *103*, 650–669.

Gillan, D. J., & Lewis, R. (1994). A componential model of human interaction with graphs: 1. Linear regression modeling. *Human Factors*, *36*, 419–440.

Gilovich, T., & Griffin, D. W. (2002). Heuristics and biases then and now. In T. Gilovich, D. W. Griffin, & D. Kahneman (Eds.), *The psychology of intuitive judgment: Heuristic and biases* (pp. 1–18). Cambridge, England: Cambridge University Press.

Goldstone, R. L., Landy, D. H., & Son, J. Y. (2010). The education of perception. *Topics in Cognitive Science*, *2*, 265–284.

Halloun, I. A., & Hestenes, D. (1985). Common-sense concepts about motion. *American Journal of Physics*, *53*, 1056–1065.

Heathcote, A., Popiel, S. J., & Mewhort, D. J. K. (1991). Analysis of response time distributions: An example using the Stroop task. *Psychological Bulletin*, *109*, 340–347.

Heckler, A. F. (2011). The ubiquitous patterns of incorrect answers to science questions: The role of automatic, bottom-up processes. In J. P. Mestre, & B. H. Ross (Eds.), *Cognition in education*. Vol. 55 (pp. 227–268). Oxford, England: Academic Press.

Hegarty, M. (2011). The cognitive science of visual-spatial displays: Implications for design. *Topics in Cognitive Science*, *3*, 446–474.

Hertwig, R., Herzog, S. M., Schooler, L. J., & Reimer, T. (2008). Fluency heuristic: A model of how the mind exploits a by-product of information retrieval. *Journal of Experimental Psychology: Learning, Memory and Cognition*, *34*, 1191–1206.

Hubbard, T. L. (1998). Representational momentum and other displacement in memory as evidence for nonconscious knowledge of physical principles. In S. Hameroff, A. Kaszniak, & A. Scott (Eds.), *Toward a science of consciousness: II. The 1996 Tucson discussion and debates* (pp. 505–512). Cambridge, MA: MIT Press.

Kahneman, D. (2003). A perspective on judgment and choice: Mapping bounded rationality. *American Psychologist*, *58*, 697–720.

Kahneman, D., & Frederick, S. (2002). Representativeness revisited: Attribute substitution in intuitive judgement. In T. Gilovich, D. Griffin, & D. Kahneman (Eds.), *Heuristics and biases: The psychology of Intuitive Judgment* (pp. 49–81). Cambridge, England: Cambridge University Press.

Kaiser, M. K., Proffitt, D. R., & Anderson, K. A. (1985). Judgments of natural and anomalous trajectories in the presence and absence of motion. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *11*, 795–803.

Kaiser, M. K., Proffitt, D. R., Whelan, S. M., & Hecht, H. (1992). Influence of animation on dynamical judgments. *Journal of Experimental Psychology: Human Perception & Performance*, *18*, 669–689.

Kellman, P. J., Massey, C. M., & Son, J. Y. (2010). Perceptual learning modules in mathematics: Enhancing students' pattern recognition, structure extraction, and fluency. *Topics in Cognitive Science*, *2*, 285–305.

Keil, F. C. (2010). The feasibility of folk science. *Cognitive science*, *34*(5), 826–862.

Kind, V. (2004). Beyond appearances: students' misconceptions about basic chemical ideas: A report prepared for the Royal Society of Chemistry, London: Education Division, Royal Society of Chemistry (2nd ed.). Available at http://www.rsc.org/images/Misconceptions_update_tcm18-188603.pdf. Accessed September 12, 2011.

Kozhevnikov, M., & Hegarty, M. (2001). Impetus beliefs as default heuristics: Dissociation between explicit and implicit knowledge about motion. *Psychonomic Bulletin & Review*, *8*, 439–453.

Kozhevnikov, M., Motes, M. A., & Hegarty, M. (2007). Spatial visualization in physics problem solving. *Cognitive Science*, *31*, 549–579.

Kriz, S., & Hegarty, M. (2007). Top-down and bottom-up influences on learning from animations. *International Journal of Human-Computer Studies*, *65*, 911–930.

Lehman, E. L., & Casella, G. (1998). *Theory of point estimation*. New York: Springer-Verlag.

Luce, R. D. (1986). *Response Times: Their Role in Inferring Elementary Mental Organization* (No. 8). New York: Oxford University Press.

MacLeod, C. M. (1991). Half a century of research on the Stroop effect: An integrative review. *Psychological Bulletin*, *109*, 163–203.

Macleod, C. M., & Dunbar, K. (1988). Training and stroop-like interference: Evidence for a continuum of automaticity. *Journal of Experimental Psychology: Learning, Memnory and Cognition*, *14*, 126–135.

MacLeod, C. M., & MacDonald, P. A. (2000). Interdimensional interference in the Stroop effect: Uncovering the cognitive and neural anatomy of attention. *Trends in Cognitive Science*, *4*, 383–391.

McCloskey, M. (1983). Naive theories of motion. In D. Gentner & A. L. Stevens (Eds.), *Mental models* (pp. 299–324). Hillsdale, NJ: Lawrence Erlbaum Associates Inc.

McDermott, L. C., & Redish, E. F. (1999). Resource Letter: PER-1 Physics Education Research, *American Journal of Physics*, *67*, 755–767.

Mcdermott, L. C., Rosenquist, M. L., & van Zee, E. (1987). Student difficulties in connecting graphs and physics: Examples from kinematics. *American Journal of Physics*, *55*, 503.

Mokros, J. R., & Tinker, R. F. (1987). The impact of microcomputer-based labs on children's ability to interpret graphs. *Journal of Research in Science Teaching*, *24*, 369–383.

Novak, J. D. (2002). Meaningful learning: The essential factor for conceptual change in limited or inappropriate propositional hierarchies leading to empowerment of learners. *Science Education*, *86*, 548–571.

Oberle, C. D., McBeath, M. K., Madigan, S. C., & Sugar, T. G. (2006). The Galileo bias: A naive conceptual belief that influences people's perceptions and performance in a ball-dropping task. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *31*, 643–653.

Ollman, R. T. (1966). Fast guesses in choice reaction time. *Psychonomic Science*, *6*, 155–156.

Peebles, D., & Cheng, P. C. (2003). Modeling the effect of task and graphical representation on response latency in a graph reading task. *Human Factors*, *45*, 28–45.

Pfundt, H., & Duit, R. (2000). *Bibliogaphy: Students' alternative frameworks and science education* (5th ed). Kiel, Germany: Institute for Education Science.

Piaget, J. (1976). *The grasp of consciousness (S. Wedgwood, Trans.)*. Cambridge, MA: Harvard University Press. (Original work published 1974).

Ratcliff, R. (1979). Group reaction time distributions and an analysis of distribution statistics. *Psychological Bulletin*, *86*, 446–461.

Ratcliff, R. (1993). Methods for dealing with reaction time outliers. *Psychological Bulletin*, *114*, 510–532.

Ratcliff, R., & McKoon, G. (2008). The diffusion decision model: Theory and data for two-choice decision tasks. *Neural Computation*, *20*, 873–922.

Ratcliff, R., & Murdock, B. B., Jr (1976). Retrieval processes in recognition memory. *Psychological Review*, *83*, 190–214.

Rohrer, D. (2003). The natural appearance of unnatural incline speed. *Memory and Cognition*, *31*, 816–826.

Schooler, L. J., & Hertwig, R. (2005). How forgetting aids heuristic inference. *Psychological Review*, *112*, 610–628.

Shah, P., & Freedman, E. G. (2011). Bar and line graph comprehension: An interaction of top-down and bottom-up processes. *Topics in Cognitive Science*, *3*, 560–578.

Simon, H. A. (1955). A behavioral model of rational choice. *Quarterly Journal of Economics*, *69*, 99–118.

Sloman, S. A. (1996). The empirical case for two systems of reasoning. *Psychological Bulletin*, *119*, 3–22.

Smith, J. P., diSessa, A. A., & Roschelle, J. (1993). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *Journal of the Learning Sciences*, *3*, 115–163.

Stanovich, K. E., & West, R. F. (2000). Individual differences in reasoning: Implications for the rationality debate. *Behavioral and Brain Sciences*, *23*, 645–665.

Stavy, R., & Tirosh, D. (2000). *How students (Mis-) understand science and mathematics: intuitive rules*. New York: Teachers College Press.

Thaden-Koch, T. C., Dufresne, R. J., & Mestre, J. P. (2006). Coordination of knowledge in judging animated motion. *Physical Review Special Topics–Physics Education Research*, *2*, 020107-1–020107-11.

Townsend, J. T., & Ashby, F. G. (1983). *Stochastic modeling of elementary psychological processes*. New York: Cambridge University Press.

Tversky, A., & Kahneman, D. (1973). Availability: A heuristic for judging frequency and probability. *Cognitive Psychology*, *5*, 207–232.

Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science*, *185*, 1124–1131.

Usher, M., & McClelland, J. L. (2001). On the time course of perceptual choice: The leaky competing accumulator model. *Psychological Review*, *108*, 550–592.

Vosniadou, S. (1994). Capturing and modeling the process of change. *Learning and Instruction*, *4*, 45–69.

Wellman, H. M., & Gelman, S. A. (1992). Cognitive development: Foundational theories of core domains. *Annual Review of Psychology*, *43*, 337–375.

## Appendix A

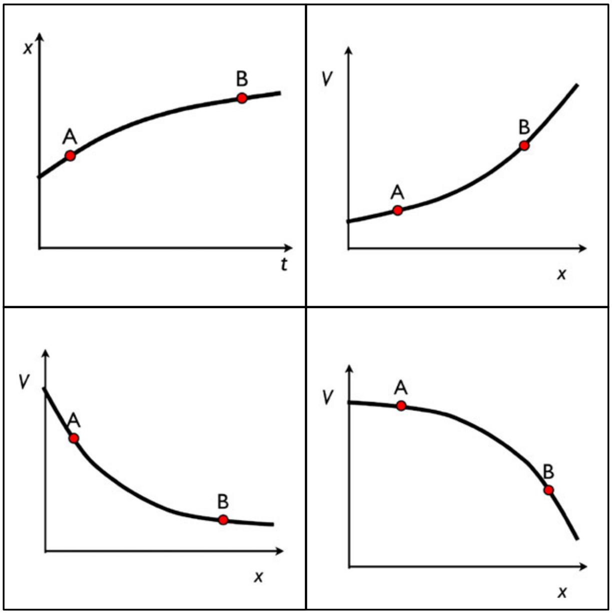


Fig. A1. Four example graphs presented for Experiment 1. Participants were asked, “Which point is higher” or “Which point has a steeper slope?” They were asked to press “A” or “B” on the keyboard as fast as they can without making a mistake.

## Appendix B

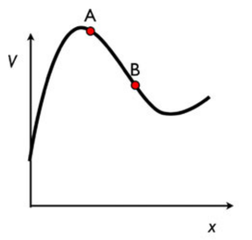
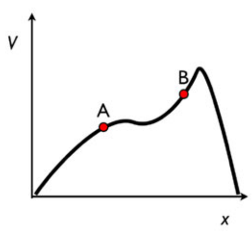
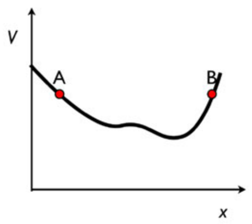
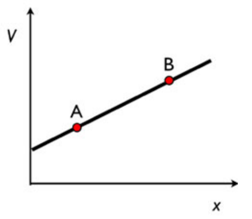
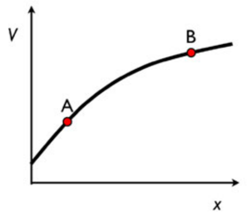
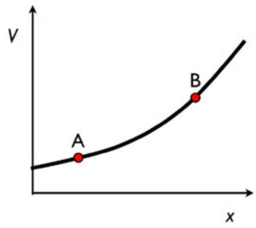
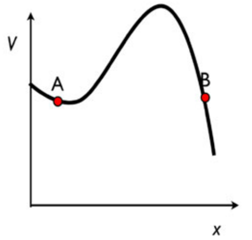
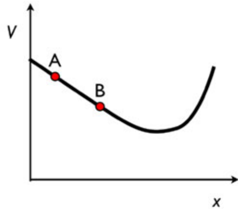
| Target questions. incongruent: Higher point, smaller slope. | Congruent: higher point, larger slope | Same height, different slope | Same slope, different height |
|---|---|---|---|
|  |  |  |  |
|  |  |  |  |

Fig. B1. Examples of the four kinds of graphs used in the test in Experiments 2b, 3, 4, and 5. I In this example, participants were asked, “At which point is the magnitude of the electric field greater?” They were instructed to press the corresponding key “A,” “B,” or “E” if the electric field magnitude was equal at both points.

## Appendix C

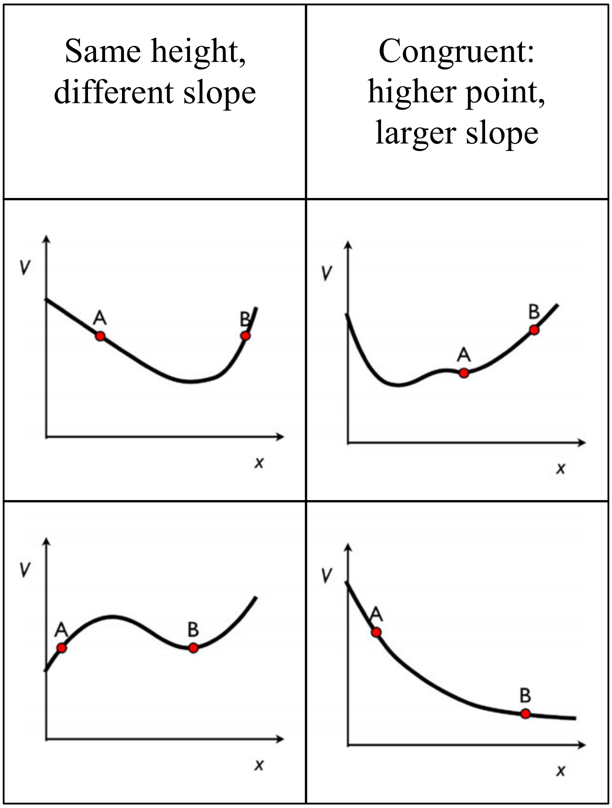


Fig. C1. Examples of the two kinds of Example training graphs used for Experiments 4 and 5. Participants were asked, "At which point is the magnitude of the electric field greater?". They were provided with feedback as to the correct answer immediately after each question.

## Appendix D: Explicit Rule training slides

Participants in the Rule training condition in Experiment 5 were presented with the following slides on a computer screen.

Slide 1

There is a very simple rule which will tell you where electric field is greater:
The electric field, E, depends on the slope of a line on a voltage, V, versus x plot.
Press 'y' if you understand the rule. Press 'n' if you do not understand the rule.

Slide 2

Which of the following rules was given on the previous slide?
a. The electric field is equal to the value of the voltage on a V versus x plot.
b. The voltage depends on the slope of a line on an E versus x plot.
c. The electric field depends on position on a V versus x plot.
d. The electric field depends on the slope of a line on a V versus x plot.

Slide 3

Answer d is correct.
You were told,
"The electric field, E, depends on the slope of a line on a V vs. x plot."
Press the space bar to continue.

Slide 4

True or False:
The electric field, E, does not depend on the slope of a line on a V versus x plot.
Press "t" for true and "f" for false.

Slide 5

FALSE
The electric field, E, DOES depend on the slope of a line on a V versus x plot.