

## Calculation of the Emergent Spectrum and Observation of Primordial Black Holes

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We calculate the emergent spectrum of microscopic black holes, taking into account the proposition that a quark and gluon photosphere forms around the black hole. We find that the limit on the average universal density of black holes is not significantly affected by the photosphere; however, we also find that gamma ray satellites such as EGRET and GLAST are well suited to look for nearby black holes out to a distance on the order of 0.3 parsec, and conclude that if black holes are clustered in our Galaxy by a factor of order  $10^8$  they may be directly detectable. [S0031-9007(97)03070-6]

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Since Hawking [1,2] first proposed that a black hole emits thermal radiation with an emission rate inversely proportional to its mass, there have been several calculations of the emergent spectrum in order to verify or at the very least constrain the presence of the smallest, hence most luminous, ones. Nominally, aside from particle spin effects and gravitational backscattering effects close to the black hole [3], one would expect the emergent spectrum to be thermal, since Hawking showed that the black hole can be thought of as a black body with a temperature  $T = (8\pi GM)^{-1}$ , where  $M$  is the mass of the black hole,  $G$  is Newton's gravitational constant, and we set the Boltzmann constant,  $k = 1$  (along with  $c = 1$  and  $\hbar = 1$ ).

However, when a detailed model of the physics of the emitted particles is considered, the emergent spectrum becomes more complicated. For example, MacGibbon and Webber [4] and Halzen *et al.* [5] have considered black holes with temperatures greater than the characteristic QCD energy scale  $\Lambda \sim 200$  MeV, where the black hole begins to emit quarks and gluons. They propose that the emitted quarks and gluons fragment into hadrons, which further decay into photons, electrons, neutrinos, etc., and they convolve a jet code with the Hawking thermal spectrum to determine the emergent spectrum. For black hole temperatures above  $\Lambda$ , the QCD degrees of freedom dominate in the standard model and the ultimate products of quark and gluon fragmentation will dominate the spectrum; thus understanding the physics of the quarks and gluons is important for determining the spectrum.

In this Letter, we reconsider an important initial assumption made by MacGibbon and Webber, namely, that the quarks and gluons emitted from the black hole directly fragment into hadrons. Rather, since the density of emitted particles around the black hole can be very high (much higher than nuclear density) for  $T > \Lambda$ , we propose that the quarks and gluons propagate through this dense plasma and lose energy via QCD bremsstrahlung and pair production interactions until the density of the outward-propagating plasma becomes low enough that the quarks finally fragment into hadrons. One can then calculate an emergent spectrum by convolving this collection of lower energy quarks with jet codes or fragmentation func-

tions, but this spectrum will be very different than the one obtained using fragmentation of quarks coming directly from the black hole. We also consider several observational consequences of this emergent spectrum, using the EGRET and GLAST satellites as exemplary detectors.

Since it is reasonable to assume that fundamental modes will be present in a thermal bath, let us assume that once above a temperature  $T > \Lambda$ , a black hole emits individual quarks and gluons. Hawking [1,2] showed that the emission rate spectrum for each particle degree of freedom is  $d\dot{N}/dE = \Gamma_s/2\pi[\exp(E/T) - (-1)^s]^{-1}$ , for particles of energy  $E$ , where  $\Gamma_s$  is the absorption coefficient which in general depends upon the spin of the particle  $s$ ,  $M$ , and  $E$  [3], and we have assumed the black hole to be uncharged and nonrotating. In the relativistic limit  $T \gg m$  the total rate at which particles are *directly* emitted from the black hole can be expressed in the form  $\dot{N}_{\text{tot}} = (10^{-2}\eta)T$ , where  $\eta$  is of order of the number of emitted relativistic particles at temperature  $T$  and can be calculated numerically [3,4]. Since this is the flux of particles crossing the Schwarzschild radius  $r_s \equiv (4\pi T)^{-1}$ , the density  $n(r)$  of emitted particles at a radius  $r$  from the black hole is then  $n = \dot{N}/(4\pi r^2)$ . Expressing this in a more illuminating form, we obtain

$$n(r) = \left( \frac{4\pi\eta}{100} \frac{r_s^2}{r^2} \right) T^3. \quad (1)$$

For the QCD,  $\eta \sim 20$ ; thus the average particle separation, defined as  $d \equiv n^{-1/3}$  is then  $d(r) \approx T^{-1}(r_s/r)^{2/3}$ .

QCD is an asymptotically free theory of interactions between quarks and gluons. In general, when particles scatter, the momentum exchanged must be at least of the order of their inverse separation  $d^{-1}$ . As a consequence, at distances smaller than  $\Lambda^{-1}$  the QCD interaction is perturbative, while at larger distances, the coupling becomes so large that vacuum fragmentation is dominant. In particular, vacuum fragmentation of quarks and gluons will occur when they are separated by a distance greater than  $\Lambda^{-1}$ . We see from the above formulas that once  $T > \Lambda$ , the quarks and gluons around the black hole will not immediately vacuum fragment into hadrons because they are closely spaced in a kind of plasma. Rather, they will

propagate for some distance in the dense quark-gluon plasma until the plasma becomes tenuous enough that vacuum fragmentation will occur. This is important because as the quarks and gluons propagate away from the black hole in the dense plasma, they can interact with each other via bremsstrahlung and pair production and decrease in energy.

This can be seen by following the arguments of Heckler [6] (which are still subject to some debate), who has proposed that since the relativistic bremsstrahlung and pair production cross sections are independent of energy, and since the density of particles around the black hole increases with temperature [Eq. (1)], there is a critical temperature for which the optical depth of an emitted particle becomes unity. At this point particles begin to scatter copiously, and a kind of photosphere forms around the black hole. The photosphere is a kind of fireball in the sense that the nearly thermalized plasma propagates outward, decreasing in temperature until (i) the electrons and positrons annihilate in the QED case or (ii) the quarks and gluons fragment or some kind of QCD phase transition occurs in the QCD case.

To simplify matters, we will assume that the plasma cools to the temperature  $\Lambda$ , at which point hadronization occurs. The emergent photon spectrum is a convolution of the quark and gluon spectrum with the pion fragmentation function [4,5] and the Lorentz-transformed neutral pion decay into photons

$$\frac{d\dot{N}_\gamma}{dE_\gamma} = \int_{m_\pi}^{(4E_\gamma^2 + m_\pi^2)/4E_\gamma} \frac{d\dot{N}_\pi}{dE_\pi} \frac{dg_{\pi\gamma}(E_\pi)}{dE_\gamma} dE_\pi, \quad (2)$$

where  $dg_{\pi\gamma}/dE = 2/(\gamma m_\pi \beta)$  is the number of photons of energy  $E$  created by a pion decaying isotropically in its rest frame,  $\gamma = (1 - \beta^2)^{-1/2}$ , and  $\beta$  is the velocity of the pion [note  $E_\gamma$  is Doppler shifted,  $m_\pi/[2\gamma(1 + \beta)] > E_\gamma > m_\pi/[2\gamma(1 - \beta)]$ ]. The pion spectrum is [4]

$$\frac{d\dot{N}_\pi}{dE_\pi} = \sum_j \int_{Q=E_\pi}^{Q=\infty} \frac{d\dot{N}_j(Q, T_0)}{dQ} \frac{dg_{j\pi}(Q, E_\pi)}{dE_\pi} dQ, \quad (3)$$

where  $d\dot{N}_j/dQ$  is the flux spectrum of the quark or gluon  $j$  of energy  $Q$  at the time of fragmentation, which is on the outer edge of the photosphere where the plasma is at a temperature of  $T_0 = \Lambda$ , and  $dg_{j\pi}/dE_\pi$  is the relative number of pions with energy  $E_\pi$  produced by  $j$  [4]. We will use  $dg/dE_\pi = (15/16)z^{-3/2}(1 - z)^2$ , where  $z = E_\pi/Q$  [5]. We approximate the quark and gluon spectrum in the observer frame by boosting a thermal spectrum at temperature  $T_0$  with the Lorentz gamma factor  $\gamma_p$  of the outer edge of the outward moving photosphere

$$\frac{d\dot{N}_j}{dQ} = \frac{\sigma_j \gamma_p^2 r_p^2 Q^2}{2\pi^2} \int_{\cos\theta=0}^1 \frac{(1 - \beta \cos\theta) \cos\theta d\Omega}{e^{[\gamma_p \omega(1 - \beta \cos\theta)]/T_0} \pm 1}, \quad (4)$$

where  $\sigma_j$  is the number of internal degrees of freedom of particle  $j$ , the sign in the denominator is for fermions or bosons, and we have integrated over the surface of the photosphere with radius  $r_p$ . Using Ref. [6], we approxi-

mate  $\gamma_p \approx (T/\Lambda)^{1/2}$  and  $r_p = \gamma_p/\Lambda$ . An accurate calculation of the spectrum would require using a Boltzmann equation to determine the exact spectrum of the nearly thermal quarks and gluons, and a jet fragmentation code for the final decay into pions and photons. We expect the approximations used will be correct within a factor of order unity.

In Fig. 1 we show the photon spectrum calculated using the above formulas both including and excluding the QCD photosphere. Without the photosphere, the quark and gluon spectrum is simply a blackbody spectrum (with spin and finite size effects) at the temperature  $T$ . We see that the main difference between these two spectra is that the photosphere spectrum has many more lower energy photons. This is physically due to the fact that the photosphere processes high energy quarks and gluon into many lower energy ones, and these eventually fragment and decay into lower energy photons. We also plot the spectrum of photons emitted directly from the black hole which peak at approximately  $5T$ . The peak height is several orders of magnitude lower than the peak from quarks and gluons, mostly because the number of QCD degrees of freedom which can decay into photons is large [4]. The total number of photons emitted from the QCD photosphere, which peaks at energies of about 100 MeV, is

$$\dot{N}_\gamma \approx 2 \times 10^{24} \left( \frac{T}{\text{GeV}} \right)^2 \text{sec}^{-1}, \quad (5)$$

which scales as  $T^2$ , which is a stronger function of black hole temperature than the  $T$  dependence of blackbody particle emission, or the  $T^{3/2}$  dependence obtained in the fragmentation model of MacGibbon and Webber [4]. Note that

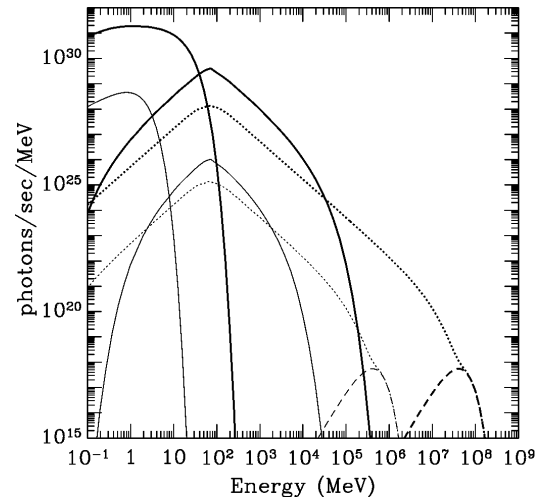


FIG. 1. Instantaneous emergent spectra. Thick lines are for a  $M = 10^9$  g ( $T = 10$  TeV) black hole. Thin lines for  $M = 10^{11}$  g ( $T = 100$  GeV). The solid lines are spectra which include photosphere: the ones peaking at about 100 MeV are the emergent spectra of the QCD photosphere; the ones peaking at about 1 MeV are for the QED photosphere. For comparison, we plot the dotted lines which are the direct fragmentation results of MacGibbon and Webber [4], and the dashed lines which are the direct photon emission spectra. The actual full spectrum is the addition of the two solid lines.

once the black hole is above the QED critical temperature of about 45 GeV, the direct photons will also be processed through a QED photosphere and be degraded to low energies [6]. We have included the QED photosphere spectrum in Fig. 1, which peaks between 1 and 10 MeV and produces  $N_\gamma \approx 5 \times 10^{28} (T/100 \text{ GeV})^{3/2} \text{ sec}^{-1}$ . The extra power of  $T^{1/2}$  in the QCD case comes from the multiplicity of fragmentation of quarks into pions.

Let us examine several observational consequences of the QCD photosphere. The most important consequence involves the search for individual nearby primordial black holes. When developing a strategy and interpreting the results of a direct search for expiring black holes, one must consider the density, emergent spectrum, and lifetime of the black holes, all of which are a function of black hole mass, and the detector sensitivity and background, which are both a function of photon energy. First, one can determine the optimum energy range to search for these black holes by considering the background: since the observed gamma ray background scales approximately as  $E^{-2.4}$  in the range 1 MeV to 10 GeV, one can show using the example of the spectrum of Fig. 1 that for black holes with  $T < 10 \text{ TeV}$  (higher  $T$  black holes have lifetimes shorter than 1 sec, and emit negligible amounts of radiation), the optimum signal to background lies in the range of 1 to 10 GeV.

Next, one can determine the optimum black hole mass to which the detector is sensitive. Naturally, if  $I_{\text{BH}}$  is the photon emission rate of the black hole, and  $\ell$  is the limiting flux to which the detector is sensitive, then  $\ell < I_{\text{BH}}/4\pi d^2$ , where  $d$  is the distance to the black hole. We know that  $I_{\text{BH}} \propto M^{-2}$ ; however, the lifetime of the black hole  $\tau \propto M^3$ . One must, therefore, also require that the total lifetime integrated number of photons incident on the detector is (at least) greater than unity. To be more realistic, let us require the observed amount of photons  $N_\gamma > 10$ . Thus,  $(I_{\text{BH}}/4\pi d^2)A\tau > 10$ , where  $A$  is the area of the detector. When both conditions are met, one finds  $\tau > 10/(\ell A)$ . The optimal observing conditions thus occur for black holes which just meet this criterion. By using Eq. (5) and noting that  $\tau \approx M^3/3\alpha$  (see, e.g., [5]), we can roughly estimate the maximum observing distance  $d_{\text{max}} \sim 0.2 \text{ pc} (A/2000 \text{ cm}^2)^{1/3} (10^{-9} \text{ cm}^{-2} \text{ sec}^{-1}/\ell)^{1/6}$ .

Let us consider two satellites: EGRET [7], which has already accumulated several years of data, and GLAST [8], which is still in its planning stages. In Fig. 2 we show the maximum distance at which each satellite can observe a small black hole, and determine that for EGRET (GLAST),  $d_{\text{max}} \approx 0.11$  (0.31) pc.

One can refashion this limit into other useful limits. MacGibbon and Carr [9], and Halzen *et al.* [5] use the Page-Hawking limit [10], which is discussed below, to find a generic limit on the local density of black holes below a mass  $M$  to be  $n_{bh} < \mathcal{N}(\zeta/3)(M/M_*)^3 \text{ pc}^{-3}$ , where  $M_* \approx 5 \times 10^{14} \text{ g}$  is the mass of a black hole which has a life of the age of the Universe,  $\mathcal{N} \approx 10^{-4} \text{ pc}^{-3}$  is the Page-Hawking limit on the average

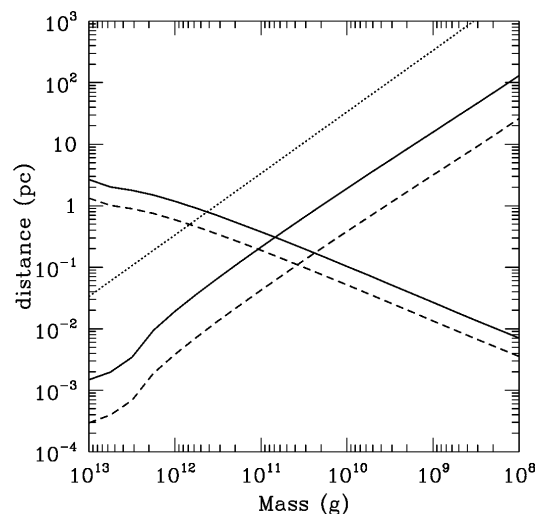


FIG. 2. Observational distance limits. The dotted line is the minimum average distance to the nearest black hole of mass  $M$ , using the Page-Hawking constraint [10] and assuming a clustering factor  $\zeta = 10^6$ . The solid (dashed) lines pertain to GLAST (EGRET). Lines sloping upward show the maximum distance a black hole can be detected for the GLAST (EGRET) point source sensitivity, and the line sloping downward shows the maximum distance at which a black hole can be and still cast ten photons on the detector, integrated over the lifetime of the hole. As the observation time of the detector increases, the intersection of the lines effectively moves to the left [see Eq. (6)], possibly intersecting with the Page-Hawking constraint.

density of  $M < M_*$  black holes, and  $\zeta$  is the local density enhancement of black holes compared to the universal average. We can make a similar limit on the value of  $\zeta \mathcal{N}$  by assuming that if EGRET and GLAST find no black holes after a time  $\tau = (\ell A)^{-1}$ , then this will optimally constrain the density black holes with masses  $M_0$  whose lifetime is  $\tau$ . However, there is a subtle beneficial effect: after observing for a time  $t > \tau$ , larger mass black holes, which have higher number densities, will have decayed to mass  $M_0$ . Since  $dM/dt = -\alpha/M^2$  (see, e.g., [5]), one can begin to constrain black holes with mass  $M^3 = 3\alpha\delta t + M_0^3$ , where  $\delta t = t - \tau$ . If the constraint on the density of  $M_0$  black holes is  $n_0 < (\Omega d_0^3/3)^{-1}$ , where  $\Omega$  is the solid angle covered by the detector, then we obtain the limit

$$\zeta \mathcal{N} < \frac{9}{\Omega d^3} \left( \frac{M_*^3}{M_0^3 + 3\alpha\delta t} \right) \approx \frac{1.5 \times 10^{10} (\text{yr})}{\Omega d^3} \left( \frac{\text{yr}}{\delta t} \right), \quad (6)$$

as long as  $t > \tau$ . Simply put, observing for a longer time allows one to observe larger, more densely populated black holes which decay to the optimum observing mass  $M_0$ , and this allows for better limits on  $\zeta \mathcal{N}$ . For EGRET (GLAST) with four years of observation one finds  $\zeta \mathcal{N} < 5.7 \times 10^{12}$  ( $8.5 \times 10^{10}$ )  $\text{pc}^{-3}$ . Notice that this limit scales as  $\sqrt{\ell}/A$ . One can also place a limit on the local rate  $R$  at which black holes expire per unit volume. Roughly,  $R \approx n(M)/\tau(M) \approx \zeta \mathcal{N} \alpha/M_*^3$ . For EGRET (GLAST) one finds  $R < 1100$  (17)  $\text{pc}^{-3} \text{ yr}^{-1}$ .

Comparison of these results to previous results is difficult because nonstandard particle models were often used. For example, there have been several limits placed on  $R$  based on the Hagedorn model [11], which presumes that the number of degrees of freedom increases exponentially with temperature, a fact that has no experimental support for energies up to about 1 TeV. It is important to note, however, that Alexandreas *et al.* [12] use the standard model of MacGibbon and Webber, and conclude from data of a TeV air shower array that  $R < 8.5 \times 10^5 \text{ pc}^{-3} \text{ yr}^{-1}$ . However, as illustrated in Fig. 3, the photosphere dramatically alters the lifetime integrated spectrum, and the expected flux of photons above 1 TeV is about 4 orders of magnitude lower than the MacGibbon and Webber result, which translates into a limit on  $R$  6 orders of magnitude weaker (higher). Note especially that when the photosphere is included, the lifetime integrated spectrum decreases as  $E^{-4}$  instead of  $E^{-3}$ . In fact the total number of photons above energy  $E_d$  produced by a black hole of initial temperature  $T$  is  $N_\gamma \approx 2 \times 10^{34} (\text{GeV}/E_d)^3$ , which is valid for energies  $E_d > (T/\text{GeV})^{1/2} \text{ GeV}$ . This is to be compared to the result of Halzen *et al.*, which finds  $N_\gamma \propto E_d^{-2}$ . This lower flux at high energies renders any search for primordial black holes with TeV air shower arrays impractical.

It is interesting to note that Wright [13] points out that the anisotropic component of the gamma ray background may be explained by the presence of primordial black holes clustered in the halo of our Galaxy. If black holes are responsible for the anisotropy, then he finds  $\zeta \mathcal{N} = (2-12)/h \times 10^9 \text{ pc}^{-3}$ , where  $h$  is the Hubble parameter, and this is only about an order of magnitude lower than the estimated detection limits of the GLAST project.

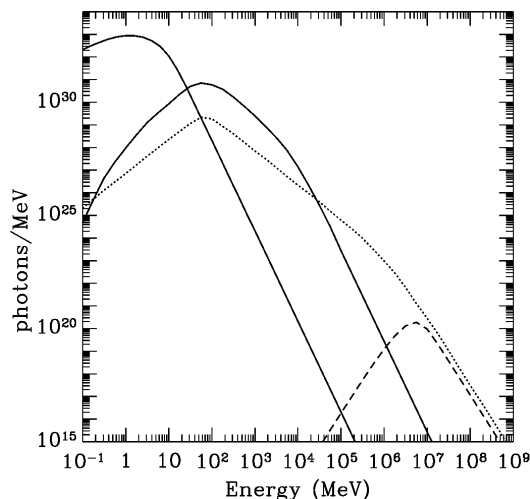


FIG. 3. Lifetime integrated emergent spectra for a  $M = 10^{10} \text{ gg}$  ( $T = 1 \text{ TeV}$ ) black hole, which has a lifetime of about 400 sec. Notation is the same as in Fig. 1. Notice that the slope of the photosphere spectra (solid line) runs as  $E^{-4}$ , whereas for the direct fragmentation picture (dotted line) the slope runs as  $E^{-3}$ .

Another consequence involves the Page-Hawking limit [10], which constrains (or possibly measures) the density of primordial black holes by comparing their expected contribution to the gamma ray background, for a given black hole density, to the actual observed background. MacGibbon and Carr [9] and Halzen *et al.* [5] have studied this approach in detail, using a model of direct fragmentation of quarks and gluons (i.e., no photosphere), and they conclude that black holes with mass less than about  $10^{16} \text{ g}$  cannot contribute more than a fraction  $\Omega_{\text{BH}} < 10^{-8}$  of the critical density of the Universe.

We have performed the same calculation including the effects of the QCD photosphere and verified, as suggested by Heckler [6], that the QCD photosphere does not significantly change (i.e., less than a few percent) the limit on  $\Omega_{\text{BH}}$  found previously [5,9]. The effect is small because the QCD photosphere becomes important only at temperatures above  $\Lambda$ , and black holes with these temperatures do not significantly contribute to the total number of photons at 100 MeV. There is one important difference in the spectrum: above energies of about 300 MeV the spectrum including the photosphere is proportional to  $E^{-4}$ , whereas previous results show a  $E^{-3}$  dependence [5,9]. The steeper slope is due to the degrading of high energy quarks into lower energies as they are processed through the photosphere.

As a final note, since the QCD photosphere will emit charged pions as well as neutral ones, there will also be a flux of neutrinos, electrons, and positrons up to several orders of magnitude larger than previously assumed, with spectra very similar to the photon spectrum in Fig. 1. This will make constraints on (or possibilities of detection of) high energy neutrinos [14] and positrons [9] from black holes much more important.

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