Physics 132: Midterm II
Afternoon Recitation
Professor Frank De Lucia
2:30 Lecture

Winter 2001

Name (1 pt): ____________________________
Recitation Instructor (1 pt): ____________________________

There are six pages to this midterm (plus an equation sheet). It is important that you write your name on each page and the name of your recitation instructor on the first page. Each name is worth one point. For many of the problems, it is possible to simply write down the answer. However, if you write down the wrong answer, there is nothing for which to give you partial credit.

1. (15 pts) Consider the circuit shown in the figure.

(a) How much current flows out of the 50V battery?
   Call this current $i$ (see figure).
   $$ \Rightarrow V_a + V_b - V_a = V_b \Rightarrow (V_a - V_b) = 50 - 60$$
   But $V_a - V_b = 30 \Rightarrow 60 - 50 = 30$
   $60 - 50 = 10 = 20 \Rightarrow i = \frac{20}{60} \Rightarrow i = 0.33 A$

(b) How much current flows through the 10 $\Omega$ resistor?
   Call this current $i'$. (see figure)
   $$ \Rightarrow i' = \frac{V_b}{R_{10}} = \frac{30}{10} \Rightarrow i' = 3 A$$

(c) How much voltage is there across the 20 $\Omega$ resistor?
   From part (a), since the current through $R = 20 \Omega$ is $i = 0.3 A$
   $$ \Rightarrow \text{Ohm's Law} \text{ gives: } V_{20} = i \times R = \frac{1}{3} \times 20 \Rightarrow V_{20} \approx 6.67 V$$
   (check: $V_{40} = \frac{1}{5} \times 40 = 13.3 V \Rightarrow -(V_{10} + V_{20}) + 50 = 30 V$)
2. (15 pts) For the circuit shown in the diagram, first draw arrows on the diagram to indicate YOUR definition of the direction of positive current flow through each of the resistors and number these currents 1 through 3.

Then write the three equations (two loop and one junction) which, when solved, would yield the currents through each of the resistors.

Note that there are several correct ways to work this problem.

For the currents as I’ve drawn them: (4 pts per equation)

junction rule – \( i_1 + i_2 = i_3 \) \((1)\)

Loop rules – 
\[
7V - (5\Omega)i_1 - (6\Omega)i_3 = 0 \quad (2)
\]
\[
9V - (7\Omega)i_2 - (6\Omega)i_3 = 0 \quad (3)
\]
\[
7V - (5\Omega)i_1 + (7\Omega)i_2 - 9V = 0 \quad (4)
\]

Must have equation \((1)\)

Choose any pair of equations \((2) - (4)\)
3. (15 pts) Consider the circuit shown in the figure.

(a) How much charge is on either of the 30 μF capacitors?

\[ q = CV \]

\[ \frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} \]

\[ C_{12} = 15 \mu F \]

\[ 40 - V_1 - 10 - V_2 = 0 \]

\[ 30 = 2V_1 \rightarrow V_1 = 15V \]

\[ q = CV = 30 \mu F (15V) \]

\[ = 450 \mu C \]

(b) How much voltage is across the 15 μF capacitor?

\[ V_8 = V_{41} = 10V \]

\[ 10 - V_3 = 0 \rightarrow V_3 = 10 \]

(c) How much voltage is across either of the 30 μF capacitors?

\[ V = 15V \quad \text{from part a (symmetry)} \]

\[ q = CV \rightarrow V = \frac{q}{C} = \frac{450 \mu C}{30 \mu F} = 15V \]
4. (15 pts) In the circuit shown, the capacitor is initially charged to a potential of 25 V. At t = 0 the switch is closed.

(a) At t = 0, what is the charge on the capacitor?
\[ q = CV = 5 \mu F \times 25 V = 125 \mu C \]

(b) At time infinity, what is the current through the resistor?
\[ I = 0 \text{ at infinity, the capacitor is fully discharged and will not supply a voltage} \]

(c) When the capacitor has been discharged to 50% of its initial voltage, how much energy has been dissipated by the resistor?

Use conservation of energy: the energy lost from the capacitor is equal to the amount of energy dissipated by the resistor.

\[ \Delta U = U_F - U_i = \frac{1}{2} C V_i^2 - \frac{1}{2} C V_c^2 = \frac{1}{2} C \left( V_i^2 - V_c^2 \right) = \frac{1}{2} C V_i^2 \left( \frac{1}{4} \right) \]

\[ \Delta U = \frac{3}{8} \times 10^{-4} \times (25 V)^2 = 1.13 \times 10^{-3} J = \text{energy dissipated} \]

(d) When the capacitor has been discharged to 50% of its initial voltage, how much current is flowing through the resistor?

\[ I = \frac{V}{R} = \frac{25 \frac{1}{1000}}{2 \times 10^{12}} \]
\[ = 12.5 \times 10^{-6} A \]
5. (15 pts) Consider the triangular current loop shown in the figure. As shown the loop carries a current \( I = 3\) A in a clockwise direction. Also as shown, there is a 3T magnetic field parallel to the hypotenuse of the triangle.

(a) What is the magnitude of the force on the vertical side of the triangle?

\[
F = iL \times B
\]

\[
|F| = i L B \sin \theta
\]

\[ L = 2\, \text{m} \]

\[ \theta = 90^\circ + 45^\circ = 135^\circ \]

\[
|F| = (3\, \text{A})(2\, \text{m})(3\, \text{T}) \sin 135^\circ
\]

\[ = (18 \sin 135^\circ) \, \text{N} \]

\[ = 9 \, \text{N} \]

\[ \approx 12.73 \, \text{N} \]

(b) Draw an arrow (or the tip or feathers of the arrow as appropriate) on the diagram to indicate the direction of this force.

\( \bigcirc \) out of page

by RHR
6. (18 pts) As shown in the figure, a hollow pipe and a thin wire both carry a current $I_0$ out of the page. The center of the pipe and the wire are separated by a distance $4d$.

(a) What is the magnitude of the magnetic field due to both currents at the center of the hollow pipe?

$$B = \frac{\mu_0 I_0}{2\pi d} \quad (\Gamma = 2\pi r)$$

(b) Draw an arrow on the diagram to indicate the direction of this field.

(c) What is the magnetic field at $x = 2d, y = 0$ (i.e., at point $P$)?

Now we have two contributions, one from the thin wire and another one from the current in the hollow pipe.

$$B_1 = \frac{k_0 I_0 (\frac{d}{2d})}{2\pi (2d)} = k_0 \frac{I_0}{4\pi d}$$

$$B_2 = \frac{k_0 I_0 (\frac{d}{2d})}{2\pi (2d)} = k_0 \frac{I_0}{4\pi d}$$

(by superposing $B_1$ and $B_2$ the total field, $B$ at $P$ is)

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = 0 \quad \text{(Note: Here we use the appropriate Amperean loops)}$$

Check the directions of the fields at $P$ using the RH rule.