Physics 132: Midterm II
Winter 2002
Name (1 pt)
Recitation Instructor (1 pt)

There are six pages to this midterm (plus an equation sheet). It is important that you write your name on each page and the name of your recitation instructor on the first page. Each name is worth one point. For some of the problems, it is possible to simply write down the answer. However, if you write down the wrong answer, there is nothing for which to give you partial credit.

1. (15 pts) Consider the circuit shown in the figure.

(a) How much current flows out of the battery?
\[ V = iR \]
\[ 20 + 30 = 50 = R_{23} \]
\[ 30 + 50 = 80 = R_{45} \]
\[ \frac{1}{40} + \frac{1}{40} = \frac{1}{20} = R_{23} \]
\[ V = iR \Rightarrow i = \frac{V}{R} \]
\[ 20 + 30 = 50 = R_{23} \]
\[ \frac{200V}{100} = 2A \]

(b) How much current flows through the 40 Ω resistor?

By symmetry, \( i_1 = i_{23} \) and \( i_1 + i_{23} = i_{45} = i \)

\[ i_1 = \frac{i}{2} = 1A \]

(c) What is the voltage across the 50Ω resistor?

\[ i_5 = i_{45} = i \]
\[ V = iR = (2A)(50Ω) \]
\[ V = 100V \]

\[ R_{1234} = 20 + 30 = 50 \] by symmetry \( V_5 = V_{1234} \) and \( V_5 + V_{1234} = V \)
2. (15 pts) For the circuit shown in the diagram, first draw arrows on the diagram to indicate YOUR definition of the direction of positive current flow through each resistor and number these currents 1 through 3. Then write the three equations (two loop and one junction) which, when solved, would yield the currents through each of the resistors.

Note that there are several correct ways to work this problem.

For the currents as I've drawn them: (4 pts per equation)

- Junction rule: \( i_1 + i_4 = i_3 \)  \( \text{Eq. 1} \)
- Loop rules:
  - \( 3V - (5\Omega)i_1 - 6V - (2\Omega)i_3 = 0 \)  \( \text{Eq. 2} \)
  - \( 4V - (7\Omega)i_2 - 6V - (2\Omega)i_3 = 0 \)  \( \text{Eq. 3} \)
  - \( 3V - (5\Omega)i_1 + (3\Omega)i_2 - 4V = 0 \)  \( \text{Eq. 4} \)

Must have equation 1

Can pick any pair from equations 2 - 4
3. (18 pts) Consider the circuit shown in the figure.

(a) What is the equivalent capacitance of the four capacitors?

(i) (ii) are in series \( C_{eq} = \frac{1}{\frac{1}{C_{12}} + \frac{1}{C_{13}}} = \frac{2}{60} \) \( \Rightarrow C_{12} = 30 \mu F \)

(iii) (iv) are in parallel \( C_{34} = 20 + 10 = 30 \mu F \)

The reduced circuit looks like \( \frac{1}{\frac{1}{C_{12}} + \frac{1}{C_{34}}} \) \( \Rightarrow \ C_{eq} = \frac{C_{12} \cdot C_{34}}{C_{12} + C_{34}} = \frac{30 \cdot 30}{30 + 30} = 15 \mu F \)

(b) How much charge will there be on the 20 \( \mu F \) capacitor?

The total charge in the system is \( Q_{tot} = \frac{V_{tot}}{V} \)

Now the voltage across (iii) is \( V_{34} = \frac{(V_{tot} \cdot \frac{C_{34}}{C_{12}}) + (V_{tot} \cdot \frac{C_{34}}{C_{12}})}{V_{1}} \) \( \Rightarrow V_{34} = 22.5 V \)

\( \Rightarrow \ C_{12} \cdot V_{34} = 20 \mu F \cdot 22.5 V \Rightarrow Q_{34} = 450 \mu C \)

(c) How much voltage will there be across either of the 60 \( \mu F \) capacitors?

From point (b) \( \Rightarrow V_{1} = \frac{Q_{tot}}{C_{12}} \) \( \Rightarrow V_{2} = \frac{Q_{tot}}{C_{2}} \)

\( \Rightarrow V_{1} = V_{2} = \frac{450 \mu C}{60} \Rightarrow \boxed{V_{1} = V_{2} = 7.5 V} \)
4. (15 pts) Consider the circuit shown in the diagram. If the capacitor is initially uncharged and the switch is closed at \( t = 0 \).

(a) At \( t = 0 \), what is the power dissipated in the resistor?

\[
P = V \cdot I = \frac{V^2}{R} = \frac{(2.5)^2}{3 \cdot 10^6}
\]

\[
P = 2.1 \cdot 10^{-4} \text{ W}
\]

(b) At \( t = \infty \), what is the energy stored in the capacitor?

At \( t \to \infty \) the capacitor is fully charged.

\[
V = \frac{1}{2} CV^2 = \frac{1}{2} (2 \cdot 10^{-6})(2.5)^2
\]

\[
V = 6.25 \cdot 10^{-4} \text{ J}
\]

(c) When the current through the resistor is 1 \( \mu \text{A} \), what is the voltage across the capacitor?

Using Ohm's law:

\[
I = \frac{V_c}{R} \Rightarrow V_c = IR = (1 \cdot 10^{-6})(3 \cdot 10^6 \Omega)
\]

\[
V_c = 3 \text{ V}
\]

\[
V_c = 25 \text{ V} - 3 \text{ V} \Rightarrow V_c = 22 \text{ V}
\]
5. (15 pts) Consider the triangular current loop shown in the figure. As shown, the loop carries a current $I_o = 5$A in a clockwise direction. Also as shown, there is a $3T$ horizontal magnetic field.

(a) What is the magnitude of the force on the side which forms the hypotenuse of the triangle?

$$\mathbf{F} = \mathbf{v} \times \mathbf{B}$$

$$|\mathbf{F}| = vL\mathbf{B} \sin \theta$$

From diagram

$$\theta = 45^\circ$$

$$L^2 = (2m)^2 + (2\sqrt{2}m)^2$$

$$L = \sqrt{8} \text{ m} = 2\sqrt{2} \text{ m}$$

$$|\mathbf{F}| = (I_o)(2\sqrt{2}m)(3T) \sin 45^\circ$$

$$= (5A)(2\sqrt{2}m)(3T) \frac{1}{\sqrt{2}}$$

$$= \boxed{30 \text{ N}}$$

(b) Draw an arrow (or the tip or feathers of the arrow as appropriate) on the diagram to indicate the direction of this force.

\[\square\text{ into page}\]

by RHR
6. (15 pts) A hollow conducting pipe with inner diameter $a$ and outer diameter $b$ carries a current $I_o$ out of the page.

(a) For $r > b$, what is the magnitude of the magnetic field?

$$B \cdot 2\pi r = \mu_0 I_o \Rightarrow B = \frac{\mu_0 I_o}{2\pi r}$$

(b) For $b > r > a$, what is the magnitude of the magnetic field?

$$\text{Density} = \frac{I_o}{\pi (b^2 - a^2)}$$

$$i_{enc} = I_o \frac{2\pi (r^2 - a^2)}{\pi (b^2 - a^2)}$$

$$\oint B \cdot ds = \mu_0 i_{enc}$$

$$B \cdot 2\pi r = \mu_0 \frac{I_o}{b^2 - a^2} \Rightarrow B = \frac{\mu_0 I_o}{2\pi r} \frac{(r^2 - a^2)}{(b^2 - a^2)}$$

(c) Draw an arrow on the diagram to indicate the direction of circulation of the magnetic field for $r > a$.

Counter clockwise