Assume that the potential is zero at infinity.

L. (23 pts) Three charges $+5Q$ are placed at three corners of a square of dimension $d$ as shown in the figure.

a. In unit vector notation, what is the electric field at point $P$ in the center of the square?

$$ E = E_1 + E_2 + E_3 = E_3 $$

$$ E_x = E_3 \cos 45^\circ = \frac{1}{4\pi \varepsilon_0} \frac{5Q}{\left(\frac{d}{2}\right)^2} = \frac{1}{4\pi \varepsilon_0} \frac{5Q \sqrt{2}}{d^2} $$

b. What is the potential at point $P$ in the center of the square?

$$ V_P = \frac{1}{4\pi \varepsilon_0} \left( -\frac{5Q}{d^2} + \frac{5Q}{d^2} + \frac{5Q}{d^2} \right) = \left( \frac{1}{4\pi \varepsilon_0} \right) \left( \frac{30Q}{d^2} \right) $$

$$ 9 \text{ points} $$

$$ 9 \text{ points} $$

If a charge of $+q_o$ were released from rest at the point $P$, what would its kinetic energy be when it reached infinity?

$$ KE = \frac{1}{4\pi \varepsilon_0} \frac{30Qq_o}{d \sqrt{2}} $$

$$ 5 \text{ points} $$
II. (24 pts) The figure shows a solid conducting sphere of radius \(a\) and net charge +8Q, surrounded by a concentric conducting shell of inner radius \(b\) and outer radius \(c\). The outer shell has a total net charge of -5Q.

(a) What is the charge on the outer surface of the outer shell?

According to Gauss Law

\[
\oint E \cdot dA = \frac{Q_{\text{enc}}}{\varepsilon_0}, \quad b < r < c
\]

\[
E_r \oint dA = \frac{Q_{\text{enc}}}{\varepsilon_0} \quad \Rightarrow \quad 0 = \frac{Q_{\text{enc}}}{\varepsilon_0}
\]

\[
E_r = 0 \quad \text{(conducting shell)} \quad \Rightarrow \quad \frac{+8Q + Q_i}{\varepsilon_0} \quad \Rightarrow \quad Q_i = -8Q
\]

\[
Q_{\text{net}} = 5Q = Q_i + Q_o \Rightarrow Q_o = Q_{\text{net}} - Q_i = -5Q - (-8Q) = +3Q
\]

(b) What is the electric field at a point \(r\) inside the shell between \(b\) and \(c\)?

Since the electric field inside the conducting shell is 0,

\[
E_r = 0, \quad b < r < c
\]

(c) Suppose an additional point charge of -2Q is placed a distance \(d\) from the center with \(d > c\). What is its potential energy?

\[
V = \frac{U}{\varepsilon_0} \quad \Rightarrow \quad U = V \cdot Q
\]

\[
V = \frac{1}{4\pi \varepsilon_0} \cdot \frac{3Q}{d}
\]

\[
U = \frac{1}{4\pi \varepsilon_0} \cdot \frac{3Q}{d} \cdot (-2Q) = -\frac{3Q^2}{2\pi \varepsilon_0 d}
\]
III. (24 pts) Three charges are located along the $x$ axis as shown.

a) What is the force in unit vector notation on a charge $+q_0$ if it is placed at point $P$?

$$\vec{F} = k \frac{4Qq_0}{(3d)^2} \vec{i} + k \frac{2Qq_0}{d^2} \vec{i} - k \frac{5Qq_0}{(2d)^2} \vec{i} = k \left( \frac{4}{9} + 2 - \frac{5}{4} \right) \frac{q_0}{d^2} \vec{i}$$

$$\vec{F} = k \frac{43}{36} \frac{Qq_0}{d^2} \vec{i}$$

b) What is the magnitude and direction (draw and label an arrow on the diagram) of the electric field at point $P$?

$$\vec{E} = \frac{\vec{F}}{q_0} = k \frac{43}{36} \frac{Q}{d^2} \vec{i}$$

\[ E = k \frac{43}{36} \frac{Q}{d^2} \vec{i} \]

b) What is the electric potential at point $P$?

$$V = k \frac{4Q}{3d} + k \frac{2Q}{d} + k \frac{5Q}{2d} = k \left( \frac{4}{3} + 2 + \frac{5}{2} \right) \frac{Q}{d}$$

$$V = k \frac{35}{6} \frac{Q}{d}$$
IV. (24 pts) The figure shows an infinite nonconducting plate of thickness $t$ with a surface charge density of $+\sigma$ on the top surface and $-\sigma$ on the bottom surface.

(a) What is the magnitude of the electric field above the plate? If the field is nonzero draw an arrow to indicate its direction.

From one plate \[ E = \frac{\sigma}{2\varepsilon_0} \]

Superposition: \[ \vec{E} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} \]

so \[ \vec{E} = 0 \]

(b) What is the magnitude of the electric field inside the plate? If the field is nonzero draw an arrow to indicate its direction.

\[ \begin{align*}
+ & + + \\
\downarrow & \downarrow \frac{\sigma}{2\varepsilon_0} & \downarrow & \downarrow \frac{\sigma}{2\varepsilon_0} & = & \frac{\sigma}{\varepsilon_0} & \downarrow \\
\end{align*} \]

(c) What is the potential difference between the top and the bottom of the plate?

\[ V = E \ t \quad \text{since } E \text{ is uniform} \]

so \[ V = \frac{\sigma}{\varepsilon_0} \ t \]

(d) Does the upper or lower plate have a higher potential?

\underline{Upper} plate is at higher potential

since \( \vec{E} \) points from upper plate to lower plate

(\( \vec{E} \) points from higher to lower potential)