Physics 132: Midterm I
Professor Frank De Lucia

Winter 2000

Name (1 pt): ____________________________

Recitation Instructor (1 pt): ____________________________

There are five pages to this midterm (plus an equation sheet). It is important that you write your name on each page and the name of your recitation instructor on the first page. Each name is worth one point.

For many of the problems, it is possible to simply write down the answers. This is fine, however, if you write down the wrong answer, there is nothing for which to give you partial credit.

1. Short Problems. Below are two different charge distributions. For each there are several questions for which quick solutions exist.

The figure shows three concentric conducting thin spherical shells of radii "a", "b", and "c"; with total charge +Q, +3Q, and -5Q on each shell, respectively.

1. (5 pts) What is the charge on the outer surface of the shell with radius "b"?
   Gauss' law requires that \( \Phi \) be on inside surface
   \[ \Rightarrow \quad \Phi_{\text{outside}} = +3Q - (-Q) = +4Q \quad \text{[conservation of charge]} \]

2. (6 pts) What is the electric potential between b and c?
   \[ V = \frac{1}{4\pi \epsilon_0} \left( \frac{+Q}{r} + \frac{+3Q}{r} - \frac{5Q}{r} \right) \quad \text{inside "c"} \]
   \[ \Rightarrow \quad \text{outside "a"} \rightarrow \text{outside "b"} \]

3. (6 pts) What would be the net flux through a Gaussian surface located between b and c?
   \[ \Phi = \oint \vec{E} \cdot d\vec{A} = \frac{\Phi}{\epsilon_0} = \frac{4Q}{\epsilon_0} \quad \text{charge enclosed by Gaussian Surface} \]
Three quarters of a complete ring of radius $R$ has a total charge $+Q$ uniformly distributed along its length.

4. (5 pts) What is the potential at the ring’s center?

$$ V = \frac{1}{4\pi \varepsilon_0} \frac{+Q}{R} $$

5. (6 pts) What is the potential a distance "R" above the plane of the ring and above the ring's center?

$$ V = \frac{1}{u \pi \varepsilon_0} \frac{+Q}{u^2 R} $$

6. (6 pts) If a charge of $+q$ were released from rest at the point "R" above the plane of the ring and the ring's center, what would its kinetic energy be when it reached infinity?

$$ E_f = U_f = V_e = \frac{1}{4\pi \varepsilon_0} \frac{+Q}{u^2 R} \frac{+Q}{u^2 R} \Rightarrow E_f = \frac{1}{4\pi \varepsilon_0} \frac{+Q}{u^2 R} $$

$$ E_f = U_f + K \Rightarrow K = \frac{1}{4\pi \varepsilon_0} \frac{+Q}{u^2 R} $$
II. Problems (20 points each). In some cases these require modestly more calculation than the Short Problems of Section I.

1. Three charges $+2Q$ are located at the corners of a square as shown in the figure. A point $P$ is located in the forth corner of the square.

   ![Diagram of three charges and point P]

   a) What is the electric field at point $P$ in unit vector notation?
   b) How much energy was required to assemble the three $+2Q$ charges from infinity?
   c) What is the electric potential at point $P$?

   \[
   \mathbf{E} = \frac{1}{4\pi\varepsilon_0} \left( \frac{2Q}{d^2} \mathbf{\hat{e}} + \frac{2Q}{d^2} \mathbf{\hat{e}} + \frac{2Q}{d^2} \mathbf{\hat{e}} \right) + \frac{2Q}{d^2} \left( \mathbf{\hat{e}} + \mathbf{\hat{f}} \right) + \frac{2Q}{d^2} \left( \mathbf{\hat{e}} + \mathbf{\hat{g}} \right) \quad \text{(this could be simplified)}
   \]

   \[
   U_1 = 0
   \]
   \[
   U_2 = U_3 = U_4 = \frac{1}{4\pi\varepsilon_0} \left( \frac{2Q}{d^2} + \frac{2Q}{d^2} + \frac{2Q}{d^2} \right) \quad U = \frac{4\pi\varepsilon_0}{4\pi\varepsilon_0} \left[ \frac{1}{d} + \frac{1}{d} + \frac{1}{d} \right]
   \]

   \[
   V = \frac{1}{4\pi\varepsilon_0} \left( \frac{2Q}{d} + \frac{2Q}{d} + \frac{2Q}{d} \right)
   \]
2. The figure shows the end view of a long rod of radius "a", which is surrounded by a thin concentric cylinder of radius "b". If both are insulators, the inner rod has a volume charge density of +\( \rho \), and the outer cylinder surface charge density of -2\( \sigma \),

\[ E \cdot dA = \frac{\rho}{\varepsilon_0} \]

\[ E = \frac{\pi a^2 \rho}{2\varepsilon_0 r} \]

a) What is the electric field between the rod and cylinder?

b) What is the electric field outside the cylinder?

c) What is the electric field inside the rod?

\[ E = \frac{\pi a^2 \rho + 2\pi b L (-2\sigma)}{\varepsilon_0} \]

\[ E = \frac{a^2 \rho - 4b\sigma}{2\varepsilon_0 r} \]
3. The figure shows two infinite planes of thickness $t$, whose separation is $2d$. Both are insulators and have volume charge density of $+\rho$.

![Diagram of two infinite planes with charge density and separation](image)

a) In unit vector notation, what is the electric field between the two planes?

b) In unit vector notation, what is the electric field inside the upper plane?

c) What is the potential difference between the lower surface ($z=d$) of the upper plane and the upper surface ($z=-d$) of the lower plane?

\[ 1^{st} \text{ consider one of the planes} \]

**Gauss Law to set $E$ inside:**

\[ 2\pi r E = \frac{2\pi r \rho}{\varepsilon_0} \quad E_i = \frac{z' \rho}{\varepsilon_0} \]

**Gauss Law to set $E$ outside**

\[ 2\pi r E = \frac{2\pi r \rho}{\varepsilon_0} \quad E_o = \frac{t \rho}{2\varepsilon_0} \]

a) \[ E = 0 \quad \text{from top + bottom planes cancel} \]

b) \[ E = E_{\text{bottom}} + E_{\text{top}} = \frac{t \rho}{2\varepsilon_0} \hat{j} + \frac{z' \rho}{\varepsilon_0} \hat{j} ; \]

\[ E = \left( \frac{t \rho}{2\varepsilon_0} + \frac{\rho}{\varepsilon_0} \left[ z - d - \frac{t}{2} \right] \right) \hat{j} \]

\[ z = d + \frac{t}{2} + z' \]

\[ z' = z - d - \frac{t}{2} \]

*Note: This is too clever for its own good.*
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Fundamental Laws and Definitions:

Coulomb’s Law (22-4): \( \vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \)

Electric Field (23-1): \( \vec{E} \equiv \frac{\vec{F}}{q_0} \)

Gauss’ Law (24-6): \( \varepsilon_0 \Phi = q_{\text{enc}} \)

Flux through a Gaussian Surface (24-4): \( \Phi \equiv \oint \vec{E} \cdot d\vec{A} \)

Electric Potential (25-7,8): \( \Delta V = V_f - V_i = -\frac{W}{q} \)

\( V = -\frac{W}{q} \)

Electric Field/Potential Relations (25-18, 41): \( \Delta V = V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} \quad E_x = -\frac{\partial V}{\partial x}; E_y = -\frac{\partial V}{\partial y}; E_z = -\frac{\partial V}{\partial z} \)

Special Cases/Derived Relations:

Dipole Moment (~23-8) \( \vec{p} = q \vec{d} \) \quad torque (23-34) \( \vec{\tau} = \vec{p} \times \vec{E} \) \quad potential energy (23-38) \( U = -\vec{p} \cdot \vec{E} \)

Electric Field/Potential due to point charge(23-3; 25-26):

\( \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r} \quad V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \)

Useful numerical quantities:

\( \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 \) \quad \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2. \)