While this problem has a simple answer (0) which can be gotten from symmetry arguments, I did not intend this to be this 'clever' a problem. I also did not intend the solution based on the integrals below.

The next page has the problem I intended.

Consider the box shown in the figure. If there exists an electric field given by

\[ \mathbf{E} = E_0 x^2 \mathbf{j}, \quad E_0 = 25 \text{ N/C}, \]

how much charge is within the box?

\[ q_{\text{enc}} = \Phi E_0 = E_0 \oint \mathbf{E} \cdot d\mathbf{A} \]

ways to solve:

method 1: notice \( \Phi = 0 \) because all field lines that go in also come out, so \( q_{\text{enc}} = 0 \)

method 2:

notice that \( \mathbf{E} \cdot d\mathbf{A} = 0 \) for all faces except 1 + 2, as the normal to the face is \( \perp \) to \( \mathbf{E} \), so dot product = 0 need \( q_1 + q_2 \)

\[ q_1 = E_0 \oint \mathbf{E} \cdot d\mathbf{A}_1 = E_0 \oint \mathbf{E} dA \cos \theta, \quad \theta = 180^\circ \text{ as } \hat{n}_1 \text{ is antiparallel to } \mathbf{E} \]

\[ = -E_0 \oint dA = -E_0 \int_0^1 dx \int_0^2 dz = E_0 x^2 \]

\[ = -E_0 \int_0^1 x^2 \mathbf{E}_0 = -E_0 \frac{x^3}{3} \bigg|_0^1 = -E_0 \frac{E_0}{3} \]

\[ q_2 = E_0 \oint \mathbf{E} \cdot d\mathbf{A}_2 = E_0 \oint \mathbf{E} dA \cos \theta, \quad \theta = 0^\circ \text{ as } \hat{n}_2 \parallel \mathbf{E} \]

\[ = E_0 \oint E_0 x^2 dA = E_0 E_0 \int_0^1 dx \int_0^2 dz = E_0 \frac{x^3}{3} \bigg|_0^1 \]

\[ q_{\text{total}} = q_1 + q_2 = -E_0 \frac{E_0}{3} + E_0 E_0 \frac{E_0}{3} = 0 \quad = E_0 \frac{E_0}{3} \]
\[ \vec{E} = E_0 y^2 \hat{j} \quad E_0 = 25 \text{N/C} \]

For this field there is no flux through any of the four 1m x 2m sides because

\[ \vec{E} \cdot \vec{A} = EA \cos 90^\circ = 0 \]

In the x-z plane, y = 0 \rightarrow \vec{E} = 0 + there is no flux through this side either.

Finally, for the x-z plane at y = 2m

\[ \vec{E} = 25 \text{N/C} \ (2m)^2 \hat{j} \]

\[ \Phi = \int_{\text{end}} (25 \text{N/C} \ (2m)^2 \hat{j}) \cdot 1 \text{m}^2 \hat{j} = 100 \text{N} \text{m} \text{C} \]

Gauss' Law

\[ E_0 \Phi = \Phi_{\text{enc}} = (8.85 \times 10^{-12}) 100 \text{ C} \]

\[ \Phi_{\text{enc}} = 8.85 \times 10^{-10} \text{C} \]