There are five pages to this midterm (plus an equation sheet). It is important that you write your name on each page and the name of your recitation instructor on the first page. Each name is worth one point.

Be sure to include the proper units in your answers.

I. Short Problems. Below are two different circuits. For each there are several questions for which quick solutions exist.

Use the circuit on the right for the first three questions.

1. (5 pts) What is the total charge from the battery required to charge the capacitors?
   \[ Q = \varepsilon \cdot V = \varepsilon \cdot 16\,\text{V} = 128\,\mu\text{C} \]

2. (6 pts) What is the electric potential across the 8 \( \mu \text{F} \) capacitor?
   \[ 8\,\text{V} \]

3. (6 pts) What is the charge on the 4 \( \mu \text{F} \) capacitor farthest to the right?
   \[ Q = \varepsilon \cdot V = 4\,\mu\text{F} \cdot 8\,\text{V} = 32\,\mu\text{C} \]
Use the circuit on the right for the following three questions.

4. (5 pts) What is the voltage across the 4 Ω resistor?

\[ \frac{5}{5R} = I_A = I_4 \]

5. (6 pts) What is the power dissipated by the 2 Ω resistor?

\[ |I_2| = \frac{15V - 10V}{2R + 5\Omega} = 1A \quad P_{2R} = |I|^2R = (1A)^2 \cdot 2\Omega = 2W \]

6. (6 pts) What is the current flow through the 10 V battery?

1 A flows in both loops, but they flow in opposite directions in the 10 V battery

\[ \Rightarrow I_{10V} = 0 \]
II. Problems (20 points each).

1. The capacitor in the figure is initially charged to $2 \times 10^4$ C. If at $t = 0$ the switch is closed

(a) What is the initial current flow through the resistor?

$$\text{At } t = 0, \quad V_c = \frac{2 \times 10^{-4} \text{C}}{2 \times 10^{-6} \text{F}} = 100 \text{V} = V_R$$

$$i = \frac{V_R}{R} = \frac{100 \text{V}}{10^6 \Omega} = 10^{-4} \text{A}$$

(b) What is the voltage across the resistor at $t = \infty$?

$$\text{At } t = \infty, \quad i = 0 \Rightarrow V_R = 0$$

(c) When the capacitor is discharged to 50% of its initial charge, how much heat energy has been produced by the resistor?

**Initial energy in capacitor**:

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{(2 \times 10^{-4})^2}{2 \times 10^{-6}} \approx 2 \times 10^{-2} \text{J}$$

**At 50% charge**:

$$U = \frac{1}{2} \cdot \left(\frac{10^{-4} \text{C}}{2 \times 10^{-6} \text{F}}\right)^2 = \frac{1}{4} \cdot 10^{-2} \text{J}$$

**Must conserve energy**

$$\Longrightarrow U_{\text{heat}} = 10^{-2} \text{J} - \frac{1}{4} \cdot 10^{-2} \text{J} = \frac{3}{4} \cdot 10^{-2} \text{J}$$
2. A spherical capacitor made of thin shells has inner and outer radii "a" and "b" as shown in the figure. Calculate its capacitance.

\[
\begin{align*}
V_a &= \frac{1}{4\pi \varepsilon_0} \left[ \frac{+\varepsilon}{a} - \frac{\varepsilon}{b} \right] \\
V_b &= 0 \\
\Delta V &= \frac{\varepsilon}{4\pi \varepsilon_0} \left( \frac{b-a}{a^2} \right) \\
C &= \frac{\varepsilon}{\Delta V} = \frac{\varepsilon}{\frac{\varepsilon}{4\pi \varepsilon_0} \left( \frac{b-a}{a^2} \right)} = 4\pi \varepsilon_0 \frac{ab}{b-a}
\end{align*}
\]
3. Consider the circuit shown in the figure.

(a) What is the current that flows in each resistor?

\[
\begin{align*}
6V - 12R \cdot i_1 - 4R \cdot i_2 &= 0 \\
4R \cdot i_3 - 6R \cdot i_2 - 3V &= 0 \\
i_1 &= i_2 + i_3
\end{align*}
\]

\[
\begin{align*}
6V - 12R \cdot (i_1 + i_3) - 4R \cdot i_3 &= 0 \\
6V - 12R \cdot i_2 - 12R \cdot i_3 &= 0 \\
3V - 6R \cdot i_2 - 8R \cdot i_3 &= 0 \\
4R \cdot i_3 - (3V - 2R \cdot i_2) - 3V &= 0
\end{align*}
\]

\[
i_3 = 0.5A
\]

\[
V_3A = i_2 + 0.5A
\]

\[
i_2 = V_3A - 0.5A = -0.167A
\]

(b) How much power is supplied by the 6 V battery?

\[
P = 6V \cdot i_1 = 2W
\]