There are five pages to this midterm (plus an equation sheet). It is important that you write your name on each page and the name of your recitation instructor on the first page. Each name is worth one point.

Be sure to include the proper units in your answers.

I. Short Problems. Below are two different circuits. For each there are several questions for which quick solutions exist.

Use the circuit on the right for the first three questions.

1. (5 pts) What is the current through the 15 Ω resistor?

\[ I_1 = \frac{10\text{V} - 5\text{V}}{15\Omega} = 0.33 \text{ A} \]

2. (6 pts) How much power is dissipated by the 20 Ω resistor?

\[ P = \frac{(5\text{V})^2}{20\Omega} = 1.25 \text{ W} \]

3. (6 pts) How much power (including sign: "+" for providing power to the rest of the circuit, ",-" for receiving power from the rest of the circuit) does the 5 V battery provide?

\[ I_1 = 0.33 \text{A} \text{ to the right} \]
\[ I_2 = 0.25 \text{A} \text{ downward} \]

Kirchhoff \[ I \Rightarrow I_3 = I_1 - I_2 = 0.083 \text{A} \text{ (opposite to battery's e)} \]

\[ P = (-0.083\text{A}) \cdot 5\text{V} = -0.417 \text{ W} \]
Use the circuit on the right for the following three questions.

![Circuit Diagram]

4. (5 pts) At $t = 0$ what is the power being dissipated by the resistor?

$$ P = \frac{(10 \text{V})^2}{2 \cdot 10^6 \Omega} = 5 \cdot 10^{-5} \text{W} $$

5. (6 pts) At $t = \infty$ what is the energy stored in the capacitor?

$$ E = \frac{1}{2} C U^2 = \frac{1}{2} \cdot 3 \cdot 10^{-6} \text{F} \cdot (10 \text{V})^2 = 1.5 \cdot 10^{-4} \text{J} $$

6. (6 pts) When the capacitor is charged to 1/3 of its final charge, what is the voltage across the resistor?

$$ Q_{\text{final}} = C \cdot U $$

when charged 1/3 only, voltage across capacitor:

$$ \frac{1}{3} Q_{\text{final}} = C \cdot U_v = \frac{U}{3} $$

$$ U_v = \frac{U}{3} $$

voltage across resistor:

$$ U_R = U - \frac{U}{3} = \frac{2}{3} U = \frac{2}{3} \cdot 10 \text{V} = 6.67 \text{V} $$
II. Problems (20 points each).

1. Consider the circuit shown in the diagram.

![Circuit Diagram]

a. If all of the capacitors were initially uncharged, how much energy was supplied by the 16 V battery during the charging of the circuit?

Equivalent capacitance seen by 16 V battery:

\[
C = \frac{1}{\frac{1}{8 \mu F} + \frac{1}{4 \mu F + 4 \mu F}} = 4 \mu F
\]

Charge delivered:

\[
Q = C \cdot U = 4 \mu F \cdot 16 V = 64 \mu C
\]
\[
E = Q \cdot U = 64 \mu C \cdot 16 V = 1.02 \text{ mJ}
\]

b. What is the voltage across the left 4 \( \mu F \) capacitor? (same as across right 4 \( \mu F \) capacitor)

Voltage across 8 \( \mu F \) capacitor:

\[
U_1 = \frac{Q}{C_1} = \frac{64 \mu C}{8 \mu F} = 8 V
\]

Leftover:

\[
U_2 = U - U_1 = 16 V - 8 V = 8 V
\]
2. A special parallel plate capacitor is made by taking two large, thin flat pieces of metal, each of surface area "A" and placing between them a thick piece of flat metal of thickness "t" and also of area "A". On both the top and bottom, the middle piece of metal is separated from the thin pieces by a distance "d".

Calculate its capacitance.

\[ C_1 = C_2 = \frac{\varepsilon_0 A}{d} \]

\[ C_{\text{equivalent}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{\varepsilon_0 A}{2d} \]

You can also think of it as:
3. Consider the circuit shown in the figure.

Write down the three loop equations and one junction equation which can be solved to find the current flow in this circuit. For your equations assume the direction of current flow indicated by the arrows in the diagram so that a positive current would represent current flow in the direction of the arrow.

My convention: "positive" loop direction = clockwise

upper left loop: \(18 \Omega \cdot I_1 - 12 \Omega \cdot I_2 = 9 \text{V} \Rightarrow 6I_1 - 4I_2 = 3I_1\)

bottom left loop: \(12 \Omega \cdot I_2 + 4 \Omega \cdot I_4 = 6 \text{V} \Rightarrow 6I_2 + 2I_4 = 3I_2\)

bottom right loop: \(6 \Omega \cdot I_3 - 4 \Omega \cdot I_4 = -3 \text{V} \Rightarrow 6I_3 - 4I_4 = -3I_3\)

(Beside these there are still 3 other possible loops!)

middle junction: \(I_1 + I_2 - I_3 - I_4 = 0\)

solution: \(I_1 = 0.65 \text{A} \quad I_2 = 0.225 \text{A} \quad I_3 = 0.05 \text{A} \quad I_4 = 0.825 \text{A}\)