(2) Given: $q = 70 \text{ pC}$  
$V = 20 \text{ V}$  

A saw and a wrench as capacitor "plates"

Find: $C$

$C = \frac{q}{V} = \frac{70 \text{ pC}}{20 \text{ V}} = \frac{7}{20} \text{ pF}$  

$\sqrt{}$

- $C \neq \varepsilon_0 \frac{A}{L}$ for this problem, but we still have a capacitance.
- $\frac{7}{20} \text{ pF}$ is a small capacitance, but it is large enough to be important in some circuits.

Given: $q = 200 \text{ pC}$  

Find: $C, V$

$C$ doesn't change. The capacitance depends only on the geometry of the capacitor. The increased charge does result in a larger potential difference.

$V = \frac{q}{C} = \frac{200 \text{ pC}}{3.5 \text{ pF}} = 57 \text{ V}$
16) Given: \( V = 10\, \text{V} \)
\( C = 10\, \mu\text{F} \) for each capacitor

Find: \( \theta_1, \theta_2 \)

\( \theta_1 = C, V = C, V = (10\, \mu\text{F})(10\, \text{V}) = 100\, \mu\text{C} \)

We'll need equivalent circuits to set \( \theta_2 \)

\( C_{23} = \frac{C_2 \cdot C_3}{C_2 + C_3} = 5\, \mu\text{F} \)

\( C_{234} = C_{23} + C_4 = 15\, \mu\text{F} \)

\( C_{2345} = \frac{C_{234} \cdot C_5}{C_{234} + C_5} = 6\, \mu\text{F} \)

We can use any interaction from any of these circuits to solve our problem.

2) \( \theta_2 = \theta_3 = \theta_{23} \)

3) \( \theta_{23} = C_{23} \cdot V_{23} \)

How do we set \( V_{23} \)?

Well, look at the 3rd circuit: \( V_{23} = V_{234} \)

But: \( V = V_5 + V_{234} \)

5) \( V_{23} = V_{234} = V - V_5 \)
Now we need \( V_5 \):
\[
V_5 = \frac{Q_5}{C_5}
\]

But \( C_5 \) and \( C_{234} \) are in series so,
\[
Q_5 = Q_{234} = Q_{2345}
\]

\( V_5 = \frac{Q_{2345}}{C_5} \)

\( C_{2345} \) \( V = (6 \times 10^{-6} \text{ F})(10 \text{ V}) = 6 \times 10^{-5} \text{ C} \)

From (4):
\[
V_5 = \frac{6 \times 10^{-5} \text{ C}}{10 \times 10^{-6} \text{ F}} = 6 \text{ V}
\]

From (6):
\[
V_{23} = 10 \text{ V} - 6 \text{ V} = 4 \text{ V}
\]

From (4):
\[
Q_{23} = (5 \times 10^{-6} \text{ F})(4 \text{ V}) = 2 \times 10^{-5} \text{ C}
\]

From (6):
\[
Q_2 = 2 \times 10^{-5} \text{ C}
\]

\( = 20 \mu \text{ C} \)
Find: \( C_e \)  \( Q_e \)  \( V_1 \)  \( Q_1 \)  \( V_2 \)  \( Q_2 \)  \( V_3 \)  \( Q_3 \)

\[
\frac{1}{C_{y_3}} = \frac{1}{C_6} + \frac{1}{C_3} \\
\Rightarrow \quad C_{y_3} = 2.0 \mu F
\]

\( C_x = C_e \times C_{y_3} + C_2 \\
\Rightarrow \quad C_x = 6.0 \mu F
\]

\( c_{xy} = C_e \times C_1 \\
\Rightarrow \quad C_e = 3.0 \mu F \checkmark
\]

\[ Q_e = C_e V \]
\[ e = 60 \mu C \checkmark \]

\( Q_1 = \frac{Q_e}{2} \Rightarrow Q_1 = 30 \mu C \checkmark \]

\[ V_1 = \frac{Q_1}{C_1} = 10 V \]
\( V_1 = V_2 = \frac{Q_2}{C_x} = \frac{Q_6}{C_x} = 10V \) 

\( Q_2 = C_x V_2 = 20 \mu C \) 

\( Q_3 = Q_6 = C_6 V_3 = C_6 V_2 = 20 \mu C \) 

\( V_3 = \frac{1}{2} V_6 \) because \( C_3 = C_6 \)

\[ = \frac{1}{2} V_2 \]
\[ = 5V \] 

There are other ways to find each of these parameters. A variety of approaches are shown here. Some are the most efficient, some are not.
23) Given: \( U = 10 \text{ kW} \cdot \text{h} \)
\( V = 1000 \text{ V} \)

Find: \( C \)

\[ U = \frac{1}{2} CV^2 \]

\[ C = \frac{2U}{V^2} \]

The units for \( U \) might look odd, but a \((\text{kW} \cdot \text{h})\) is a unit of energy. The idea is this:

\[ \text{energy} = \left( \text{rate \ energy \ is \ delivered} \right) \times \left( \text{time} \right) \]

\[ = \text{power} \times \text{time} \]

and the standard unit of power is the Watt \((W)\). We just need to convert to Joules \((J)\):

\[ U = \left( 10 \cdot 10^3 \text{ W} \cdot \text{hr} \right) \left( \frac{3600 \text{ seconds}}{1 \text{ hr}} \right) \]

\[ = 3.6 \cdot 10^7 \text{ J} \]

\[ C = \frac{2 \left( 3.6 \cdot 10^7 \text{ J} \right)}{(1000 \text{ V})^2} = 72 \text{ F} \]
55) Given: \( V = 12 \text{ V} \)
\( d = 1.0 \text{ mm} \)
\( C = 10 \text{ pF} \)

Find: (a) \( A \)

(b) \( C' = \text{capacitance for a spacing of} \ d = d - 0.1 \text{ mm} \)

(c) \( \Delta V = V' - V = \text{potential change going from} \ d \ \text{ to} \ d' \) with the charge held constant.

For a parallel plate capacitor:

\[
C = \frac{\varepsilon_0 A}{d} \Rightarrow A = \frac{C d}{\varepsilon_0} = \frac{(1.1 \times 10^{-3}) (10.10^{\text{F}})}{(8.85 \times 10^{-12} \text{ F/m})} = 1.1 \times 10^{-3} \text{ m}^2 \sqrt{\text{ new units for} \ \varepsilon_0!}
\]

\[
C' = \frac{\varepsilon_0 A}{d'} = \frac{(8.85 \times 10^{-12} \text{ F/m})(1.1 \times 10^{-3})}{(1.0 \times 10^{-3} \text{ m} - 0.1 \times 10^{-3} \text{ m})} = 11.1 \text{ pF} \sqrt{\text{ new units for} \ \varepsilon_0!}
\]

(e) The only way to keep the charge constant is to disconnect the battery before moving the plates — then there is no where for the charge to go.

Before move: \( q = CV \)  \( \Rightarrow \delta' = \delta \) so \( CV = C'V' \)

After move: \( q' = C'V' \)
\( V' = V \frac{C}{C'} = 12 \text{ V} \frac{(10 \text{ pF})}{(11.1 \text{ pF})} = 10.8 \text{ V} \)
\( \Delta V = -1.2 \text{ V} \sqrt{\text{ new units for} \ \varepsilon_0!} \)

You could rattle a microphone using this technique (although it won’t work very well). Charge up the capacitor in the problem and then measure the voltage across it as someone talks into it. The sound will shake the plates, changing the voltage. The trouble is, the charge will drain off — no capacitor is perfect. A better approach is to leave the battery connected. Then,
Given: \( C_1 = 6.00 \ \mu F \)
\( C_2 = 4.00 \ \mu F \)
\( V = 200 \ \text{V} \)

Find: \( C_e \)
\( \varphi_1 \), \( \varphi_2 \)
\( V_1 \), \( V_2 \)

\[
\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2}
\]

\[
C_e = \frac{C_1 \cdot C_2}{C_1 + C_2} = \frac{(6.00 \ \mu F)(4.00 \ \mu F)}{6.00 \ \mu F + 4.00 \ \mu F} = 2.40 \ \mu F \ \checkmark
\]

\[
\varphi_1 = \varphi_2 = \varphi_e = C_e V = (2.40 \ \mu F)(200 \ \text{V}) = 480 \ \times 10^{-6} \ \text{C} \ \checkmark
\]

\[
V_1 = \frac{\varphi_1}{C_1} = \frac{480 \times 10^{-6} \ \text{C}}{6.00 \ \times 10^{-6} \ \text{F}} = 80 \ \text{V} \ \checkmark
\]

\[
V_2 = V - V_1 = 200 \ \text{V} - 80 \ \text{V} = 120 \ \text{V} \ \checkmark
\]

As a check, let's calculate \( V_2 \) the way I calculate \( V_1 \):

\[
V_2 = \frac{\varphi_2}{C_2} = \frac{480 \times 10^{-6} \ \text{C}}{4.00 \ \times 10^{-6} \ \text{F}} = 120 \ \text{V} \ \checkmark
\]
567) \( V = 300 \text{V} \) is applied to \( C_1 = 2.0 \mu \text{F} \) and \( C_2 = 80 \mu \text{F} \) in series. What are \( \bar{\varepsilon}_1, \bar{\varepsilon}_2, V_1, V_2 \)?

\[
\bar{\varepsilon}_1 = \bar{\varepsilon}_2 = \bar{\varepsilon}_{12} = \text{charge on equivalent capacitance}
\]

\[
\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} \\
C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(2.0)(80)}{2.0 + 80} \mu \text{F} = 1.6 \mu \text{F}
\]

\[
\bar{\varepsilon}_{12} = C_{12} V = (1.6 \times 10^{-6} \text{ F})(300 \text{ V}) = 4.8 \times 10^{-4} \text{ C} = \frac{480 \mu \text{C}}{80 \mu \text{F}} = \bar{\varepsilon}_1 = \bar{\varepsilon}_2
\]

\[
V_1 = \frac{\bar{\varepsilon}_1}{C_1} = \frac{480 \times 10^{-6} \text{ C}}{2.0 \times 10^{-6} \text{ F}} = 240 \text{ V} \checkmark
\]

\[
V_2 = \frac{\bar{\varepsilon}_2}{C_2} = \frac{480 \times 10^{-6} \text{ C}}{8.0 \times 10^{-6} \text{ F}} = 60 \text{ V} \checkmark = V - V_1 \checkmark
\]

\( C_1 \) and \( C_2 \) are connected together, positive plate to positive plate. What are the new \( \bar{\varepsilon}'s \) and \( V's \)? \( \bar{\varepsilon}_1', \bar{\varepsilon}_2', V_1', V_2' \)

\[
C_1 + C_2 \quad \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \quad \left( C_1 \text{ and } C_2 \text{ are in parallel now} \right)
\]

\[
\frac{\bar{\varepsilon}_1}{C_1} = \frac{\bar{\varepsilon}_2}{C_2}
\]

but, \( \bar{\varepsilon}_1 + \bar{\varepsilon}_2 = \bar{\varepsilon}_1 + \bar{\varepsilon}_2 \) (charge is conserved)

Let \( \bar{\varepsilon}_1 = \bar{\varepsilon}_1' + \bar{\varepsilon}_2' = 480 \mu \text{C} + 480 \mu \text{C} = 960 \mu \text{C} \)

\[
\bar{\varepsilon}_1' = \frac{\bar{\varepsilon}_1}{C_1} = 96 \text{ V} \checkmark
\]

\[
V_1' = \frac{\bar{\varepsilon}_1'}{C_1} = 96 \text{ V} \checkmark
\]

\[
V_2' = \frac{\bar{\varepsilon}_2'}{C_2} = 96 \text{ V} \checkmark
\]

\[
\bar{\varepsilon}_1' = \frac{C_1}{C_1 + C_2} \bar{\varepsilon}_T = 192 \mu \text{C} \checkmark
\]

\[
V_1' = \frac{\bar{\varepsilon}_1'}{C_1} = 96 \text{ V} \checkmark
\]

\[
\bar{\varepsilon}_2' = \frac{C_2}{C_1 + C_2} \bar{\varepsilon}_T = 768 \mu \text{C} \checkmark
\]

\[
V_2' = \frac{\bar{\varepsilon}_2'}{C_2} = 96 \text{ V} \checkmark
\]

\[
V_1' = V_2' \checkmark
\]
Starting from the original circuit, connect \( C_1 \) and \( C_2 \), positive plate to negative plate. What are the charges, \( q_1' \) and \( q_2' \), and the voltages, \( V_1' \) and \( V_2' \), on \( C_1 \) and \( C_2 \) now?

Now we go from this:

![Original Circuit Diagram]

To this:

![Modified Circuit Diagram]

The charge \( q_1' \) cancels \( -q_2' \) because \( q_1 = q_2 \).

\[
q_1' = q_2' = 0 \text{ C} \quad \checkmark \quad V_1' = V_2' = 0 \text{ V}
\]
$V = 1/2 V$
$c_1 = c_2 = c_3 = c_4 = 7.00 \mu F$

$\xi_1 = \xi V = c_1 V = 24.0 \mu F$ \checkmark

$V = V - \frac{\xi}{c_2} = V - \frac{\xi e}{c_2} = V - \frac{V c_e}{c_2}$

$= V (1 - \frac{c_e}{c_2}) = \frac{1}{3} V = 4.00 V$ \checkmark