Q3) Given: Zero potential reference at infinity 
charged particles as shown 

Find: \( V_p = \text{potential at point } P \)

The potential reference means we can use the special formula for the potential from a point charge:

\[
V = \frac{kQ}{r}
\]

\[
V_p = \sum \frac{kQ_i}{r_i}
\]

\[
= k \left( \frac{-28}{d} + \frac{8}{d} - \frac{58}{d} + \frac{48}{d} - \frac{28}{d} - \frac{8}{d} + \frac{58}{d} - \frac{48}{d} \right) 
\]

\[
= -\frac{4kQ_0}{d} \checkmark
\]
\[ E = \frac{-dV}{dx} \] (in one dimension) or \[ E = \frac{-dV}{dx} \] for a uniform field.

Let the distance between equipotentials be \( d \) for case (1). Then, the distance between must be \( 2d \) for (2) and (3).

**Rank the field magnitudes.**

Over a distance \( 2d \), the change in potential is:

- Case (1): \( \Delta V = 60V - 20V = 40V \)
- Case (2): \( \Delta V = -140V - (-120) = 20V \)
- Case (3): \( \Delta V = -30V - (-10) = -20V \)

Case 1 has the largest change in potential. Cases 2 and 3 have the same change in magnitude.

\[ |E_1| > |E_2| = |E_3| \checkmark \]

**Which case has a downward directed field?**

Potential decreases down a field line so we are looking for a filling potential as we go down the page: case 3 \( \checkmark \)
4) Given: Field does work

\[ W_{AB} = 3.94 \times 10^{-19} \text{J} \]

on an electron that moves from A to B along a field line.

Find: \( V_B - V_A \), \( V_C - V_A \), \( V_C - V_B \)

\[ V_B - V_A = -\frac{W_{AB}}{q} = -\frac{3.94 \times 10^{-19}}{-1.6 \times 10^{-19} \text{C}} = 2.46 \text{V} \]

(This is like Q29. The potential increased as we went up a field line.)

Now \( V_B = V_C \) because \( O \) and \( C \) lie on the same equipotential.

\[ V_C - V_A = V_B - V_A = 2.46 \text{V} \]

\[ V_C - V_B = 0 \]

\[ V_C - V_B = 0 \]
5) Given: infinite nonconductive sheet
\( \sigma = 0.10 \cdot 10^{-6} \ \text{s/m} \)
\( \Delta V = 50 \text{V} \)

Find: \( \Delta x \)

We have a uniform field, so

\[ E = \frac{\Delta V}{\Delta x} \]

\[ \Delta x = \frac{\Delta V}{E} \]

but \( E = \frac{\sigma}{2\varepsilon_0} \) for a nonconductive sheet

\[ \Delta x = \frac{\Delta V}{\frac{\sigma}{2\varepsilon_0}} = \frac{2\varepsilon_0 \Delta V}{\sigma} \]

\[ = \frac{2 \left( 8.85 \cdot 10^{-12} \ \text{C}^2/\text{Nm}^2 \right) (50 \text{V})}{(0.10 \cdot 10^{-6} \ \text{s/m})} \]

\[ = 8.8 \cdot 10^{-3} \ \text{m} \]
8) Given: \( E(r) = \frac{\Phi}{4\pi \varepsilon_0 R^3} \)
\( V(r=0) = 0 \)

Find: \( V(r) \)

\[ V(R) - V(0) \]

\[ V_s - V_e = \int_s^e \vec{E} \cdot d\vec{s} \]

By symmetry, \( \vec{E} \) is radial. Integrate along a radial vector from the center (initial position) to a distance \( R \) (final position):

\[ V(R) - V(R=0) = \int_0^R \vec{E} \cdot d\vec{r} \]

\[ V(R) = \frac{\Phi}{4\pi \varepsilon_0} \frac{1}{R^3} \int_0^R r \, dr = \frac{\Phi}{4\pi \varepsilon_0} \frac{1}{R^3} \left[ r^2 \right]_0^R = \frac{\Phi}{8\pi \varepsilon_0} \frac{R^2}{R^3} \checkmark \]

\[ V(0) - V(R) = \frac{\Phi}{8\pi \varepsilon_0} \frac{1}{R} \checkmark \]

If \( \Phi > 0 \), then \( V(0) > V(R) \) \( \checkmark \)
21) Given: zero potential reference at infinity
 Charges as shown
 Find: potential at point P

The potential reference means we can use the special formula for the potential from a point charge:

\[ V = \frac{kq}{r} \]

\[ V_p = \sum \frac{V}{\text{charges}} \]

\[ = K \left( \frac{5.08}{d} + \frac{5.08}{d} - \frac{5.08}{d} - \frac{5.08}{2d} \right) \]

\[ = K \frac{5.08}{2d} = 2.5 \frac{kq}{d} \checkmark \]
Given: \( V(x, y) = \alpha x^2 + 6y^2 \)

\( \alpha = 2.0 \ \text{V/m}^2 \)

\( b = -3.0 \ \text{V/m} \)

Find: \( \mathbf{E} \) at \( (x, y) = (3.0 \text{m}, 2.0 \text{m}) \)

\[
\begin{align*}
E_x &= -\frac{\partial V}{\partial x} = -2\alpha x = -2(2.0 \ \text{V/m}^2)(3.0 \text{m}) = -12 \ \text{V/m} \\
E_y &= -\frac{\partial V}{\partial y} = -2b y = -2(-3.0 \ \text{V/m})(2.0 \text{m}) = +12 \ \text{V/m}
\end{align*}
\]

\[
E = \sqrt{E_x^2 + E_y^2} = \sqrt{288 \ \text{V}^2/\text{m}^2} = 17 \ \text{V/m}
\]

\[
\tan \theta = \frac{E_y}{E_x} = -1
\]

\[
\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = 135^\circ
\]

(Note: If you evaluate \( \tan^{-1}(1) \) on your calculator, you get \( \tan^{-1}(1) = 45^\circ \), which is not the correct answer for this problem. Your calculator always provides an answer in the range \( -90^\circ < \theta < 90^\circ \).)
38) Given: \( V = 1500 \, x^2 \)
Find: \( E \) at \( x = 0.3 \, \text{cm} \).

\[
\begin{align*}
E_x &= -\frac{\partial V}{\partial x} = -3000x \\
E_y &= -\frac{\partial V}{\partial y} = 0 \\
E_z &= -\frac{\partial V}{\partial z} = 0
\end{align*}
\]

At \( x = 0.013 \, \text{m} \), then:
\[ \vec{E} = -34 \, \text{N} \, \text{m} \, \text{C} \quad \checkmark \]

**A little extra if you are curious**

What does the \( \vec{E} \) field look like? Let's use field lines.

First, \( \vec{E} \) points left, so the field lines must also.

Now, as \( x \) gets smaller, \( \vec{E} \) gets smaller until it \( x = 0 \), \( \vec{E} = 0 \).

This means the field lines must get **further apart** as you go to the left. But, \( E_x = E_z = 0 \), so they *can't* spread apart:

\[ \text{NO! We can't have a non-zero y component.} \]

The only other option is for some field lines to **suddenly disappear**, like this:

Field lines can only start or stop **on a charge**. We conclude, then, that the region between the plates is filled with charge.
Given: charges as shown
Find: \( V_A \) \( \Delta U \)

\[ \Delta U = \text{work done by you} \]

\[ \text{having } q_3 = 30 \mu C \text{ from } A \text{ to } B \]

\[ \Delta U = \text{change in total electrical energy} \]

\( c) \) \( V_A = V_A^A + V_A^B = \frac{kq_3}{r_A} + \frac{kq_2}{h} \)

\( \text{(We're given } V = 0 \text{ at } r = \infty \text{ so we can just use } U(r) = \frac{kq}{r} \) \)

\[ = \left( 8.85 \times 10^{-12} \frac{N\cdot m^2}{C^2} \right) \frac{-5.0 \times 10^2 \mu C}{0.15 m} + \left( 8.85 \times 10^{-12} \frac{N\cdot m^2}{C^2} \right) \frac{2.0 \times 10^2 \mu C}{0.05 m} = 6.0 \times 10^{-4} V \]

\( d) \) \( V_0 = V_B + V_A^B = \frac{kq_3}{h} + \frac{kq_2}{w} = -7.8 \times 10^{-5} V \)

\( e) \) The work done by you \( W_{BA} \) is the negative of the work done by the field \( W_{BA} \) since \( q_1 \) and \( r_2 \) resist the move from \( B \) to \( A \).

\[ W_{BA} = \Delta U_3 = q_3 \Delta V = q_3 (V_A - V_B) = (3.0 \times 10^{-10} C) \left[ 6.0 \times 10^{-4} V - (-7.8 \times 10^{-5} V) \right] = +7.5 J \]

\( f) \) \( \text{Work is independent of path, for electrostatic problems.} \)

\[ \Delta U_{total} = U_f - U_i = q_3 \left( \frac{kq_1}{w} + \frac{kq_2}{h} \right) - q_3 \left( \frac{kq_1}{w} + \frac{kq_2}{h} \right) \]

\[ = q_3 (V_A - V_B) = \Delta U_3 = +7.5 J \]
a) Potential increases to the left.
   For a uniform field: \( \Delta V = -E \cdot \Delta \vec{z} = -Ed \cos \theta \)

   \( \vec{z} \) is a displacement vector. For the potential to increase, we
   want \( \Delta V > 0 \) after the displacement, so we need \( \cos \theta = -1 \)
   or \( \theta = 180^\circ \): \( \vec{z} \) needs to point to the left.

   In general, the potential always increases as you
   go up a field line (That is, opposite the arrow).

b) see figure. \( V_{\text{left}} = -50 \, \text{V} \)

c) An external force \( \vec{F} \) moves an electron to the right.
   There is also a force on the electron from the
   uniform field.

   Work done by \( f = \vec{F} \cdot \Delta \vec{z} = Fd \cos 90^\circ = 0 \) \( \Rightarrow \vec{F} \)

   Work done by field \( = F_{\text{field}} \cdot \Delta \vec{z} = Fd \cos 180^\circ = -Fd < 0 \)

* I assumed \( \vec{F} \) was constant, otherwise \( W = \int \vec{F} \cdot d\vec{s} \), which is still positive
Given: \( V(x) \) as shown.

Final: \( E(x) \) for
- \( a < x < b \)
- \( b < x < c \)
- \( c < x < d \)
- etc

\[ E_x = -\frac{dV}{dx} \]

Since, in the regions of interest, \( V \) only changes at a constant rate, we can write:

\[ \frac{dV}{dx} = \frac{\Delta V}{\Delta x} \implies E_x = -\frac{\Delta V}{\Delta x} \]

**Region I**

\( a < x < b \)

\( \Delta V = V_b - V_a = 12V \)

\( \Delta x = x_b - x_a = 2m \)

\[ E = -\frac{\Delta V}{\Delta x} = -\frac{12V}{2m} = -6 \text{ V/m} \uparrow \]

**Region II**

\[ E = -\frac{0V}{2m} = 0 \text{ V/m} \uparrow \]

**Region III**

\[ E = -\frac{-(6V)}{2m} = 3 \text{ V/m} \uparrow \]

**Region II**

\[ E = -\frac{-(6V)}{2m} = 3 \text{ V/m} \uparrow \]

**Region III**

\[ E = -\frac{-(7.5V)}{0.5m} = 15 \text{ V/m} \uparrow \]

**Region II**

\[ E = -\frac{0V}{2m} = 0 \text{ V/m} \uparrow \]

**Region III**

\[ E = -\frac{-(7.5V)}{2.5m} = -3 \text{ V/m} \]

If you're curious, here's a circuit that could create the potential \( V(x) \) graphed above. We have 7 conducting plates: 11111111

Plates at \( x = a, x = c, x = b \) are grounded and so have \( V = 0 \).

Plates 6 and 7 are shorted together and have the same potential.

I used two batteries to provide the potential differences.
Given: 3 quarks as shown

Find: Potential energy of the two up quarks, P.E. of entire system.

P.E. of up quarks

\[ U_{uu} = \frac{k \cdot \frac{2}{3} e \cdot \frac{2}{3} e}{\frac{8}{3} e} \cdot \frac{\frac{8}{3} e}{\frac{8}{3} e} = \frac{\frac{8}{9} k e^2}{L} \]

\[ = \frac{4}{9} \left( 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 \right) \left( 1.60 \times 10^{-19} \text{ C} \right)^2 \]

\[ \frac{1.32 \times 10^{-15} \text{ m}}{1.32 \times 10^{-15} \text{ m}} \]

\[ = 7.8 \times 10^{14} \text{ J} \]

\[ = (7.8 \times 10^{14} \text{ J}) \left( 1.60 \times 10^{-19} \text{ eV/J} \right) = 4.8 \times 10^5 \text{ eV} \]

\[ = 480 \text{ KeV} \checkmark \]

P.E. of entire system

\[ U_{total} = U_{uu} + U_{ud} + U_{ud} = U_{uu} + 2U_{ud} \]

\[ \frac{k \left( \frac{2}{3} e \right)(\frac{2}{3} e)}{L} + 2 \frac{k \left( \frac{2}{3} e \right)(-\frac{1}{3} e)}{L} = \frac{\frac{8}{9} k e^2}{2} - \frac{\frac{8}{9} k e^2}{2} = 0 \checkmark \]