Physics 8820 **Homework 7** **Oct. 25**

(1) (Fox 8.8) Consider a beamsplitter with input ports #1 and #2 and output ports #3 and #4, as usual.

(a) Show that: $\hat{a}_3 = t\hat{a}_1 - r\hat{a}_2$ and $\hat{a}_4 = t\hat{a}_1 + r\hat{a}_2$.
(Note, I only treated the 50:50 case in lecture for the quantum HBT.)

(b) Use the various commutation relations for the output port quantities to show that:
$|r|^2 + |t|^2 = 1$ and $r^*t - rt^* = 0$. These are the relations we obtained classically at the beginning of class.

(2) (Loudon 5.7) Calculate the range of phase angles over which the noise band crosses the horizontal axis in the figure on notes page 92 top (Loudon Fig. 5.5) when $|\alpha| \gg 1$ and compare your result with the phase uncertainty: $\Delta\phi = \frac{1}{2|\alpha|} = \frac{1}{2\langle n\rangle^{1/2}}$.

(3) (Fox 7.7) A ns ruby laser operating at 693 nm emits 1 mJ pulses. What is the quantum uncertainty in the phase?

(4) (Fox 7.10) Explain why light with very strong quadrature squeezing will not exhibit amplitude squeezing, no matter how the axes of the uncertainty ellipse are chosen.

(5) (Fox 7.15) Calculate the quadrature squeezing expected for 1064 nm vacuum modes in a nonlinear crystal being used for degenerate downconversion with $\chi^{(2)} = 4 \times 10^{-12}$ m/V, n = 1.75, and L = 10 mm for a pump intensity of 2×10^{10} W/m^2.

(6) (Loudon 5.13) **(a)** Show that the squeezed vacuum state is quadrature-squeezed for all values of q, θ_s and χ that satisfy: $\cos(2\chi - \theta_s) > \tanh q$. **(b)** Show that for strong squeezing (q >> 1), the smallest phase differences that satisfy this condition are given approximately by:
$\left|\chi - \frac{1}{2}\theta_s\right| < e^{-q}$.

The range of phase angles for which squeezing occurs thus diminishes with increasing magnitude of the squeeze parameter.