Physics 8820 **Homework 6** **Oct. 18**

(1) *A problem from Prof. Lafyatis following up on his last lecture.*

The red color in auroras is the result of spontaneous emission transitions in oxygen atoms: from the lowest energy excited state, $|e\rangle \Leftrightarrow {}^1D$, to the atomic oxygen ground state, $|g\rangle \Leftrightarrow {}^3P$.

This transition is forbidden under electric dipole selection rules and occurs because of the magnetic dipole coupling between the atomic electrons and the electromagnetic field: $\hat{H}_{MD} = -\vec{\hat{\mu}} \cdot \hat{\mathbf{B}}$.

(We are using arrows for the magnetic dipole moment vector simply because we don't have bold-face Greek fonts.) Assume you are given the (time independent) magnetic dipole transition matrix element, $\vec{\mu}_{eg} = \langle e|\vec{\hat{\mu}}|g\rangle$, and the ground and excited state energies. Find the spontaneous emission rate (Einstein A coefficient) for this transition.

(2) Prove the commutation relation: $\left[cos\hat{\phi}, sin\hat{\phi}\right] = \frac{1}{2i}[\hat{a}^{\dagger}(\hat{n}+1)^{-1}\hat{a} - 1]$

(3) Show that the mean and variance of the phase of the photon number state are, respectively, $\langle\phi\rangle = \pi$ and $(\Delta\phi)^2 = \pi^2/3$, independent of n. The range of phase angles should be taken to be 0 to 2π. (Loudon 5.3)

(4) Derive the result shown at the bottom of the notes on page 94: $\langle n^2\rangle = 3\ \sinh^4 q + 2\ \sinh^2 q$. (I see the notes use a small letter 's' where it should be 'q'. The expression above is correct.)

(5) Read the Physics Today article by Serge Haroche and Dan Kleppner on Carmen (Content::Handouts). Answer the following questions. (Your answers needn't be long to be correct. Plain words are preferred over equations where possible.)

(a) On p26, what is the significance of the changes to the radiation damping rate seen by Gabrielse, et al? Note the article doesn't say what the changes were, only that there were changes, so that alone is apparently interesting.

(b) P26: In the various experiments, why was the orientation of the atomic dipole important?

(c) Assume a cavity with V = 1000 μm^3 and a D_{ef} = 3 x 10^{-29} Cm. What is Ω_{ef} for the single mode vacuum field?

(d) What would you expect for $g^{(2)}(0)$ for the one-photon maser EM field once it reaches steady-state? Explain your reasoning.

(e) P30: What is the significance of the time $(N\Gamma_o)^{-1}$?