Physics 8820 **Homework 4** **Sept. 28**

(1) Show that $g^{(2)}(0) > 1$. (Fox, Loudon)

(a) By considering the expression $(I(t_1) + I(t_2))^2$ prove Cauchy's inequality: $I(t_1)^2 + I(t_2)^2 \geq 2I(t_1)I(t_2)$.

(b) Show that: $(\sum_{i=1}^{N} I(t_i))^2 \leq N \sum_{i=1}^{N} I(t_i)$

(c) Defining the average intensity as $\langle I(t) \rangle = \frac{1}{N}\sum_{i=1}^{N} I(t_i)$ for a discrete number of measurements and $\langle I(t)^2 \rangle$ similarly, show that: $\langle I(t) \rangle^2 \leq \langle I(t)^2 \rangle$ and hence that $g^{(2)}(0) > 1$.

(2) Derive the uncertainty relation on notes p. 46: $(\Delta X)^2 (\Delta Y)^2 \geq 1/16$. Do this from first principles, using ladder operator properties (in other words, you can't just use the commutator given on that page).

(3) Derive the relation: $\mathcal{E}_R = \sum_{\vec{k}\lambda} \mathcal{E}_{\vec{k}\lambda}$ (see notes p 51 and preceeding).

(4) Show that: $\left[\hat{H}_R, \hat{\vec{A}}_T(\vec{r}, t)\right] = i\hbar \hat{\vec{E}}_T(\vec{r}, t)$

(5) What is the Lamb shift for the hydrogen 1s state?

Sorry for all the proofs, but we need these results!