

(1) Values

This being an election year, values have come to the fore. Believe it or not, I regularly encounter graduate students that do not know their constants. This is serious and has gotten more than a few students in trouble during their general exams. Further, in any area of study, there are values and orders of magnitude you should know and preferred units systems in which you should know them. Take this seriously both in classes and in your research. It will save you public embarrassment. This problem is here to make sure you have a good internal scale for light of common experience:

Enter the correct or, at least, a reasonable value for each entry.

- (a) The visible range of the EM spectrum (nm):
- (b) The visible range is rarely specified using frequencies, but a “typical” frequency is useful to know (Hz):
- (c) Like (b), but energy (eV):
- (d) Longest wavelength that can single-photon ionize hydrogen:
- (e) Wavelengths of the sodium D lines (and what transitions do these correspond to?):

Wavelength or typical wavelength for:

- (f) red pointer laser:
- (g) green pointer laser:
- (h) fiber optics communications:

(2) Wein's Displacement Law (Loudon 1.1)

- (a) Prove that the maximum value $\langle W_T(\omega) \rangle_{\max}$ of the energy density and the frequency ω_{\max} at which it occurs are related by:

$$\langle W_T(\omega) \rangle_{\max} = (\omega_{\max}^2 / \pi^2 c^3) (3k_B T - \hbar \omega_{\max})$$

(This value of the frequency is roughly: $\omega_{\max} = 2.8 k_B T / \hbar$. This is Wien's displacement Law.)

- (3) (a) Show that the total energy density of the photons in the cavity is:

$$\int_0^{\infty} d\omega \langle W_T(\omega) \rangle = \frac{\pi^2 k_B^4 T^4}{15c^3 \hbar^3}$$

- (b) Find a similar expression for the number of photons per unit volume excited in a cavity at temperature T. The cosmic EM background spectrum is that of a blackbody with a temperature of $T \approx 2.7$ K. This corresponds to about 5×10^5 photons per liter (Loudon 1.2).

- (4) Prove that the rth factorial moment of the Planck probability distribution is (Loudon 1.3):

$$\langle n(n-1)(n-2)\dots(n-r+1) \rangle = r! \langle n \rangle^r$$

The rth factorial moment is defined in the class notes. We have two expressions for the Planck probability distribution $P(n)$, one in terms of U and the other in $\langle n \rangle$, also in the class notes.