## Saturation (text section 2.8)

Let's go back to a homogeneously broadened, 2-level system in a monochromatic field so we can focus on the main effects. We start with absorption, so assume all atoms are in the lower level (such as would be the case for an unpumped medium where the energy separation between the levels is large compared to kT).

As time passes, the light will excite atoms into the upper level (absorption) and atoms will fall back into the lower level through stimulated emission and decay (spontaneous & non-radiative). The equation of motion for the upper state is:

[$dN_2/dt$ = change due to stimulated emission + change due to absorption + change due to decays]

$$\frac{dN_2}{dt} = -W(N_2 - N_1) - \frac{N_2}{\tau}$$

[W = single atom transition rate = $\sigma F$, $N_i$ = (#atoms in level i)/(unit volume),
photon flux = I/hν = #photons per unit time per unit area]

If $N_2$ becomes close $N_1$, absorption/emission will go to zero. This is saturation. Since it's the population difference we care about, let's rewrite the equation to focus on that. Define:

$$N_t = N_1 + N_2 \qquad \text{(atom density)}$$
$$\Delta N = N_1 - N_2 \qquad \text{(population difference density)}$$

Now,

$$N_2 = \tfrac{1}{2}(N_t - \Delta N)$$
$$dN_2/dt = d/dt\ \tfrac{1}{2}(N_t - \Delta N) = -\tfrac{1}{2}\, d/dt\ \Delta N$$

since $dN_t/dt = 0$. So,

$$\frac{d\Delta N}{dt} = -\Delta N\left(1/\tau + 2W\right) + N_t/\tau$$

In steady state (the relevant case for absorption of a cw laser), $d/dt\ \Delta N = 0$, and:

$$\Delta N = \frac{N_t}{1 + 2W\tau}$$

This is the steady state population difference. It is a non-thermal equilibrium value.

The steady state net power absorbed per unit volume is:

$$\frac{dP}{dV} = (\text{\# absorptions per unit time per unit volume}) * (\text{energy lost per absorption})$$

$$\frac{dP}{dV} = (W\,\Delta N) * (h\nu) = h\nu \frac{N_t W}{1 + 2W\tau}$$

Sanity check: If W = 0, dP/dV = 0. If we don't drive any transition, we don't lose any power to the medium.

You can think of $W\tau$ as the (time for a decay) / (time for a stimulated transition).
Now, suppose we have $W\tau >> 1$, say by making F large. <u>This is the case for saturation.</u>

$$\left(\frac{dP}{dV}\right)_{sat} = h\nu \frac{N_t}{2\tau}$$

The most power that can be absorbed is that given by: ½ (1 photon per atom per decay time)(# atoms/volume). If there were no decay ($\tau$ goes to infinity), there wouldn't be any absorption in steady state. Each absorption would be balanced by a stimulated emission. It's the decays that throw away energy, either by converting our light photons to fluorescence via spontaneous emission or by converting them to phonons and heat via non-radiative decays.

As promised in class, we want to keep our most important equations in terms easily used by the experimenter. We know:

$$W = \sigma F = \sigma \frac{I}{h\nu}$$

$$\Delta N = \frac{N_t}{1 + \frac{2I\sigma\tau}{h\nu}}$$

Define the saturation intensity:

$$I_{sat} = h\nu / 2\sigma\tau$$

This intensity is ½ (1 photon per decay time per cross-section). The population difference as a fraction of the total population is:

$$\frac{\Delta N}{N_t} = \frac{1}{1 + \frac{I}{I_s}}$$

For I=0, $\Delta N / N_t$ = 1 or $N_1 = N_t$ (makes sense!).

For I small, $\Delta N / N_t \approx 1 - \frac{I}{I_{sat}}$ so if we double the intensity we double the population transferred to the 2$^{nd}$ level.

For $I = I_{sat}$, $\Delta N / N_t$ = ½ or $N_1 = 3N_2$.

For I very large, $\Delta N / N_t$ goes to zero or $N_1 = N_2$.

Recall that if the $N_1 = N_2$ for a two level system, the absorption coefficient is zero.

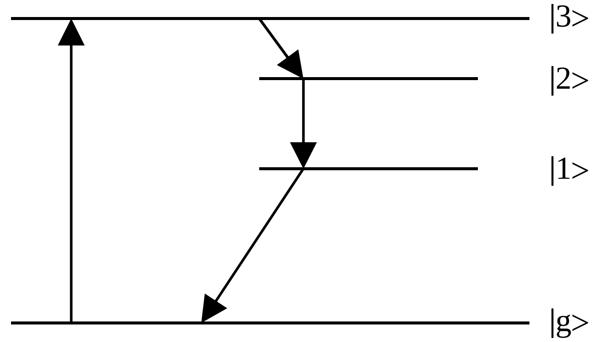
We had for an absorbing system: $F = F_o e^{-\alpha z}$, with $\alpha = \sigma(N_1 - N_2)$.

For I large, the power absorbed per unit volume become $(dP/dV)_{sat}$, derived above, *independent of I*. The medium can only absorb a few photons per atom per decay time. Any extra photons get through. Once you reach this point, the medium is said to be *bleached*.

The text goes on to consider saturation for the cases of: absorption of pulsed laser in a homogeneously broadened medium; gain of a cw and pulsed laser in a homogeneously broadened medium; and the inhomogeneous case. These are all important cases, but the main goal of this discussion was to introduce the idea of saturation – a medium cannot absorb or provide an arbitrary number of photons in an arbitrary time. The limit is characterized by a saturation parameter. The parameter will be somewhat different for each case, but is always something like 1 photon per cross-section per decay time.

## Gain Saturation in a 4-level laser system

$$\frac{dN_2}{dt} = R_p - WN_2 - \frac{N_2}{\tau}$$




At steady state: $0 = R_p - W N_2 - N_2/\tau$

$$N_2 = \frac{R_p}{W + {}^1/_\tau} = \frac{R_p \tau}{1 + W\tau}$$

Now, we have $W = \sigma I/h\upsilon$, so

$$N_2 = \frac{N_{20}}{1 + {}^I/_{I_{sat}}}$$ where $N_{20} \equiv R_p\, \tau$ and $I_{sat} = h\upsilon/\sigma\tau$.

# Amplified Spontaneous Emission (text section 2.9.2)

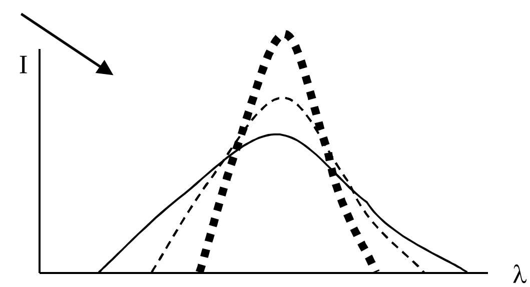
So, we have considered the interaction of light with a medium composed of two-level atoms. We have realistically treated the atom's response to light by considering homogeneous and inhomogeneous broadening and combinations of the two. In doing this we introduced the cross-section and obtained an equation of motion for the light. We have treated the case where the two levels are really two bands of levels and shown that the cross-section approach can be generalized. And just now, we considered the case where the light becomes intense.

As our first application of these ideas, we now discuss Amplified Spontaneous Emission (ASE). It is the bane of many laser systems.

The basic idea is straightforward. Suppose we have a laser medium which has been pumped somehow so that a population inversion exists: $N_2 > N_1$, assuming a two level system. The medium provides gain.

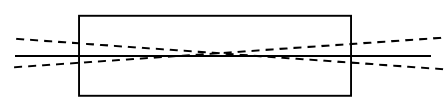
*Aside*: Although I've said "two level" system, there must be at least one other level to provide a pumping pathway. For now, we ignore it. When it comes time to treat this we will now need to modify our two-level results much. For example, in the rate equations we just add extra terms to represent the pumping process similar to what I described when I discussed non-radiative decays.

Now, imagine a photon is spontaneously emitted somewhere in the medium. As it travels out of the medium it will get amplified stealing precious energy from our laser beam. Remember, until the intensity gets near $I_{sat}$, the fluence increases exponentially with distance. Initially, this "noise" light will have a spectrum similar to the lineshape of the medium, but the center of the line have a larger gain coefficient than the wings so the spectrum starts to narrow. The noise light becomes more temporally coherent!




ASE sits in between fluorescence (unamplified spontaneously emitted light) and laser light. It has some temporal and spatial coherence and directionality.

The directionality comes about because the ASE that follows the axis of the medium (usually cylindrically shaped) sees a longer pathway and so much larger gain than off-axis ASE. Typically, only a small deviation from the laser axis is necessary for the ASE to be sufficiently misaligned that it misses the cavity mirrors or doesn't hit them properly so that the ASE never becomes part of the laser mode. This small deviation still allows it to enjoy the full length of the medium, however. The ASE is beamlike.



solid line – laser
dashed line – ASE paths

Finally, ASE exhibits a soft-threshold. We discussed previously that lasers have a threshold inversion or gain level. Below this level nothing happens, above this level the laser intensity increases abruptly. ASE behaves similarly, but the transition is less abrupt. (Sketch figure 2.26, dark curve only. Remind them that this is on a log scale and sketch what it would look like on a linear scale. Basically, there will be nothing below a value of 10 on the x-axis, and then the ASE intensity rises quickly.)

The text derives the ASE intensity in Appendix C. For the case of a system with a Gaussian line shape, it is given by:

$$I_{ASE} = I_{sat}\phi \frac{\Omega}{4\pi}\frac{(G-1)^{3/2}}{(G\ln G)^{1/2}}$$

Definitions:

$I_{sat}$ is the peak saturation intensity – the saturation intensity at the peak of the line. It is given by, $I_{sat} = h\nu_o/\sigma_{peak}\tau$ (note it differs by a factor of two from the $I_{sat}$ we used above, but is still of the form 1 photon per cross-section per decay time).

$\phi$ is the "fluorescence quantum yield". This is a parameter that characterizes the medium. It is the ratio of the number of photons emitted to the number of atoms in the upper level. A low quantum yield means that decay of the upper state is dominated by non-radiative processes. Naturally, a low yield means reduced ASE. It is given by $\phi = \tau / \tau_R$ (see equation 2.6.22). [$\tau$ is the decay time due to radiative processes (spontaneous emission) and non-radiative processes (e.g. collisional deexcitation). $\tau_R$ is the decay time due to radiative processes alone.]

$\Omega$ is the ASE emission solid angle and is determined by the gain medium geometry. For the case where the medium is a long cylinder, it is given eqn 2.9.1.

$\phi$ and $\Omega$ are given to us at some level. It is G that concerns the discussion here. G is the gain at the peak of the lineshape. It is given, of course, by $G = \exp(\sigma_{peak}\,[N_2\text{-}N_1]\,L)$ where L is the length of the medium. Note that G is the actual gain, not the gain coefficient. We can control G by choosing the pumping rate. ASE is generally not an issues unless G is well above 1.

Suppose we have a gain medium without a cavity. There are two basic modes into which it can emit light: fluorescence and ASE. The ASE will vastly dominate the emission if the gain is such that it can reach the saturation intensity before leaving the crystal. This defines the threshold. Specifically, a reasonable figure of merit is to define the threshold gain to be that for which $I_{ASE} = I_{sat}$ after a spontaneously emitted photon launched from one side of the medium reaches the other side. From the equation at the top of the page, we get:

$$G^{3/2} = \frac{4\pi}{\phi\Omega}(G\ln G)^{1/2}$$

$$G = \frac{4\pi}{\phi\Omega}(\ln G)^{1/2}$$

Even if the medium is in a laser cavity, this is still a good figure of merit. Now there is a third mode into which energy can go: the laser light. Both ASE and lasing start from a “few” spontaneously emitted photons. However, the ASE usually does a better job of filling the gain medium then the laser mode does, so the ASE has more atoms available for gain. If the ASE can sweep out the gain in a single pass, the multiple passes available to the laser mode won’t help it.

Moral of the story: Increasing the gain can actually result in **less** laser light if you exceed the threshold for ASE.

There are many ways to combat ASE. One possibility is two separate the gain medium into two pieces with some distance between them. The laser mode will still see a total path length of L. Since the ASE leaves each piece in a relatively large solid angle compared to the laser, the ASE from one piece does not all reach the second piece. This approach in used in some solid state lasers.  For dye lasers, the main approach is to keep the gain below the threshold.