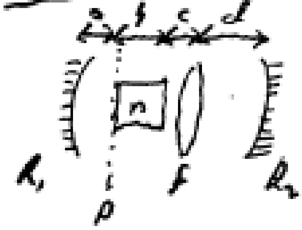
Recap:



Find $\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{RT}$ $\Rightarrow$ $-1 \le \frac{A+D}{2} \le 1$

For stability

(notes p61)

You get the laser mode at "p" for free:

$$R = \frac{2B}{D-A}$$

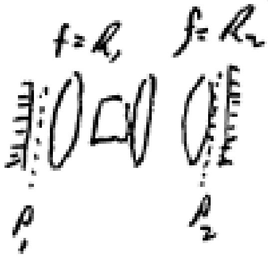
$$w = \sqrt{\frac{\lambda}{\pi n}}\, \frac{|B|^{1/2}}{\left[1-\left(\frac{A+D}{2}\right)^2\right]^{1/4}}$$

For any other q:

$$q = \frac{A q_p + B}{C q_p + D}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{RT} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{2}{R_1} & 1 \end{bmatrix}\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & b/n \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & c \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \times$$

$$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{2}{R_2} & 1 \end{bmatrix}\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}\begin{bmatrix} 1 & c \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & b/n \\ 0 & 1 \end{bmatrix}$$



Find $\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}$ going from p_1 to p_2 $\Rightarrow$ $0 \le A_1 D_1 \le 1$

for stability

(notes p67)

At the left mirror of the real cavity:

$$R = R_1$$

$$w = \sqrt{\frac{\lambda}{\pi}} \left[-\frac{B_1 D_1}{A_1 C_1}\right]^{1/4}$$

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{R_2} & 1 \end{bmatrix}\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}\begin{bmatrix} 1 & c \\ 0 & 1 \end{bmatrix} \times$$

$$\begin{bmatrix} 1 & b/n \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{1}{R_1} & 1 \end{bmatrix}$$

At the right mirror:

$$R = R_2$$

$$w = \sqrt{\frac{\lambda}{\pi}} \left[-\frac{A_1 B_1}{C_1 D_1}\right]^{1/4}$$

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2 mirror cavity $\Rightarrow$ $g_i \equiv 1 - \frac{L}{R_i}$

special case: empty cavity (notes p 64,70)

stability condition: $0 \le g_1 g_2 \le 1$

$$w_1 = \sqrt{\frac{L\lambda}{\pi}} \left[\frac{g_2}{g_1(1-g_1 g_2)}\right]^{1/4}$$

$$w_0 = \sqrt{\frac{L\lambda}{\pi}}\, \frac{\left[g_1 g_2 (1-g_1 g_2)\right]^{1/4}}{\left[g_1 + g_2 - 2 g_1 g_2\right]^{1/2}} \quad \text{(waist)}$$

$$w_2 = \sqrt{\frac{L\lambda}{\pi}} \left[\frac{g_1}{g_2(1-g_1 g_2)}\right]^{1/4}$$

$$l = L\, \frac{g_2 (1-g_1)}{g_1 + g_2 - 2 g_1 g_2} \quad \text{(waist location)}$$

measured with respect to mirror #1

Problem Set #1, Problem 5

5) (Text problem 2.5) The R_1 laser transition of ruby has, to a good approximation, a Lorentzian lineshape of FWHM width 330 GHz at room temperature. The measured peak transition cross-section is $\sigma = 2.5 \times 10^{-20}$ cm^2. The index of refraction of ruby is n = 1.76. Calculate the radiative lifetime, τ_R. (Recall that $A = 1/\tau_R$.)

Discussion:

The problem asks for the radiative lifetime, meaning the lifetime due to spontaneous emission. However, you're given the <u>actual</u> $g(\upsilon-\upsilon_o)$ lineshape which has both radiative and other processes contributing. Thus, you can't directly get the *radiative* lifetime from the lineshape width alone using $\tau_R = 1/\pi\Delta\upsilon$.

The radiative lifetime depends on the cross-section which depends on g and you're given enough information about these to work the problem. Note, however, that it is the full linewidth, containing radiative and other contributions, that determines radiative lifetime. This isn't a contradiction. A spontaneous emission event must yield a photon in the spectral line of the system and if that line is broadened by, for example, non-radiative transitions that will affect the radiative rate.

(2.5) Ruby R_1 laser transition: $\Delta\nu = 330$ GHz, Lorentzian

$\sigma_p = 2.5 \cdot 10^{-20}$ cm^2

$n = 1.76$ $\quad \lambda_0 = 694.3$ nm

$\tau = 3$ ms

$$\sigma = \frac{2\pi^2}{3n\varepsilon_0 ch} |\mu|^2 \nu\, g_t(\nu-\nu_0)$$

$$\sigma_p = \sigma(\nu=\nu_0) = \frac{2\pi^2}{3n\varepsilon_0 ch} |\mu|^2 \nu_0 \left(2\tau_c\right)$$

since we have a Lorentzian (see eqn. 2.5.10).

$$\tau_R = 1/A = \frac{3h\varepsilon_0 c^3}{16\pi^3 \nu_0^3 n |\mu|^2}$$

$$= \frac{c^2}{4\pi n^2 \nu_0^2} \frac{\tau_c}{\sigma_p} = \frac{\lambda_0^2}{4\pi n^2} \frac{\tau_c}{\sigma_p} = \frac{\lambda_0^2}{4\pi^2 n^2} \frac{1}{\Delta\nu_0 \sigma_p}$$

(using eqn 2.5.11)

$\tau_R = 4.8$ ms ✓