Homework Set #6 **Due: 2-27-12**

(1) Text 6.4

(2) Text 6.5

(3) Text 6.6 (A sunlight pumped laser! Is it possible?)

First, I = 1 kW/cm^2 is crazy. Do you see this? Try I = 1 kW/m^2 instead. [According to one reference from NASA, $I_{peak} = 1.3\ kW/m^2$.]

Finally, to calculate the focal lengths, remember that the sun is so far away that its light is plane wave-like. Think of the sun's light as consisting of an ensemble of plane waves whose angle of incidence upon the lenses varies about normal incidence over an angle of 9.3 mrad. Use this model to calculate the focal lengths. Make sure to explain your reasoning.

(4) Text 6.7

By threshold, we mean that the laser just starts to lase at the end of the lamp pulse.
Why don't we find that the inversion at threshold is $R_{pc}\,\tau$ as is the case for CW lasers?

(5) A homogeneously broadened laser transition at $\lambda = 10.6\ \mu m$ (CO_2) has the following characteristics: $A_{21} = 0.34\ s^{-1}$, $g_2 = 43$, $g_1 = 41$, $\Delta\nu = 1.0$ GHz, $\tau_1 = 0.1\ \mu s$ and $\tau_2 = 10\ \mu s$.
(τ_2 is the decay from the upper laser level to the lower and τ_1 is the decay of the lower level.)
(a) In general, are A_{21}, $\Delta\nu$, τ_1, τ_2 effectively independent quantities or can one of them be derived from the other three? **(b)** What must be the population inversion density ($N_2 - g_2/g_1\, N_1$) to obtain a gain coefficient of 0.05/cm? **(c)** What is I_{sat}? (You can simply add the reciprocal lifetimes to get an effective reciprocal lifetime for the transition here.)

(6) The Anti-Reflection (AR) Coating.

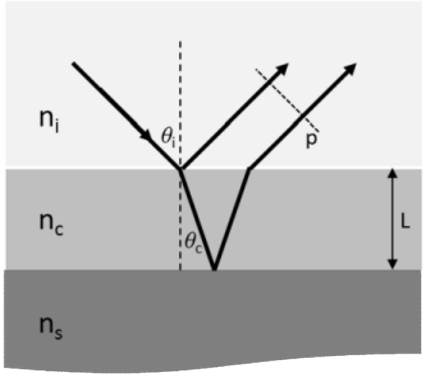
When light travels from air into glass (or vice versa) it will suffer reflection losses at the interface. It turns out that coating the glass with a thin dielectric can significantly reduce the reflection loss. Since reflection losses can be costly or even catastrophic in some applications, AR coatings are commonly used. The AR coatings are often more complex than what is treated here, but this problem captures the general idea. Also, rather than reduce the reflection, it is possible to apply coatings that enhance the reflection, for example, to make beam splitters or mirrors with close to 100% reflectivity, in which case, the coating is sometimes described as HR. Note, a flat with a beamsplitter coating on one side requires an AR coating on the opposite side. Other interesting optics include flats that reflect one wavelength and transmit another (often the second harmonic).

The index of refraction plays two roles in this kind of problem: it determines the coefficient of reflection at an interface through the Fresnel relations and it determines the phase shift suffered after traveling a given distance in a medium. Since the index varies with wavelength, it may be surprising to learn that AR and HR coatings can be constructed that operate over a large bandwidth (exceeding 100 nm). Even more surprising, mirrors can be constructed with high reflectivity and uniform net phase shift over a large bandwidth. These are helpful in short pulse applications.

A very common restriction is that the optic will only perform well for a narrow range of incident angles, typically centered around 0^o or 45^o (retroreflection, 90^o turn, respectively). There may be restrictions on polarization, as well ("s" or "p"). Thus, you might see a mirror described as something like "HR @ 45^o p, 750-850 nm."

These coatings, essentially made from various glasses, can be tough; generally *much* harder to scratch than metals. For example, even a lens tissue can scratch gold. Since these glasses have very low absorption in the optical, whereas most metals absorb ~5%, their damage thresholds are very high. Coated optics tend to be more expensive than metal optics (mirrors coated with gold, aluminum, or silver), but they are often far superior. Interestingly, even metal optics may get a dielectric coating to improve performance and durability.

A nice reference from a major supplier of coated optics is:
http://www.cvimellesgriot.com/Products/Documents/TechnicalGuide/Optical-Coatings.pdf



The figure shows light incident on a coated surface from a medium with index n_i. We restrict to $n_i < n_c < n_s$. The first two reflections from the coating are shown, and are the only ones we'll consider. Let the wavelength of the input light in vacuum be λ_o. In the following don't forget that light suffers a π phase shift when reflecting off a medium with a higher index.

(a) Show that the phase difference between the two reflections at the observing plane, p, is given by: $\Delta\varphi = (4\pi\, n_c\, L/\lambda_o)\cos\theta_c$.

(b) For 532 nm light incident on an AR coated flat at 45^o, what should the AR coating index of refraction be if the coating thickness is 300 nm?

An analysis accounting for all reflections (preferably by introducing fields and matching boundary conditions shows that a good AR coating can be made by setting $L = \lambda_o/4$ and $n_c = n_s^{1/2}$. Finding materials with specific indices of refraction is impossible, so some compromise is made.