Homework Set #3 **Due: 2-1-10**

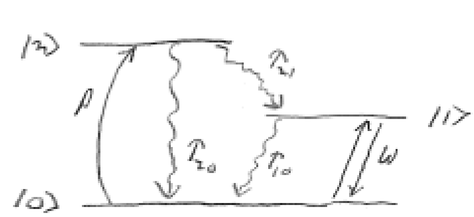
1) The spontaneous emission profile of a transition has an unusual lineshape: the curve is an isosceles triangle with a base width of 3.0 GHz. The transition states are 2.0 eV apart and the A coefficient for this transition is 6.0 MHz. Take the index of refraction to be unity. What is the cross-section for stimulated emission at resonance?

2) A three level system (states #0,#1,#2 in ascending order in energy) is pumped from |0> to |2> at rate P by some unspecified mechanism, perhaps an electrical discharge. Consider all decay time-constants τ_{ij} and cross-section σ_{10} given. Let F be the photon flux coupling states |0> and |1>. Assume that the transition frequencies for the other possible transitions are so different from that for |0> and |1> that F does not drive these transitions.

(a) Write population rate equations for this system: dN_i/dt.

(b) Verify $dN_{total}/dt = 0$.

(c) Assume that the groundstate is not significantly depleted, τ_{20} is very large and stimulated emission/absorption can be neglected. Use $P = 10^{20}$ cm^{-3} s^{-1}, $\tau_{21} = 1.0$ μs, and $\tau_{10} = 2.0$ μs. What are the steady state populations of |1> and |2>?

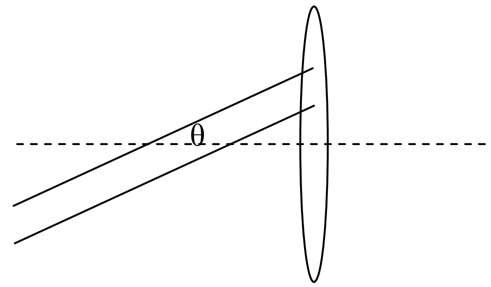


3) For a two-level, homogeneously broadened system in a CW field, we found that: $\frac{dN_2}{dt} = -W(N_2 - N_1) - \frac{N_2}{\tau}$. Assume that the field intensity, I, can be taken to be a constant, such as would be the case if the medium is very thin. Numerically integrate this equation in time to find $N_2(t)$ for the upper level for the cases $I = 0.10\, I_{sat}$, $I = I_{sat}$ and $I = 3I_{sat}$. You must integrate long enough to see the correct steady state behavior. Your answer should be in the form of one or more well labeled graphs and a clear description of your integration method. What is the fractional error in the steady state value? Note that you do not need to integrate the corresponding coupled equation for N_1.

Parameters: The Nd:YAG laser transition isn't a two-level system, but let's use its cross-section, lifetime and resonance frequency to get a feel for what these numbers mean (see Table 2.2 in the text). For initial conditions, take $N_1(0) = 10^{20}$ cm^{-3} and $N_2(0) = 0$.

Mathematica can numerically integrate this equation and is probably the best tool for this assignment. Matlab is also good. However, you are free to use any package at your disposal. Note there will be later assignments that are more complicated. A "quick and dirty" approach for a related problem is described on the next page and an Excel example file that illustrates this approach is provided as well. Although quick and dirty numerical approaches tend to actually be slow and dirty because they don't work well, please read the following if you are new to numerical integration to learn a little about what is involved.

4) **(a)** Two parallel rays making an angle θ with respect to the horizontal are incident upon a thin lens with focal length f, as shown. Take advantage of matrix methods to find the r and z where they converge. Take z=0 to be at the lens. **(b)** Five collimated beams are incident on a 100 mm focal length lens. (As we'll see, the word "collimated" has a fuzzy meaning, but generally means the beam diameter isn't changing as the beam propagates. In other words, the beam is neither focusing or defocusing.) Their angles with respect to the horizontal are -20^o, -10^o, 0^o, 10^o, 20^o. Assuming the beams can be accurately represented as a bundle of parallel rays, what would you observe on a card held at the image plane? Be specific. (This result is important for the design of spectrometers and some kinds of image analysis.)



Quick and dirty integration:

Consider the equation $dN_2/dt = -1/\tau\ N_2$. Let $N_2(0) = 10^{10}$ cm^{-3} and $\tau = 2.0$ μs. Although we know the solution is $N_2(t) = N_2(0)\ e^{-t/\tau}$, let's try solving it numerically. We can represent the solution using two arrays $N_2(i)$ and $t(i)$ for the population and the time where i is an integer with $i = 1, 2, \ldots, i_{max}$. In other words, we are only going to try to determine N_2 at successive discrete values of time so, for example, at time t(6) the population is $N_2(6)$. So long as the time intervals are small compared to the characteristic time or times of the problem, this can actually work.

Let's guess that $\Delta t \equiv t(i+1) - t(i) = 0.05\ \tau$ is a good starting point for the time between evaluations - the decay time should be well resolved. Let's also guess that integrating out to $t_{max} = 4\tau$ is sufficient. One algorithm that might work is the following:

Initialize: $N_2(1) = 10^{10}$ cm^{-3} and t(1) = 0 s, i = 1.

Evaluate the rate at time t(i): $R \equiv dN_2/dt = -1/\tau\ N_2(i)$
Evaluate the population at $t(i+1) = t(i) + \Delta t$: $N_2(i+1) = N_2(i) + R\Delta t$.
i = i+1
Repeat until $i = i_{max} = t_{max}/\Delta t$

After trying your first integration you should generally try a different value of Δt. If the approximation is to be valid, the result should be independent of Δt over some reasonable range. Also, you can usually tell by inspection if your initial choice for t_{max} is correct and adjust accordingly.

Although this approach can work, in general it is a <u>disaster</u> because the error scales with Δt. There are methods whose error scales as $(\Delta t)^4$ and that permit an automatically adjustable time-step. In addition, the method given above is not robust with respect to numerical error that arises from the way computers represent numbers. You can find yourself in a situation where making the time step smaller *decreases* the accuracy. For the needs of this course and the equations we are going to encounter, the results using this "quick and dirty" approach will be acceptable. I strongly encourage you to use this assignment to find an approach that will assist you in your research by using a standard software package. Having to integrate equations of motion is a common task.