Physics 570 **Midterm** **Schumacher**

April 30, 2012 ____________________

Rules of thumb and basic facts test.

All numerical answers should be given to two significant digits except question (f).
This section carries ~~10%~~ of the weight of the exam. No partial credit. ~~6~~ minutes.
12 points 5

(a) What is the value of the elementary charge, e?

$e = 1.6 \cdot 10^{-19}\ C$

(b) What is the photon energy of 488 nm light?

$E = \frac{hc}{\lambda} = \frac{1240\ eV\cdot nm}{488\ nm} = 2.5\ eV$ $\left(4.0 \cdot 10^{-19}\ J\right)$

(c) Write the Lorentz force equation.

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

(d) It is observed that one of these wavelengths gets significantly absorbed in a type of glass used to make windows. Circle the most likely wavelength: **300 nm** **500 nm** **700 nm**

(e) Blue and green laser pulses enter a slab of fused silica at the same time, at normal incidence. Which gets out first?

The green pulse

(f) A 10 W laser goes through a thick plastic block at normal incidence. What is a reasonable estimate of the power of the first reflection off the front surface?

$P_R \approx 4\% \times 10W = 400\ mW.$

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General Exam

Short problems. Each question below is worth 5 points.

(A) Light is incident on a 50/50 beamsplitter. This means half the power is reflected and half is transmitted in beams with the same cross-sectional area. What is the ratio of the peak electric field of the transmitted beam to the peak field of the incident beam?

$$\frac{P_t}{P_i} = \frac{I_t}{I_i} = \frac{E_t^2}{E_i^2} \qquad \frac{E_t}{E_i} = \sqrt{\frac{I_t}{I_i}} = \frac{1}{\sqrt{2}} \simeq 71\% \checkmark$$

(B) What is the magnitude and phase angle of $z = 2 + 4e^{i\pi/3}$?

$$z = 2 + 4\cos\frac{\pi}{3} + i\,4\sin\frac{\pi}{3} = 4 + i\,2\sqrt{3}$$

$$|z| = \sqrt{16 + 12} = 5.3 \checkmark \qquad \theta = \tan^{-1}\left(\frac{2\sqrt{3}}{4}\right) = 41^\circ \checkmark \quad (0.71\ \text{rad})$$

(C) The dielectric permittivity of water is about 80 times that of free space for the microwave radiation used in ovens. With what speed do such waves travel in water?

$$v = \frac{c}{n} = \frac{c}{\sqrt{\frac{\varepsilon}{\varepsilon_0}}} = \frac{c}{\sqrt{80}} = 3.4 \cdot 10^7\ \text{m/s} \checkmark$$

(D) 550 nm light has a wavelength of 340 nm in medium #1 and a wavelength of 400 nm in medium #2. What is n_1/n_2?

$$\lambda_i = \lambda / n_i \qquad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{400\ \text{nm}}{340\ \text{nm}} = 1.2 \checkmark$$

(E) A medium has a density of 4.0 g/cm^3 and a molar mass of 20 g/mole. How many molecules are there per cm^3?

$$N = (4.0\ \text{g/cm}^3)\left(\frac{1}{20}\ \frac{\text{mole}}{\text{g}}\right)\left(6.02 \cdot 10^{23}\ \frac{\text{molecules}}{\text{mole}}\right) = 1.2 \cdot 10^{23}\ \text{molecules/cm}^3 \checkmark$$

(F) According to the text, silver has n = 0.12 + i3.45 at 560 nm. What is the normal incidence reflectivity of silver in water (n_{water} = 1.33)?

$$R = \left|\frac{n - n_{water}}{n + n_{water}}\right|^2 = \frac{(n_r - n)^2 + n_{i}^2}{(n_r + n)^2 + n_{i}^2}$$

(using example 3.14, replacing $n_{air} = 1$ with n_{water}.)

$$= \frac{13.37}{14.00} = 95.5\% \checkmark$$

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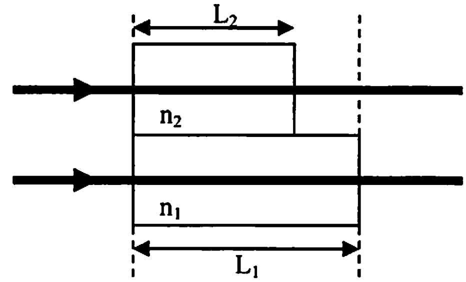
Longer problems, 15 points each. Use your own paper. Make sure to sign your name.

(1) Two waves of 0.600 μm light are initially in phase. They pass through plastic blocks immersed in water ($n = 1.33$) as shown in the figure with $L_1 = 4.00$ μm, $L_2 = 3.50$ μm, $n_1 = 1.40$ and $n_2 = 1.60$. **(a)** What is the optical path length traveled by each beam between the dashed lines? **(b)** What is the phase difference between them, $\phi_2 - \phi_1$, at the 2[nd] dashed line?

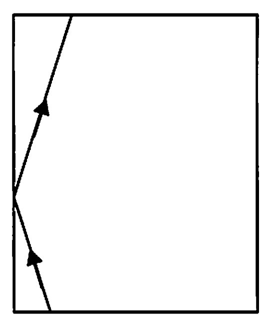


[Hint: Taking the time to be t=0, the phase of a beam in a single medium can be written as kz. Write the phase in terms of the optical path length, instead.]

(2) A 100 W, p-polarized, laser beam is incident on a 5.0 cm thick glass slab ($n = 1.42$) at 40° incidence. After passing through the slab it enters centered into a power meter. **(a)** Sketch a figure showing the path the light travels going from air on one side of the slab to the meter on the other side. **(b)** The slab is removed. What is the minimum distance one must move the meter to recenter the laser?

(3) For the case of s-polarized light incident on an interface and undergoing total internal reflection, use the real fields we found previously to: **(a)** Find the Poynting vector of the evanescent wave; **(b)** Find the time average and show that, on average, no energy is transmitted across the interface.

(4) A green laser beam travels inside a plastic block ($n = 1.2$) along the path shown (not to scale): it reflects from the left side and then travels into the air above.



(a) If the beam undergoes total internal reflection from the left side at the critical angle, what angle does the beam make with the normal to the surface once it refracts through the top?

(b) Redo part (a) but with the beam hitting the left side at Brewster's angle. (Assume the beam has some s-pol, so there will be reflected light.)

(5) An EM wave can be approximated as a plane wave in a region of vacuum. It is propagating in a direction given using direction cosines as $\alpha = 0$, $\beta = \gamma = 1/\sqrt{2}$. Its frequency is 1.5×10^{14} Hz and the power through a cross-sectional area of 1 cm^2 is 100 kW. The magnetic field vector lies somewhere in the yz plane. Write possible expressions for the electric and magnetic fields. (A good approach is to answer questions like these symbolically, making sure to provide numerical values for each symbol.)

(1) (a) $L_o = \sum_i l_i n_i$

$L_{o1} = L_1 n_1 = 5.600 \text{ mm}$

$L_{o2} = L_2 n_2 + (L_1 - L_2) n = 6.265 \text{ mm}$ } ✓

(b) $\phi = kz = \frac{2\pi}{\lambda} z = \frac{2\pi}{\lambda_o/n} z$ where $\lambda_o \equiv$ vacuum wavelength

$= \frac{2\pi}{\lambda_o} nz = 2\pi \frac{L_o}{\lambda_o}$

$\Delta\phi = 2\pi \frac{\Delta L_o}{\lambda_o} = 2\pi \frac{(L_{o2} - L_{o1})}{\lambda_o}$

$\Delta\phi = 1.108 \times 2\pi = 6.96 \text{ rad}$ ✓

close to constructive interference

(2)

$$\sin\theta_t = \frac{n_i \sin\theta_i}{n_t} = \frac{\sin 40^\circ}{1.42}$$

$$\theta_t = 26.9^\circ$$

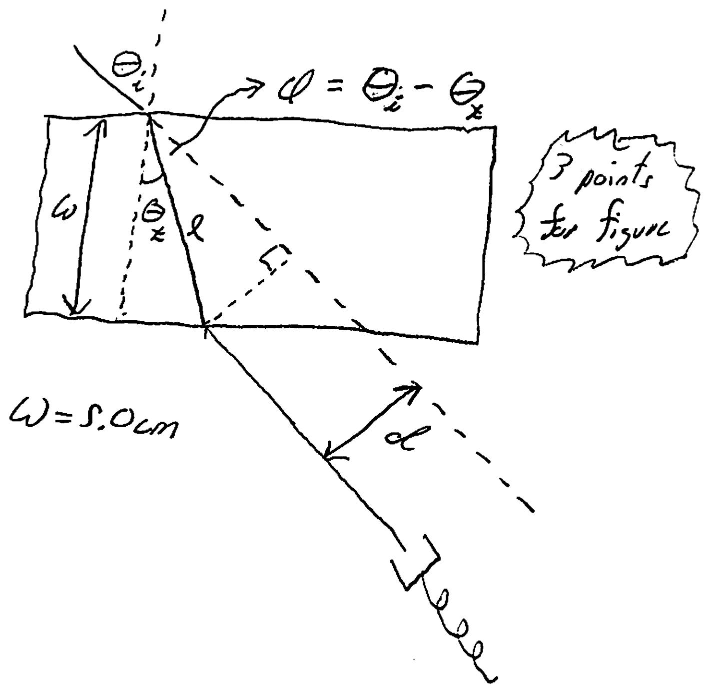
$$\cos\theta_t = \frac{w}{l}$$

$$l = \frac{w}{\cos\theta_t} = 5.61\text{ cm}$$

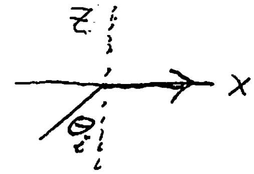
$$\alpha = \theta_i - \theta_t = 13.1^\circ$$

$$\sin\alpha = \frac{d}{l}$$

$$d = l\sin\alpha = 1.3\text{ cm}$$ ✓



③ $\vec{E}_t = E_{0t}\, e^{-\kappa z} \cos(kx-\omega t)\,\hat{y}$



$$\vec{B}_t = \frac{E_{0t}}{\omega}\, e^{-\kappa z}\left[\kappa \sin(kx-\omega t)\,\hat{x} + k\cos(kx-\omega t)\,\hat{z}\right]$$

$$\mu_0 \vec{S} = \vec{E}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & E_y & 0 \\ B_x & 0 & B_z \end{vmatrix} = E_y B_z\,\hat{x} - E_y B_x\,\hat{z}$$

$$\vec{S} = \frac{E_{0t}^2}{\mu_0\omega}\, e^{-2\kappa z}\left[k\cos^2\phi\,\hat{x} - \kappa\cos\phi\sin\phi\,\hat{z}\right]$$ with $\phi \equiv kx-\omega t$

$$= \frac{E_{0t}^2}{\mu_0\omega}\, e^{-2\kappa z}\left[k\cos^2\phi\,\hat{x} - \frac{\kappa}{2}\sin 2\phi\,\hat{z}\right] \checkmark$$

$\langle\cos^2\phi\rangle = \frac{1}{2}$ $\quad\langle\sin 2\phi\rangle = 0$

$$\langle\vec{S}\rangle = \frac{E_{0t}^2 k}{2\mu_0\omega}\, e^{-2\kappa z}\,\hat{x} \checkmark$$

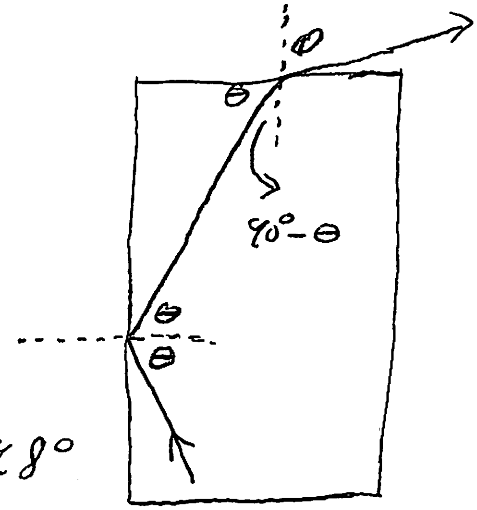
No $\hat{z}$ component. Energy is transmitted to the right.

(4)

(a) $\theta = \theta_c = \sin^{-1}\left(\frac{1}{1.2}\right) = 56.4^\circ$

$\phi = \sin^{-1}\left(1.2 \cdot \sin(90^\circ - \theta)\right)$

$= 41.6^\circ$ ✓



(b) $\theta = \theta_B = \tan^{-1} \frac{n_t}{n_i} = \tan^{-1} \frac{1}{1.2} = 39.8^\circ$

$\phi = 67.2^\circ$ ✓

(5) $\alpha = 0$ so $k_x = 0$ $\Rightarrow$ $\vec{k}$ lies in the yz plane.

Since $\vec{k}$ & $\vec{B}$ lie in the yz plane, $\vec{E} \perp$ yz plane, or:

$\vec{E}$ is polarized along $\hat{x}$

$$I = 100\ \mathrm{kW/cm^2} = 10^9\ \mathrm{W/m^2}$$

$$E_0 = \sqrt{\frac{2I}{c\varepsilon_0}} = 8.68\cdot 10^5\ \mathrm{V/m}$$

$$k = \frac{\omega}{c} = \frac{2\pi f}{c} = 3.14\cdot 10^6\ \mathrm{rad/m}$$

$$\omega = 2\pi f = 9.4\cdot 10^{14}\ \mathrm{Hz}$$

$$\vec{E} = E_0\,\hat{x}\,e^{i[k(\beta y+\gamma z)-\omega t]}$$

$$B_0 = E_0/c = 2.9\ \mathrm{mT}$$

$$\vec{B} = B_0(\gamma\hat{y} - \beta\hat{z})\,e^{i[k(\beta y+\gamma z)-\omega t]}$$