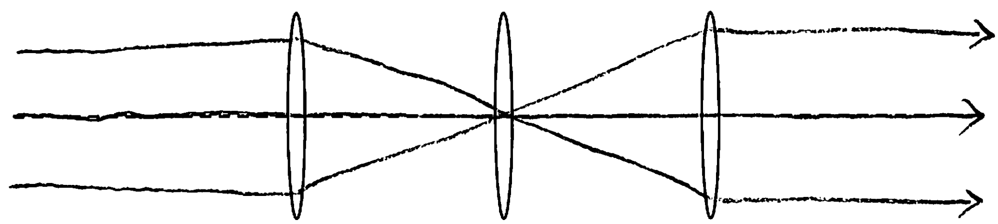
Physics 570 **Midterm II** **Schumacher**

May 30, 2012 ____________________

(1) [10 points] Three lenses with focal length f are in a line, as shown. The spacing between them is also f. Suppose we have a point source located on-axis, to the left at infinity. Use ray tracing to find its image. Identify the image location clearly on the figure or in words.



location: infinity, to the right ✓

(2) [15 points] The objective lens of a compound microscope has a focal length of 3.56 mm and the length of the microscope body is 17.0 cm. The eyepiece uses a lens with a 30.0 mm focal length.

(a) If you view the image with your eye in a relaxed configuration (no accommodation) what is the magnification of the microscope?

$$M = \frac{L}{f_{ob}} \frac{25}{f_e} = \frac{170\text{ mm}}{3.56\text{ mm}} \frac{25\text{ cm}}{3\text{ cm}} = 398$$ ✓

(b) If you view the image at the N.P. (eye maximally strained) what is the magnification?

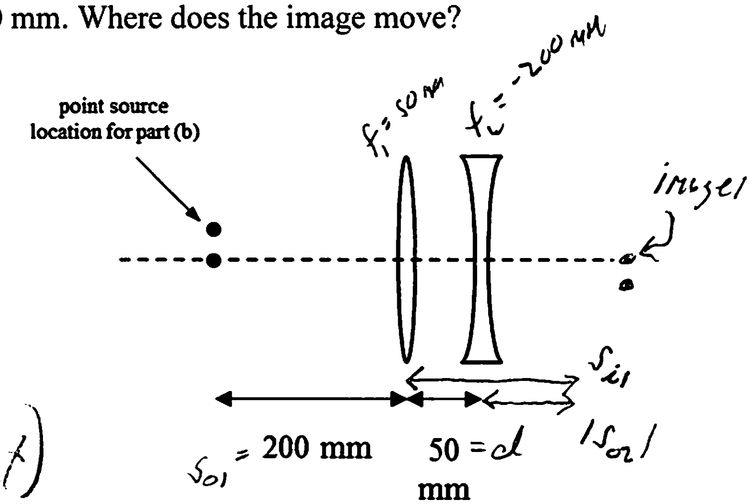
The eye piece, which is a magnifying glass, must now be modeled using 4.53:

$$M = \frac{L}{f_{ob}}\left(\frac{25}{f_e} + 1\right) = 446$$ ✓

(3) [20 points] The point source (indicated with a dot) is initially on-axis, the positive lens has a focal length of 50 mm, and the negative lens a focal length of -200 mm.

(a) Where is the image?

(b) The point source is moved upwards by 2.0 mm. Where does the image move?



(a) $$\frac{1}{S_{i1}} = \frac{1}{f_1} - \frac{1}{S_{o1}}$$

$S_{i1} = 66.7$ mm

$|S_{o2}| = |S_{i1} - d| = 16.7$ mm

$S_{o2} = -16.7$ mm (virtual object)

$$\frac{1}{S_{i2}} = \frac{1}{f_2} - \frac{1}{S_{o2}}$$

$S_{i2} = 18.2$ mm

image is on-axis 18 mm to the right of the negative lens. ✓

(b) $m = m_1 m_2 = \left(-\frac{S_{i1}}{S_{o1}}\right)\left(-\frac{S_{i2}}{S_{o2}}\right) = -0.36$ ✓

The image moves down $|m|(2.0\text{ mm}) = 0.73$ mm ✓

(4) [10 points] Find the **(a)** phase velocity and **(b)** group velocity for 100 nm light traveling through copper, a good conductor with a plasma frequency of 1.64 x 10^{16} rad/s. Express both answers as a multiple of c.

$\lambda_0 = 100\,nm$ (wavelength in vacuum) $\omega = \frac{2\pi c}{\lambda_0} = 1.88 \cdot 10^{16}$ rad/s

$$n = \sqrt{1 - \omega_p^2/\omega^2} = 0.493$$

$$v_p = \frac{c}{n} = 2.03\,c \checkmark$$

$$v_g = c\sqrt{1 - \omega_p^2/\omega^2} = 0.493\,c \checkmark$$

(see example 5.10 for equations for n, v_g.)

Many of you used $k = \frac{2\pi}{\lambda_0}$. This is only true in vacuum. In a medium, like copper, $k = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda_0} n$.

(5) [10 points] We have seen two similar schemes for making a high reflecting (HR) mirror using dielectric coatings: (1) that described in section 5.11.2; and (2) that treated in problem 5.45. For a given choice of n_H, n_L, and the desired reflectivity, which scheme requires fewest layers? Explain your answer. (Working an example with your calculator does not constitute an explanation. This can be answered simply using a sentence or two.)

Approach #1 requires fewest layers. ✓

#1 requires $n_t \left(\frac{n_H}{n_L}\right)^{2N} \gg 1$ or $\left(\frac{n_H}{n_L}\right)^{2N} \gg \frac{1}{n_t}$

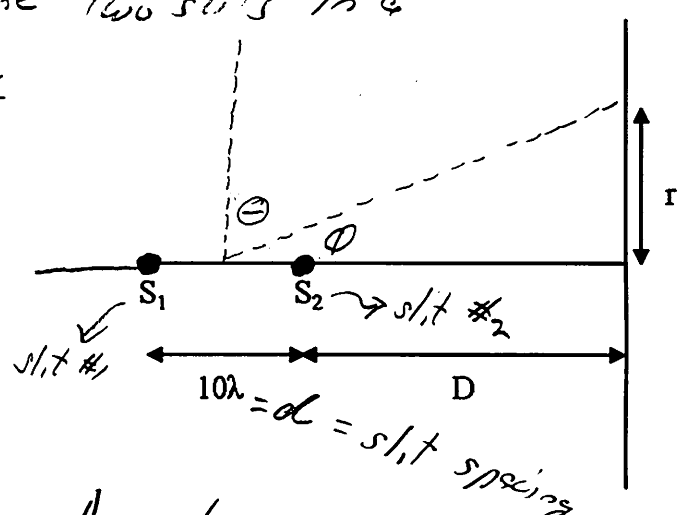
#2 requires $\left(\frac{n_H}{n_L}\right)^{2} \gg n_t$

Now $n_t > 1$, #1 is clearly easier to satisfy.

(6) [20 points] Two point sources of EM radiation, 10 wavelengths apart, illuminate an observing screen, as shown. D = 4 m (exact) and λ = 2.0 mm. Find the three lowest values of r for which there are intensity maxima on the screen.

Hints: (1) This is a little messy, but be careful making approximations. You may use D >> λ, but if you apply this too soon you'll throw the answer away. (2) If you introduce an integer m, as in mλ, think very carefully about the values of m needed for this problem.

Here's the quick solution:
Point sources S_1 & S_2 are like the two slits in a Young's experiment, so maxima are specified by*:



$$d \sin\Theta = m\lambda$$

$$\sin\Theta = \frac{m}{10} \checkmark$$

Large m yields large Θ which yields small r. No small angle approx.

$m = m_{max} = 10 \Rightarrow \Theta = 90^0$ and $r = 0 \checkmark$

$m = 9 \Rightarrow \Theta = 64.2^0$ so $\Phi = 25.8^0$

$$r = \tan\Phi\,(D + d/2) \approx D\tan\Phi = 1.9\,m \checkmark$$

$m = 8 \Rightarrow \Theta = 58.1^0$ so $r = 3.0\,m \checkmark$

* In a Young's experiment, the viewing screen is usually parallel to the slits ↕D ⊥ but the angle Θ points to the maxima in any case so long as $D \gg d$.

(6) **[20 points]** Two point sources of EM radiation, 10 wavelengths apart, illuminate an observing screen, as shown. D = 4 m (exact) and λ = 2.0 mm. Find the three lowest values of r for which there are intensity maxima on the screen.

Hints: (1) This is a little messy, but be careful making approximations. You may use D >> λ, but if you apply this too soon you'll throw the answer away. (2) If you introduce an integer m, as in mλ, think <u>very</u> carefully about the values of m needed for this problem.

"Messy" solution:

Maxima occur when: $L_1 - L_2 = m\lambda$

$L_1 = \sqrt{D_1^2 + r^2}$ $\quad L_2 = \sqrt{D_2^2 + r^2}$

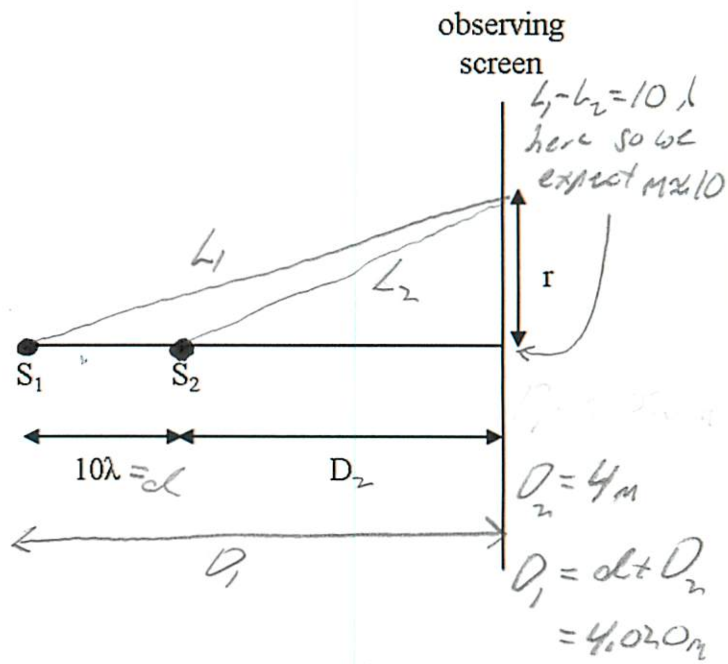
$L_1 = L_2 + m\lambda$

$D_1^2 + \cancel{r^2} = D_2^2 + \cancel{r^2} + 2L_2 m\lambda + m^2\lambda^2$

$L_2 = \left(D_1^2 - D_2^2 - m^2\lambda^2\right)/2m\lambda$

$D_2^2 + r^2 = \left[\left(D_1^2 - D_2^2 - m^2\lambda^2\right)/2m\lambda\right]^2$

$r^2 = \left[\left(D_1^2 - D_2^2 - m^2\lambda^2\right)/2m\lambda\right]^2 - D_2^2$ ✓



Not too bad, actually, and we <u>can</u> work with this on, since $D_1^2 = D_2^2 + 2dD_2 + d^2$:

$r^2 = \left[\left(2dD_2 + d^2 - m^2\lambda^2\right)/2m\lambda\right]^2 - D_2^2$

Now $dD_2 \gg d^2 - m^2\lambda^2 = (100 - m^2)\lambda^2$ so

$r^2 \approx \left(\frac{2dD_2}{2m\lambda}\right)^2 - D_2^2 = D_2^2\left(\frac{d^2}{m^2\lambda^2} - 1\right) = D_2^2\left(\frac{100}{m^2} - 1\right)$

$r \approx D_2\left(\frac{100}{m^2} - 1\right)^{1/2}$ ✓

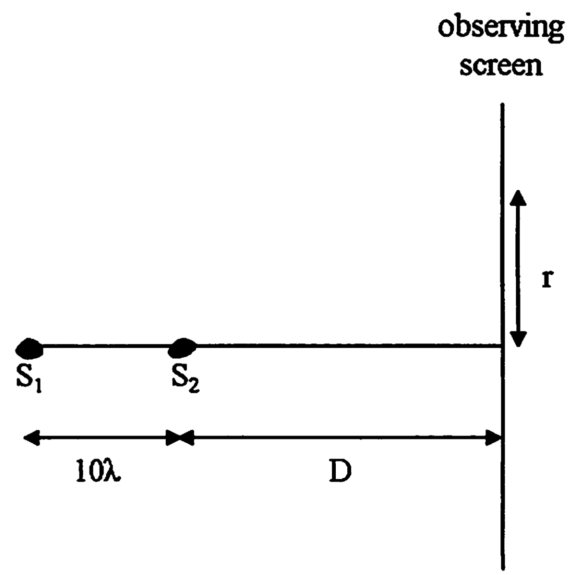
$m = 10 \rightarrow r = 0$ ✓

$m = 9 \rightarrow r = 1.9m$ ✓

$m = 8 \rightarrow r = 3.0m$ ✓

(6) [20 points] Two point sources of EM radiation, 10 wavelengths apart, illuminate an observing screen, as shown. D = 4 m (exact) and $\lambda = 2.0$ mm. Find the three lowest values of r for which there are intensity maxima on the screen.

Hints: (1) This is a little messy, but be careful making approximations. You may use $D \gg \lambda$, but if you apply this too soon you'll throw the answer away. (2) If you introduce an integer m, as in $m\lambda$, think very carefully about the values of m needed for this problem.



Some of you tried a third approach, using the fringe geometry analysis of section 5.3.5. Specifically, the plane wave/spherical wave case. That works, too! (Not my preference, but that's okay.)

There weren't any plane waves in this problem, just two spherical waves. Still, you could imagine a plane wave being present and then use it as a reference.

That is, you could write the phase difference of each spherical wave with respect to the plane wave, and the phase difference we want is the difference of the differences. This allows you to borrow from p. 211, but don't use the last equation with the m. You don't want to use the maxima condition from this section (it's the wrong problem), just the expressions for the path lengths.

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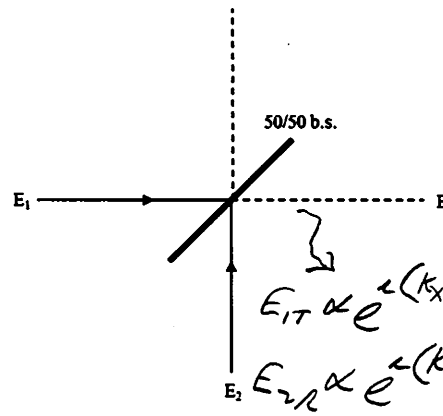
(7) [25 points] The figure shows a 50/50 beamsplitter immersed in water (n = 1.33) with two laser beams, E_1 and E_2, incident. One of the two output beams is labeled E_3. Assume the beamsplitter is extremely thin and does not change the relative phases between any fields after reflection or transmission. The fields just before the beamsplitter are:

$$E_1 = E_o\, e^{i(kx - \omega t + \varphi_1)},\; E_2 = 2E_o\, e^{i(ky - \omega t + \varphi_2)}$$

with E_o = 1700 V/m. +x is to the right and +y is upwards with the origin at the point of intersection. E_1 and E_2 have incident angles of 45°.



$E_{1T} \propto e^{i(kx-\omega t+\varphi_1)}$

$E_{2R} \propto e^{i(kx-\omega t+\varphi_2)}$

(a) Find I_3.

(b) What are the minimum and maximum possible values for I_3?

(c) Suppose φ_1 = 0.500 rad and φ_2 = 3.5 t, with t in seconds. How long would it take for I_3 to oscillate from minimum to maximum?

(a) $E_3 = E_{1T} + E_{2R}$ where $E_{1T} = E_1$ transmitted & $E_{2R} = E_2$ reflected.

$I_1 = \frac{\varepsilon v}{2} E_o^2 = \frac{n\varepsilon_0 c}{2} E_o^2 = 5100\ W/m^2$ $\qquad n^2 = \frac{\varepsilon}{\varepsilon_0}$ so $\varepsilon = n^2\varepsilon_0$, $v = c/n$

$I_2 = 4 I_1$ b/c $E_2 = 2E_1$

$I_{1T} = \frac{1}{2} I_1$ and $I_{2R} = \frac{1}{2} I_2 = 2 I_1$

$I_3 = I_{1T} + I_{2R} + 2\sqrt{I_{1T} I_{2R}}\cos\left[(kx-\omega t+\varphi_2) - (kx-\omega t+\varphi_1)\right]$

$= \frac{1}{2} I_1 + 2 I_1 + 2 I_1 \cos(\varphi_2 - \varphi_1)$

$= I_1(2.5 + 2\cos(\varphi_2-\varphi_1))$ ✓

(b) $I_{3,min} = (2.5-2) I_1 = 2550\ W/m^2$ ✓ (close to 0)

$I_{3,max} = (2.5+2) I_1 = 22{,}950\ W/m^2$ ✓ (close to $I_1 + I_2$)

(c) I_3 goes from min to max in the time it takes for $(\varphi_2-\varphi_1)$ to change by π. $\Delta(\varphi_2-\varphi_1) = \Delta\varphi_2$ b/c φ_1 = constant.

$\Delta\varphi_2 = \pi \Rightarrow 3.5\Delta t = \pi \Rightarrow \Delta t = \frac{\pi}{3.5} = 0.898\,s$ ✓