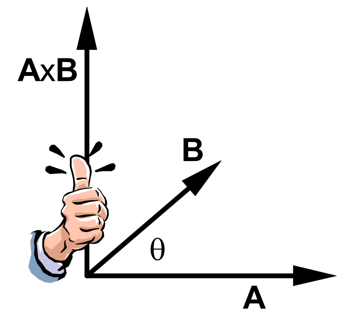
CROSS-PRODUCT REVIEW

The cross product (or vector product) between two vectors **A** and **B** is written as **A**x**B**. The result of a cross-product is a ***new vector***. We need to find its magnitude and direction. (See section 3-7 in the text for more review.)

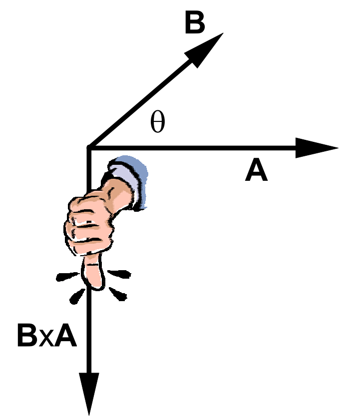


Magnitude: |**A**x**B**| = A B sinθ. Just like the dot product, θ is the angle between the vectors **A** and **B** when they are drawn tail-to-tail.

Direction: The vector **A**x**B** is perpendicular to the plane formed by **A** and **B**. Use the right-hand-rule (RHR) to find out whether it is pointing into or out of the plane.

Right-hand-rule (RHR): Here's how it works. Imagine an axis going through the tails of **A** and **B**, perpendicular to the plane containing them. Grab the axis with your *right* hand so that your fingers sweep **A** into **B**. Your outstretched thumb points in the direction of **A**x**B**.

Note that **B**x**A** gives you a new vector that is opposite to **A**x**B**. Why? Because, now you have to sweep **B** into **A**.



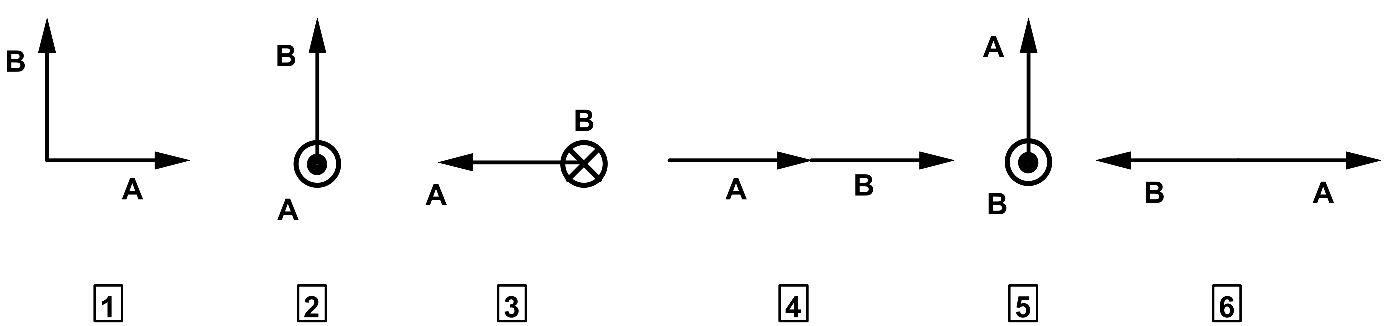
Cross-product facts:

Bx**A** = -**A**x**B**

|**A**x**B**| = 0 if **A** and **B** are *parallel*, because then θ = 0° or θ = 180° degrees. This gives the *minimum magnitude.*

|**A**x**B**| = AB if **A** and **B** are *perpendicular*, because then θ = 90° or θ = 270° degrees. This gives the *maximum magnitude*.

Here's a test to see if you understand how to use the RHR (answers are on the back of this page). In each case, decide whether **A**x**B** points up, down, left, right, into the page, or out of the page. ⊙ The symbol means a vector pointing out of the page, ⊗ and a vector pointing into it.



Finally, there is another way to evaluate the cross-product, given **A** and **B** in component form:

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - B_yA_z)\,\hat{i} - (A_xB_z - B_xA_z)\,\hat{j} + (A_xB_y - B_xA_y)\,\hat{k}$$

Differences between the dot- and cross-products: The biggest difference, of course, is that $\vec{A} \bullet \vec{B}$ is a number and $\vec{A} \times \vec{B}$ results in a new vector. Also, when the magnitude of the dot product is a maximum, the magnitude of the cross-product is zero and *vice versa*.

Moreover, because $\vec{A} \bullet \vec{B} = AB\cos\theta$, the dot product is proportional to:

The magnitude of $\vec{A}$ times the *component* of $\vec{B}$ that is *parallel* to $\vec{A}$.

On the other hand, the cross-product magnitude is given by $AB\sin\theta$, so it is proportional to:

The magnitude of $\vec{A}$ times the *component* of $\vec{B}$ that is *perpendicular* to $\vec{A}$.

Solutions to practice RHR problems on the front page. (1) out of page, (2) left, (3) down, (4) zero magnitude so the direction is undefined, (5) right, (6) zero magnitude.