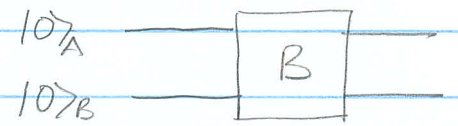
Quantum Teleportation

If Alice and Bob have qubits $|0\rangle_A$ and $|0\rangle_B$, they can produce an entangled state by using a Bell gate:



$$\frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

Alice also has another qubit, which is in an unknown spin state:

$$|\chi\rangle_C = \alpha|0\rangle_C + \beta|1\rangle_C$$

where α, β are complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$.

The 3-qubit state is

$$|\chi\rangle_C \cdot \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

$$= \frac{\alpha}{\sqrt{2}}\left(|000\rangle + |011\rangle\right) + \frac{\beta}{\sqrt{2}}\left(|100\rangle + |111\rangle\right)$$

where $|ijk\rangle \equiv |i\rangle_C|j\rangle_A|k\rangle_B$.

Alice operates on qubits C and A using a Bell gate, whose effect on the basis states is as follows:

$$|0\rangle|0\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + |1\rangle|1\rangle\right)$$

$$|0\rangle|1\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle + |1\rangle|0\rangle\right)$$

$$|1\rangle|0\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle - |1\rangle|0\rangle\right)$$

$$|1\rangle|1\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle - |1\rangle|1\rangle\right)$$

The 3-qubit state is now

$$\frac{\alpha}{2}\left(|000\rangle + |110\rangle + |011\rangle + |101\rangle\right)$$

$$+ \frac{\beta}{2}\left(|010\rangle - |100\rangle + |001\rangle - |111\rangle\right)$$

Now Alice measures qubits C and A, getting one of 4 possible results: 00, 01, 10, 11.
The measurement collapses the 3-qubit state to one of the following possibilities:

$$00: \ \alpha|000\rangle + \beta|001\rangle = |0\rangle_C|0\rangle_A\left(\alpha|0\rangle_B + \beta|1\rangle_B\right)$$

$$01: \ \alpha|011\rangle + \beta|010\rangle = |0\rangle_C|1\rangle_A\left(\alpha|1\rangle_B + \beta|0\rangle_B\right)$$

$$10: \ \alpha|101\rangle - \beta|100\rangle = |1\rangle_C|0\rangle_A\left(\alpha|1\rangle_B - \beta|0\rangle_B\right)$$

$$11: \ \alpha|110\rangle - \beta|111\rangle = |1\rangle_C|1\rangle_A\left(\alpha|0\rangle_B - \beta|1\rangle_B\right)$$

Alice communicates the result of her measurement to Bob, who applies a different gate to his qubit for each outcome

00 : no gate $\quad |0\rangle \rightarrow |0\rangle$

$|1\rangle \rightarrow |1\rangle$

01 : NOT gate $\quad |0\rangle \rightarrow |1\rangle$

$|1\rangle \rightarrow |0\rangle$

10 : NOT gate followed by Z gate $\quad |0\rangle \rightarrow -|1\rangle$

$|1\rangle \rightarrow |0\rangle$

11 : Z gate $\quad |0\rangle \rightarrow |0\rangle$

$|1\rangle \rightarrow -|1\rangle$

In each of the 4 cases, the final 3-qubit state is as follows

00 : $|0\rangle_C |0\rangle_A \left(\alpha|0\rangle_B + \beta|0\rangle_B\right)$

01 : $|0\rangle_C |1\rangle_A \left(\alpha|0\rangle_B + \beta|1\rangle_B\right)$

10 : $|1\rangle_C |0\rangle_A \left(\alpha|0\rangle_B + \beta|1\rangle_B\right)$

11 : $|1\rangle_C |1\rangle_A \left(\alpha|0\rangle_B + \beta|1\rangle_B\right)$

In each case, Bobs cubit is in the same unknown spin state as the original cubit C.