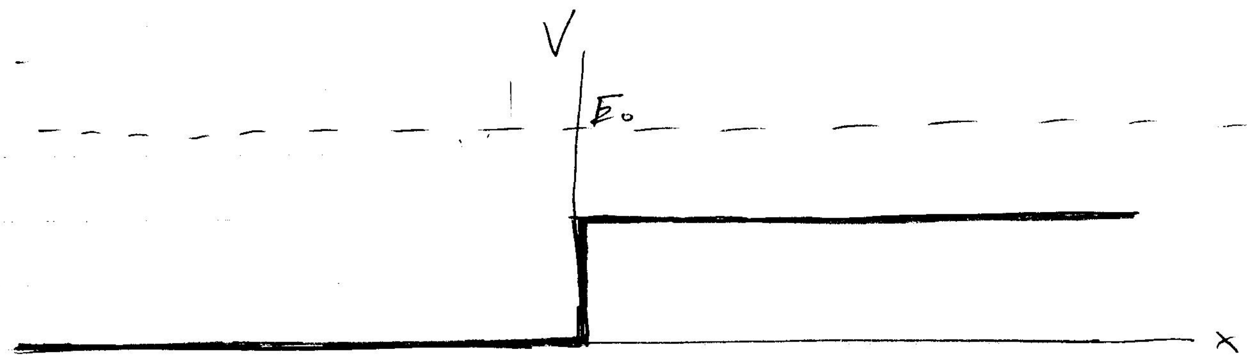
Scattering: Schroedinger Equation

step potential: $V(x) = 0 \qquad x < 0$
$\qquad\qquad\qquad\qquad\quad = V_0 \qquad x > 0$

particle with energy $E_0 > V_0$ approaching from left



The wavefunction of the particle will be
partly reflected by the step
and partly transmitted

Suppose its position x is subsequently measured

reflection and transmission probabilities

R = probability for $x < 0$

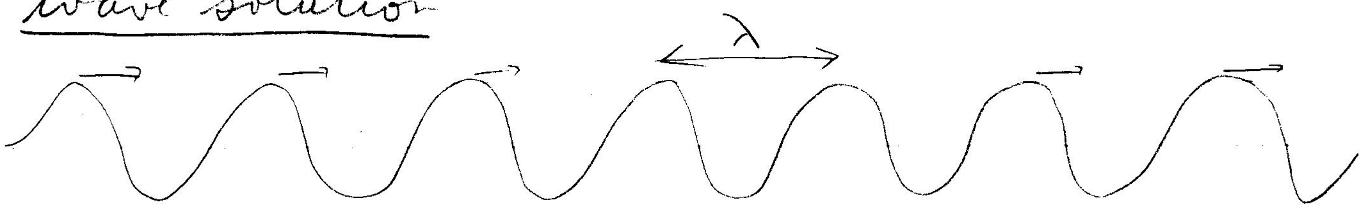
T = probability for $x > 0$

$R + T = 1$

Schroedinger equation with no potential

$$i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m}\left(\frac{\partial}{\partial x}\right)^2 \psi$$

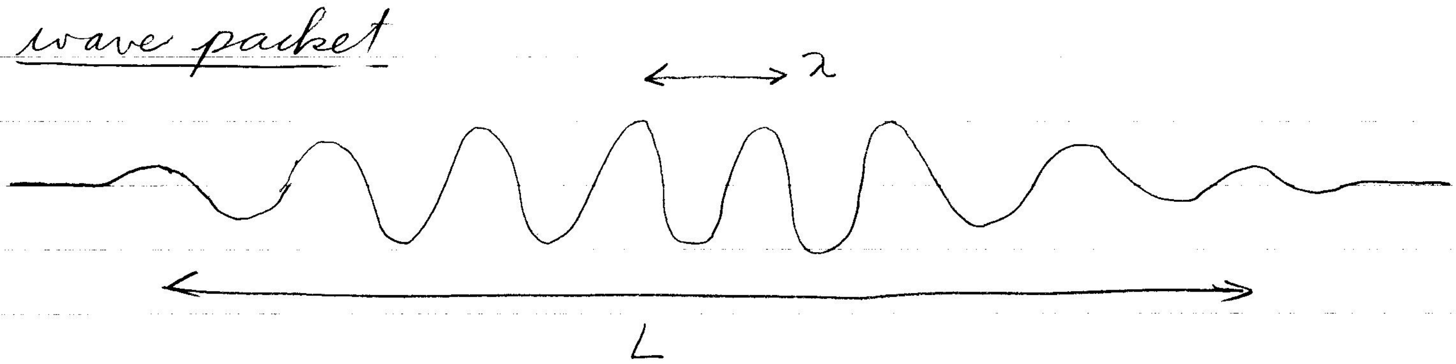
<u>wave solution</u>



$$\psi(x,t) = A\, e^{i p_0 x/\hbar}\, e^{-i (p_0^2/2m) t/\hbar}$$

- wavelength $\lambda = \frac{h}{p}$
- infinite spatial extent
- definite energy: $E = \frac{p_0^2}{2m}$
- wave flows to right with speed $\frac{p_0}{m}$
- uniform probability density. $|\psi(x,t)|^2 = |A|^2$
- not normalizable: $\int_{-\infty}^{\infty} dx\, |\psi(x,t)|^2 = +\infty$

wave packet



wavelength : $\lambda = \frac{h}{p_0}$
length of wavepacket : L

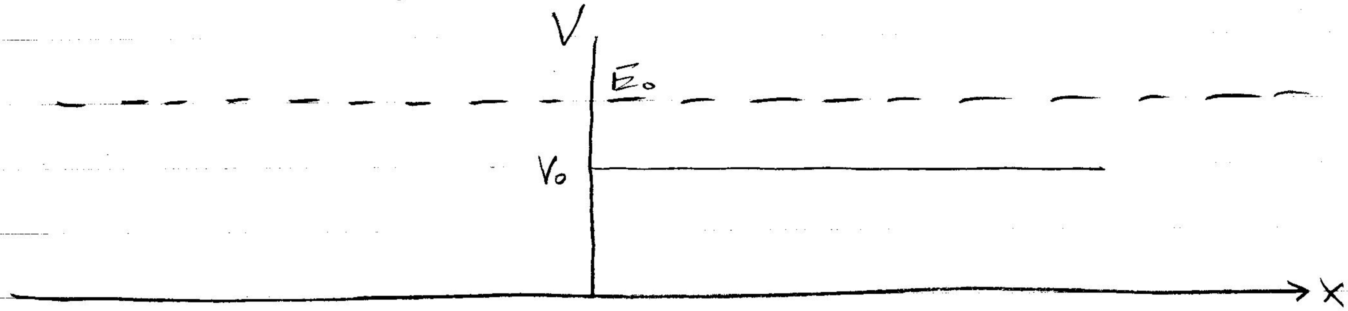
$$\Psi(x,t) = \int_{-\infty}^{\infty} dp\, \phi(p)\, e^{ipx/\hbar}\, e^{-i(p^2/2m)t/\hbar}$$

can choose $\phi(p)$ so

- wavepacket is centered at x_0
  and has length $\Delta x \sim L$
- energy has narrow distribution
  centered at $E_0 = p_0^2/2m$
  with width $\Delta E \sim \frac{p_0}{2m} \frac{\hbar}{L}$
- wavefunction is normalized : $\int_{-\infty}^{\infty} dx\, |\psi(x,t)|^2 = 1$
- probability flows to right with speed $\frac{p_0}{m}$

explicit solution : $\phi(p) = A\, e^{-L^2(p-p_0)^2/\hbar^2}\, e^{-ipx/\hbar}$

Scattering from step potential

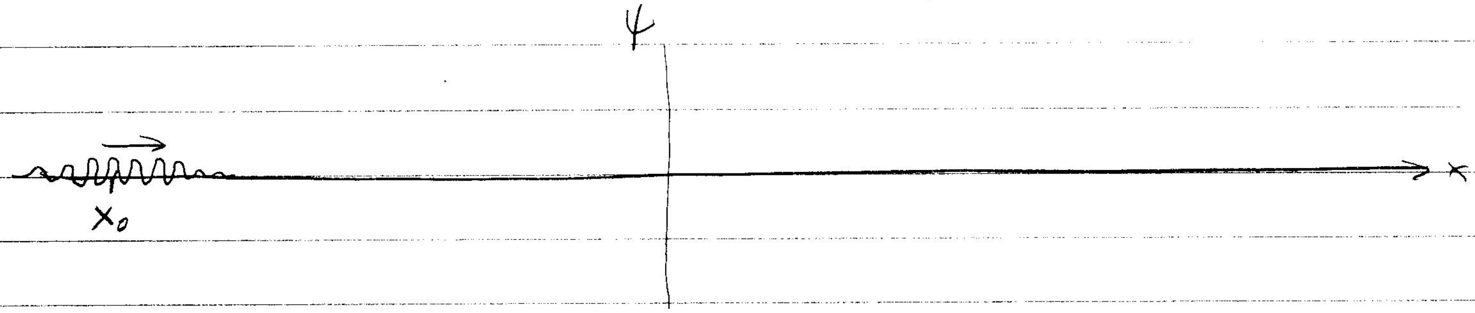


initial wavefunction at $t=0$: $\Psi(x,0) = \psi_{initial}(x)$

wavepacket with center x_0 far to left of step

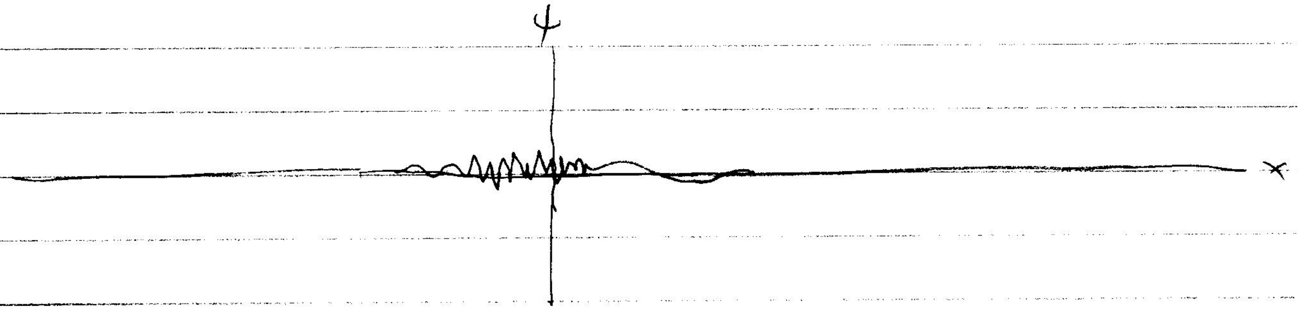
energy distribution near E_0

moving right with speed $\sqrt{\frac{2E_0}{m}}$

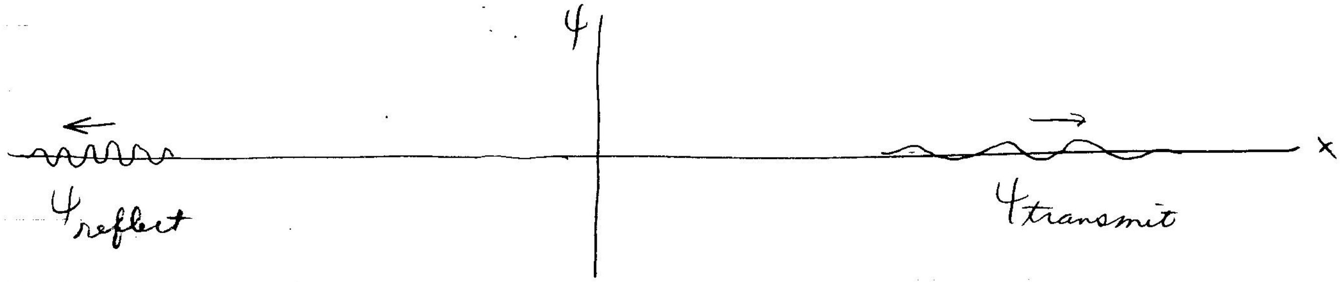


for $t>0$, wavefunction evolves according to Schroedinger equation

becomes very complicated when it reaches the step



at much later time T,
wavefunction is a superposition
of a reflected wavepacket
and a transmitted wavepacket



$$\psi(x,T) = \psi_{reflect}(x) + \psi_{transmit}(x)$$

reflection probability:

$$R = \int_{-\infty}^{0} dx \, |\psi_{reflect}(x)|^2$$

transmission probability

$$T = \int_{0}^{\infty} dx \, |\psi_{transmit}(x)|^2$$