Measurement of Position

free particle in 1 dimension

wavefunction $\psi(x,t)$

Schroedinger equation

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = -\frac{\hbar^2}{2m}\left(\frac{\partial}{\partial x}\right)^2 \psi(x,t)$$

simple solution:

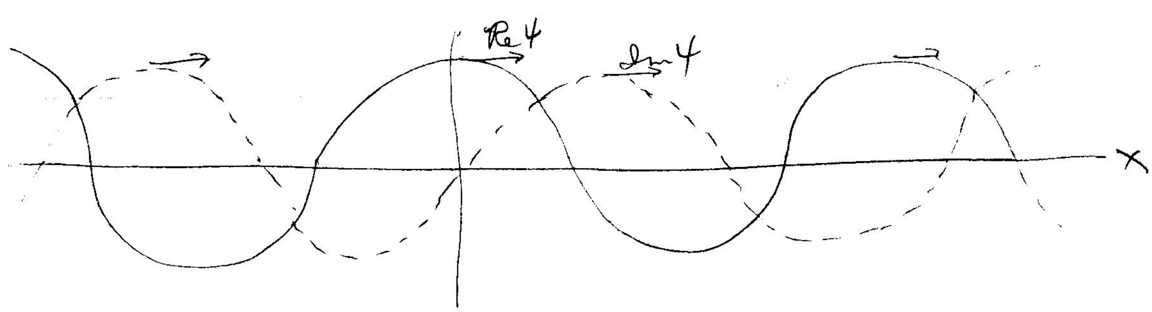
$$\psi(x,t) = A\, e^{ip_0 x/\hbar - i(p_0^2/2m)t/\hbar}$$ p_0, A real constants

$i\hbar \frac{\partial}{\partial t}\psi = \frac{p_0^2}{2m}\psi \implies$ particle has energy $p_0^2/2m$

$-i\hbar \frac{\partial}{\partial x}\psi = p_0 \psi \implies$ particle has momentum p_0

$$\psi(x,t) = A\, e^{ikx - i\omega t}$$ $k = \frac{p_0}{\hbar}$ $\omega = \frac{p_0^2}{2m\hbar}$

$$= A\left[\cos(kx-\omega t) + i\sin(kx-\omega t)\right]$$ wave!



$|\psi(x,t)|^2$ is proportional to the probability that a measurement at time t will find the particle at position x

$$|\psi(x,t)|^2 = \left|A\, e^{ip_0x/\hbar - i(p_0^2/2m)t/\hbar}\right|^2$$

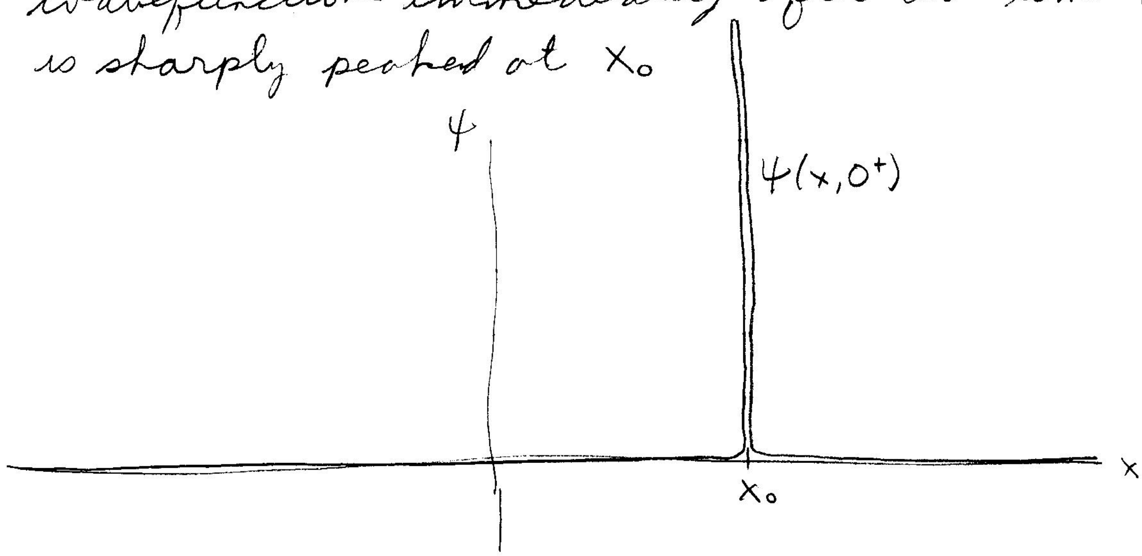
$$= |A|^2 \cdot \left|e^{ip_0x/\hbar}\right|^2 \cdot \left|e^{-i(p_0^2/2m)t/\hbar}\right|^2$$

$$= |A|^2 \cdot 1^2 \cdot 1^2 = |A|^2$$

constant $\Rightarrow$ equal probability everywhere

BUT if the position is actually measured wavefunction collapses so that $\psi = 0$ everywhere except at the measured position

If measurement at time $t=0$ gives position x_0 wavefunction immediately after at time $t=0^+$ is sharply peaked at x_0



collapsed wave function can be expressed in terms of Dirac delta function $\delta(x)$

whose properties are

$$\delta(x) = 0 \qquad \text{if } x \neq 0$$

$$\delta(0) = +\infty$$

$$\int_{-\epsilon}^{+\epsilon} dx\ \delta(x) = 1 \quad \text{for any } \epsilon > 0$$

integral representation : $\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, e^{ikx}$

collapsed wave function

$$\psi(x, 0^+) = \delta(x - x_0)$$

$$= \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} dp\ e^{ip(x-x_0)/\hbar}$$

subsequent time evolution

is described by Schroedinger equation

$$\psi(x,t) = \frac{1}{2\pi\hbar}\int_{-\infty}^{+\infty} dp\ e^{-ipx_0/\hbar} \cdot e^{ipx/\hbar - i(p^2/2m)t/\hbar}$$

- quantum superposition of all possible momenta p
- expanding wave spreading out from point x_0