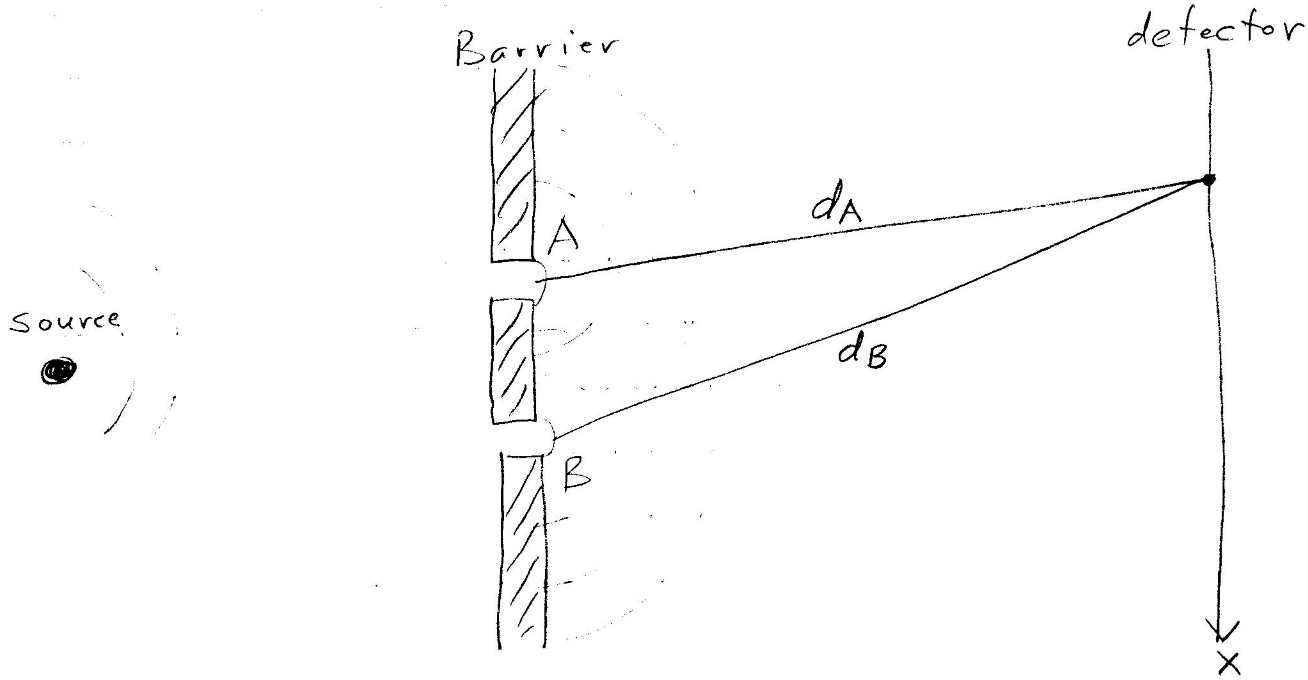
Interference



Slits A and B act as secondary sources for wave at detector

distance from source A : d_A
" " B : d_B

path length difference : $\Delta d = d_B - d_A$
depends on x

wavelength λ

constructive interference: $\Delta d = n\lambda$ n = integer

destructive interference : $\Delta d = (n + \frac{1}{2})\lambda$ n = integer

Classical Wave

amplitude h
frequency ν
wavelength λ
speed c $\qquad \nu\lambda = c$

wave at detector from single source A

$$h(x,t) = h_A \cos(2\pi\nu t - 2\pi d_A/\lambda)$$

h_A and d_A can depend on x

wave at detector from sources A and B

$$h(x,t) = h_A \cos(2\pi\nu t - 2\pi d_A/\lambda)$$
$$+ h_B \cos(2\pi\nu t - 2\pi d_B/\lambda)$$

interference depends only on phase difference: $\delta = \dfrac{2\pi(d_B - d_A)}{\lambda}$

simpler wave with same interference

$$h(x,t) = h_A \cos(\omega t + \delta) \qquad \omega = 2\pi\nu$$
$$+ h_B \cos(\omega t)$$

intensity: energy delivered by wave is proportional to the time average of the square of the amplitude

$$I(x) \propto \langle h(x,t)^2 \rangle_{\text{average over } t}$$

time average: (over many periods) $\langle f(t) \rangle = \frac{1}{T}\int_0^T dt\, f(t)$

$$\langle \cos^2(\omega t) \rangle = \frac{1}{2}$$

$$\langle \sin^2(\omega t) \rangle = \frac{1}{2}$$

$$\langle \cos(\omega t)\sin(\omega t) \rangle = 0$$

single source A

$$h(x,t) = h_A \cos(\omega t - 2\pi d_A/\lambda)$$

$$\langle h(x,t)^2 \rangle = h_A^2 \langle \cos^2(\omega t - 2\pi d_A/\lambda) \rangle$$

$$= h_A^2 \langle \cos^2(\omega t) \rangle$$

$$= h_A^2 \cdot \frac{1}{2}$$

<u>two sources: A and B</u>

$$h(x,t) = h_A \cos(\omega t + \delta) + h_B \cos(\omega t)$$

$$= h_A [\cos(\omega t) \cos\delta - \sin(\omega t) \sin\delta]$$

$$+ h_B \cos(\omega t)$$

$$= (h_A \cos\delta + h_B) \cos(\omega t) - h_A \sin\delta \sin(\omega t)$$

$$\langle h(x,t)^2 \rangle = (h_A \cos\delta + h_B)^2 \langle \cos^2(\omega t) \rangle$$

$$- 2 h_A \sin\delta (h_A \cos\delta + h_B) \langle \cos(\omega t) \sin(\omega t) \rangle$$

$$+ h_A^2 \sin^2\delta \langle \sin^2(\omega t) \rangle$$

$$= (h_A^2 \cos^2\delta + 2 h_A h_B \cos\delta + h_B^2) \cdot \tfrac{1}{2}$$

$$+ 0 + h_A^2 \sin^2\delta \cdot \tfrac{1}{2}$$

$$= \tfrac{1}{2} (h_A^2 + h_B^2 + 2 h_A h_B \cos\delta$$

intensity: $I_{AB} = I_A + I_B + 2\sqrt{I_A}\sqrt{I_B} \cos\delta$

Quantum Particle / Wave

quantum particle propagates like a wave
that can be represented by
complex number ψ
("wavefunction", "probability amplitude")

probability of observing the particle
is proportional to $|\psi|^2$

$$P \propto |\psi|^2$$

wave at detector from source A

$$\psi(x,t) = R_A e^{-i(2\pi\nu t - 2\pi d_A/\lambda)}$$

where R_A and d_A depend on x

$$|\psi(x,t)|^2 = R_A^2$$

wave at detector from two sources A, B

$$\psi(x,t) = R_A e^{-i(2\pi\nu t - 2\pi d_A/\lambda)}$$
$$+ R_B e^{-i(2\pi\nu t + 2\pi d_B/\lambda)}$$

simpler wave with same interference

$$\psi(x) = R_A + R_B e^{i\delta}$$

phase shift $\delta = 2\pi \frac{d_B - d_A}{\lambda}$

$$|\psi(x)|^2 = \psi(x)^* \psi(x)$$

$$= \left(R_A + R_B e^{-i\delta}\right)\left(R_A + R_B e^{+i\delta}\right)$$

$$= R_A^2 + R_A R_B \left(e^{i\delta} + e^{-i\delta}\right) + R_B^2 e^{-i\delta} e^{+i\delta}$$

$$= R_A^2 + R_A R_B (2\cos\delta) + R_B^2$$

probability of observing particle at x
if the slit it passes through is not determined

$$P_{A+B} \propto |\psi(x)|^2$$

$$P_{A+B} = P_A + P_B + 2\sqrt{P_A}\sqrt{P_B}\cos\delta$$