Compton Scattering

Planck (1900)
In black-body radiation,
light of frequency ν is emitted
in discrete packets of energy $h\nu$

Einstein (1905
In photoelectric effect,
electron absorbs light of frequency ν
in discrete packets of energy $h\nu$

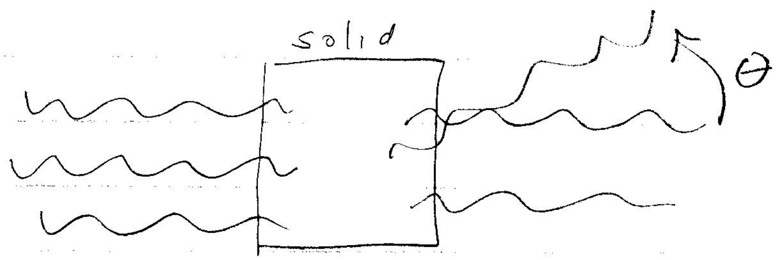
Maxwell's Equation
Light wave carries energy and momentum
that is distributed continuously in space

$$\text{energy density} \propto \vec{E}^2 + \vec{B}^2 c^2$$

$$\text{momentum density} \propto \vec{E} \times \vec{B}$$

Einstein
In light wave of frequency ν (and wavelength $\lambda = \frac{c}{\nu}$)
energy and momentum are concentrated
in particles (photons)
with energy $E = h\nu$ $\quad \nu\lambda = c$
momentum $p = \frac{h}{\lambda}$ $\quad E = pc$

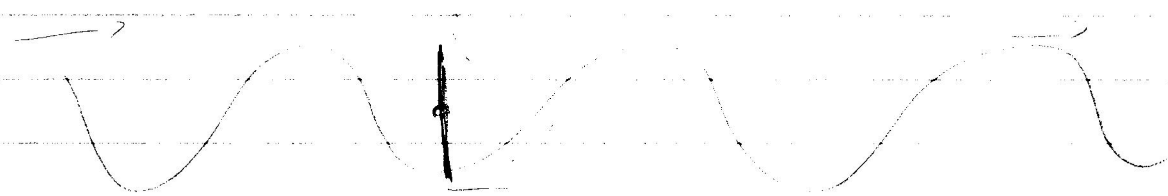
X-rays hitting solid
some pass through undeflected
some are scattered at various angles



scattered X-rays could be
scattering off electron
or scattering from atomic nuclei

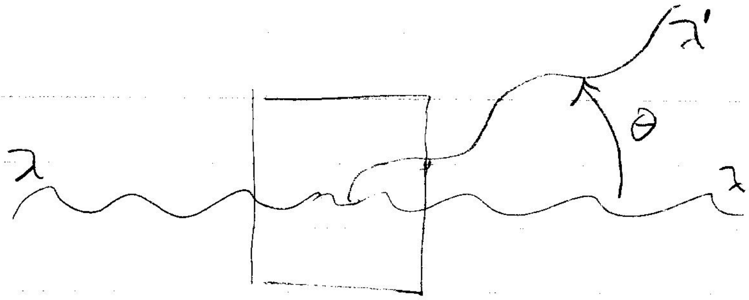
<u>Maxwell's equation</u>

In EM wave of frequency ν
charged particle oscillates at frequency ν
oscillation $\Longrightarrow$ acceleration
accelerated particle radiate at frequency ν



$\Longrightarrow$ radiated waves should have
same frequency ν
same wavelength λ
as original waves

But scattered X-rays
were observed to have longer wavelengths
that depend on scattering angle



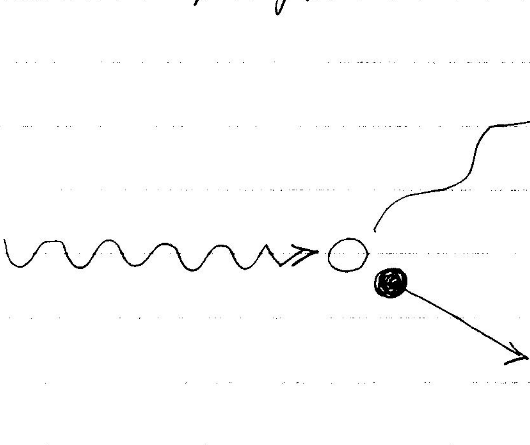
careful measurement by Compton

$$\lambda' - \lambda = \lambda_c(1-\cos\theta)$$

$$\lambda_c \approx 2.4\times10^{-12}\ m$$

explanation by Compton 1922

scattering of photons from electrons
conservation of energy and momentum
quantum physics + relativity



initial state:

photon: $E_\gamma = \frac{hc}{\lambda}$

$\vec{P}_\gamma = \frac{h}{\lambda}\hat{z}$

electron: $E_e = m_e c^2$

$\vec{P}_e = 0$

final state:

photon: $E_\gamma' = \frac{hc}{\lambda'}$

$\vec{P}_\gamma' = \frac{h}{\lambda'}(\cos\theta\,\hat{z} + \sin\theta\,\hat{x})$

electron: $E_e' = \sqrt{(m_e c^2)^2 + (p_x c)^2 + (p_z c)^2}$

$\vec{P}_e' = p_z\hat{z} + p_x\hat{x}$

conservation laws

$$\left.\begin{array}{ll} \text{energy:} & \frac{hc}{\lambda} + m_e c^2 = \frac{hc}{\lambda'} + E_e' \\ \text{momentum:} & \frac{h}{\lambda} + 0 = \frac{h}{\lambda'}\cos\theta + p_z \\ & 0 + 0 = \frac{h}{\lambda'}\sin\theta + p_x \end{array}\right\} \text{3 equation}$$

knowns: λ, θ (and h, c, m_e)

unknowns: λ', p_x, p_z

Solution

① Solve conservation laws for E_e', P_x, P_z.

$$E_e' = \frac{hc}{\lambda} - \frac{hc}{\lambda'} + m_e c^2$$

$$P_z = \frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta$$

$$P_x = -\frac{h}{\lambda'}\sin\theta$$

② Set the two expressions for E_e^2 equal:

$$(m_e c^2)^2 + (p_x c)^2 + (p_y c)^2 = \left(\frac{hc}{\lambda} - \frac{hc}{\lambda'} + m_e c^2\right)^2$$

③ Eliminate p_x and p_y

$$(m_e c^2)^2 + \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta\right)^2 c^2 - \left(-\frac{h}{\lambda'}\sin\theta\right)^2$$

$$= \left(\frac{hc}{\lambda} - \frac{hc}{\lambda'} + m_e c^2\right)^2$$

④ Simplify

$$\cancel{m_e^2 c^4} + \cancel{\frac{h^2c^2}{\lambda^2}} - \frac{2h^2c^2}{\lambda\lambda'}\cos\theta + \cancel{\frac{h^2c^2}{\lambda'^2}(\cos^2\theta + \sin^2\theta)}$$

$$= \cancel{m_e^2 c^4} + 2 m_e c^2\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right) + \cancel{\frac{h^2c^2}{\lambda^2}} - 2\frac{h^2c^2}{\lambda\lambda'} + \cancel{\frac{h^2c^2}{\lambda'^2}}$$

$$2m_ec^2\left(\frac{hc}{\lambda}-\frac{hc}{\lambda'}\right) = 2\frac{h^2c^2}{\lambda\lambda'}(1-\cos\theta)$$

5 Multiply by $\frac{\lambda\lambda'}{2m_ec^2 \cdot hc}$

$$\boxed{\lambda'-\lambda = \frac{h}{m_ec}(1-\cos\theta)}$$

Compton's equation

with the Compton wavelength expressed in terms of fundamental constants

$$\boxed{\lambda_C = \frac{h}{m_eC}}$$

Nobel prize: 1927