EXERCISES

1.1 Assume that a human body emits blackbody radiation at the standard body temperature.

(a) Estimate how much energy is radiated by the body in one hour.

(b) At what wavelength does this radiation have its maximum intensity?

1.2 A distant red star is observed to have a blackbody spectrum with a maximum at a wavelength of 3500 Å [1 Å $= 10^{-10}$ m]. What is the temperature of the star?

1.3 The universe is filled with blackbody radiation at a temperature of 2.7 K left over from the Big Bang. [This radiation was discovered in 1965 by Bell Laboratory scientists, who thought at one point that they were seeing interference from pigeon droppings on their microwave receiver.]

(a) What is the total energy density of this radiation?

(b) What is the total energy density with wavelengths between 1 mm and 1.01 mm? Is the Rayleigh–Jeans formula a good approximation at these wavelengths?

1.4 Over what range in frequencies does the Rayleigh–Jeans formula give a result within 10% of the Planck blackbody spectrum?

1.5 Let $\rho(<\nu_0)$ be the total energy density of blackbody radiation in all frequencies less than ν_0, where $h\nu_0 \ll kT$. Derive an expression for $\rho(<\nu_0)$.

1.6 Suppose we want to measure the total energy density in blackbody radiation above some cutoff frequency ν_0. Let $\rho(>\nu_0)$ be the total radiation density in all frequencies greater than ν_0. Using the Planck blackbody spectrum show that $\rho(>\nu_0) = (8\pi/c^3)kT\nu_0^3 e^{-h\nu_0/kT}$ is a good approximation when $h\nu_0$ is much larger than kT.

1.7 **(a)** Express the Planck spectrum (Equation 1.7) as a function of the wavelength λ of the radiation, rather than the frequency ν.

(b) Use this expression to derive the wavelength λ_{max} at which the spectrum is a maximum.

(c) Does $\lambda_{max}\nu_{max} = c$?

1.8 In a photoelectric experiment, electrons are emitted from a surface illuminated by light of wavelength 4000 Å, and the stopping potential for these electrons is found to be $\Phi_0 = 0.5\ V$. What is the longest wavelength of light that can illuminate this surface and still produce a photoelectric current?

1.9 A lightbulb emits 40 W of power at a wavelength of 6×10^{-7} m.

(a) What is the total number of photons emitted per second?

(b) What is the energy of each photon?

1.10 **(a)** Using the Planck blackbody spectrum, and the fact that a photon with a frequency ν has an energy of $h\nu$, derive an expression for $n(\nu)\,d\nu$, the total number density of photons with frequencies between ν and $\nu + d\nu$ in blackbody radiation.

(b) Using the expression from part (a), show that the total number density of photons in blackbody radiation is given by

$$n = \beta(kT/hc)^3$$

where β is a constant given by $\beta \approx 60$. [Note that the integral $\int_0^\infty x^2\,dx/(e^x - 1)$ cannot be done analytically, so use the numerical result that $\int_0^\infty x^2\,dx/(e^x - 1) \approx 2.4$.]

1.11 A gamma ray with energy 1 MeV is scattered off of an unknown particle which is at rest. The gamma ray is reflected directly backward with a final energy of 0.98 MeV. What is m_0c^2 for the unknown particle? (Express your answer in MeV.)

1.12 Calculate the de Broglie wavelength of a proton ($mc^2 = 938$ MeV) with

(a) a kinetic energy of 0.1 MeV

(b) a total energy of 3 GeV.

1.13 The Balmer series (the $m = 2$ case in Equation 1.18) was discovered before the other series of spectral lines ($m = 1$, $m = 3$, etc.). Why? (Hint: Plug in some numbers and calculate wavelengths for $m = 1$, $m = 2$, and $m = 3$.)

1.14 Verify that $\hbar$ has units of angular momentum.

1.15 Beginning with the Bohr energy levels (Equation 1.24), derive the expression for the wavelengths of the spectral lines in hydrogen (Equation 1.18) and use this result to express R as a function of m, e, $\hbar$, c, and ϵ_0. Plug in values for these constants and verify that the correct result for R is obtained.

1.16 Suppose that the attractive force between the electron and proton in the hydrogen atom as given by some power law other than the inverse square law, i.e., assume that the force is given by $F = kr^\beta$, where k is a constant, and β is an arbitrary number with $\beta \neq 1$. [For example, the ordinary Coulomb law corresponds to the case $\beta = -2$. The harmonic oscillator corresponds to $\beta = 1$.] Use the Bohr quantization rule to show that for $\beta \neq -1$, the energy levels of the atom are give by

$$E = \left(\frac{\hbar^2 n^2}{m}\right)^{(\beta+1)/(\beta+3)} k^{2/(\beta+3)} \left(\frac{1}{2} + \frac{1}{\beta+1}\right)$$

This formula gives an absurd answer when $\beta = -3$; why?