Schwartz Chapter 28: Problem 4*

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The Lagrangian can be expressed more conveniently as

$$\mathcal{L} = \sum_i \frac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^i - \frac{1}{4}\lambda\Big(\sum_i \phi^i\phi^i - v^2\Big)^2.$$

(a) What are the global symmetries? Identify the group G of matrices $g^i{}_j$ such $\phi^i(x) \to \sum_j g^i{}_j\, \phi^j(x)$ is a symmetry of $\mathcal{L}$.

(b) What are all the possible vacua? Each of the fields can have a vacuum expectation value (VEV): $\langle\phi^i(x)\rangle = v^i$. Identify all the sets of VEVs $(v^i, i = 1, ..., n)$ that minimize the energy.
Are all the vacua equivalent? For any two sets of VEVs (v^i) and $(v^{i\prime})$, is there a symmetry transformation that maps one into the other?

(c) Write down the Lagrangian for excitations around a specific vacuum: $(v^i = 0, i = 1, \ldots, n-1,\ v^n = v)$. Identify the mass of each of the excitation fields. Write down the Feynman rules for each interaction term.

(c') How many Goldstone bosons are there? Determine the symmetry group of the specific vacuum. Verify Goldstone's Theorem, which states that the number of Goldstone bosons is $\dim(G) - \dim(H)$.