Schwartz Chapter 23: Problem 4*

Problem 4*

(0) The Lagrangian for N real scalar fields with an $O(N)$ symmetry is given in Eq. (23.132). Verify that the Feynman rule for the propagator of a scalar with momentum p and indices i, j is

$$\frac{i\delta^{ij}}{p^2 - m^2 + i\epsilon}.$$

Verify that the Feynman rule for the 4-scalar vertex with indices i, j, k, l is

$$-2i\lambda\big(\delta^{ij}\delta^{kl} + \delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}\big).$$

In the case $N = 1$, how is the coupling constant λ related to the parameter λ in the Lagrangian in Eq. (23.85).

(a1) Calculate $\beta(\lambda)$. Check that your result for $N = 1$ is consistent with Eq. (23.95).

Calculate the ultraviolet divergent part of each of the three one-loop vertex correction diagrams using dimensional regularization. Determine the renormalization constant Z_4 for the 4-point vertex to order λ using minimal subtraction. The renormalization constant for λ is $Z_\lambda = Z_4/(\sqrt{Z_\phi})^4$. The wavefunction renormalization constant to order λ is $Z_\phi = 1$. The relation between the bare and renormalized coupling constants is $\lambda_0 = \mu^{d-4} Z_\lambda \lambda$. Apply $\mu d/(d\mu)$ to the logarithm of both sides and use the fact that λ_0 does not depend on μ.

(a2) Calculate $\gamma_m(\lambda)$. Check that your result for $N = 1$ is consistent with Eq. (23.96).

Treat the $-\frac{1}{2}m^2\phi^i\phi^i$ term in the Lagrangian as an interaction term whose vertex has two incoming lines and the Feynman rule $-im^2\delta^{ij}$. Calculate the ultraviolet divergent part of the one-loop vertex correction diagram using dimensional regularization. Determine the renormalization constant Z_2 for the 2-point vertex to order λ using minimal subtraction. The renormalization counterterm for the mass parameter m^2 is $Z_m = Z_2/(\sqrt{Z_\phi})^2$. The relation between the bare and renormalized masses is $m_0^2 = Z_m m^2$. Apply $\mu d/(d\mu)$ to the logarithm of both sides and use the fact that m_0^2 does not depend on μ.

(b) Express the equations for $\beta(\lambda)$ and $\gamma_m(\lambda)$ in the forms in Eqs. (23.104) and (23.105). Identify the nontrivial fixed point analogous to that in Eqs. (23.106).

(c) Predict the critical exponent ν defined in Eq. (23.103) in $d = 3$ dimensions.