Schwartz Chapter 23: Problem 2*

Problem 2*

The 4-fermion interaction term in Eq. (23.40) that allows the decay of the muon has anomalous dimension 0. Consider instead the 4-fermion interaction term that allows the weak decay of the strange quark with electric charge $-\frac{1}{3}$ into an up quark with electric charge $+\frac{2}{3}$:

$$\mathcal{L}_{4F} = G\,\bar{\psi}_c \gamma_\mu P_L \psi_b\,\bar{\psi}_e \gamma^\mu P_L \psi_{\nu_e},$$

where $P_L = \frac{1}{2}(1-\gamma_5)$. The Feynman rule for the 4-fermion vertex is

$$iG\left(\gamma_\mu P_L\right)_{ji}\left(\gamma^\mu P_L\right)_{kl},$$

where i, j, k, l are the spinor indices for the b, c, e, ν_e lines. The Feynman rules for the QED vertices are $iQe\gamma^\mu$, where Q is the electric charge.

A. Draw the tree diagram for $b(P) \longrightarrow c(q_1)e^-(q_2)\bar{\nu}_e(q_3)$. Write down the expression for the diagram.

B. Draw the three one-loop diagrams for $s(P) \longrightarrow u(q_1)e^-(q_2)\bar{\nu}_e(q_3)$ with a photon of momentum k exchanged between two charged lines. Write down the expression for the diagram.

C. Express the ultraviolet divergent term in each of the three diagrams in as simple a form as possible as as the product of two spinor factors and a tensor loop integral.

D. Determine the pole in $d-4$ for the dimensionally regularized tensor loop integral.

E. Use Dirac algebra to put the product of the two spinor factors into the same form as the tree diagram. For some of the diagrams, this will require using the Fierz identity in Eq. (23.131) (with P_L replaced by P_R).

F. Identify the counterterm δG that is needed to cancel the ultraviolet divergence. Express it in terms of a vertex renormalization factor Z_v defined by $G + \delta G = Z_v G$.

G. Express the bare coupling constant G_0 in terms of the renormalized coupling constant G, the vertex renormalization factor Z_v, and the wavefunction renormalization constants Z_{2s}, Z_{2s}, and Z_{2e}. Use the known results for Z_{2e} to deduce Z_{2s} and Z_{2s}.

H. Deduce the anomalous dimension of the 4-fermion operator. Write down the renormalization group equation for $G(\mu)$, which involves the running QED coupling constant $\alpha(\mu)$. Write down the renormalization group equation for $\alpha(\mu)$, assuming that its running comes from e, μ, u, d, and s loops, for which the sum of Q^2 adds up to 5.

I. Express the solution of the renormalization group equation for $G(\mu)$ as the product of $G(m_s)$ and $\alpha(\mu)/\alpha(m_s)$ raised to some power. Express the solution as the product of $G(m_s)$ and a function of $\alpha(m_s)\log(\mu/m_s)$.