Schwartz Chapter 18: Problem 1*

Problem 1*

(a) Draw the two one-loop diagrams and the counterterm diagram for the scalar self-energy, labeling the momenta and Lorentz indices. The sum of the three diagrams is $-i\Pi(p^2)$, where $\Pi(p^2)$ is the self-energy function.

(a') Calculate the one-loop self-energy diagrams.
1. Use the Feynman rules to express them in terms of integrals over a loop momentum k.
2. In the diagram with two propagators, combine them using a Feynman parameter and shift the loop momentum to reduce the denominator to a function of k^2.
3. Evaluate the integrals over the loop momentum using dimensional regularization in d dimensions.
Add the three diagrams to obtain an expression for $\Pi(p^2)$ that involves a Feynman parameter integral.

(a") Calculate $\Pi(m^2)$ and $\Pi'(m^2)$.
1. Set $p^2 = m^2$ and simplify the expressions.
2. Evaluate the integral over the Feynman parameter.
3. Expand in powers of $\epsilon = 4 - d$, keeping only the $1/\epsilon$ and ϵ^0 terms.
For $\Pi'(m^2)$, use a photon mass to regularize the infrared diveergence.

(b) Express the renormalization conditions for the on-shell scheme in the form of equations involving the self-energy function $\Pi(p^2)$. Using the results of part (a), determine the mass counterterm δm^2 and the field renormalization counterterm δZ to order α. Give the pole mass m^2_{pole} to order α. Give the residue Z_s of the pole in the propagator $i/[p^2 - m^2 - \Pi(p^2)]$ to order α.

(c) Using the results of part (a), determine the counterterms δm^2 and δZ in the $\overline{\text{MS}}$ renormalization scheme. Give the pole mass m^2_{pole} to order α. Give the residue Z_s to order α.